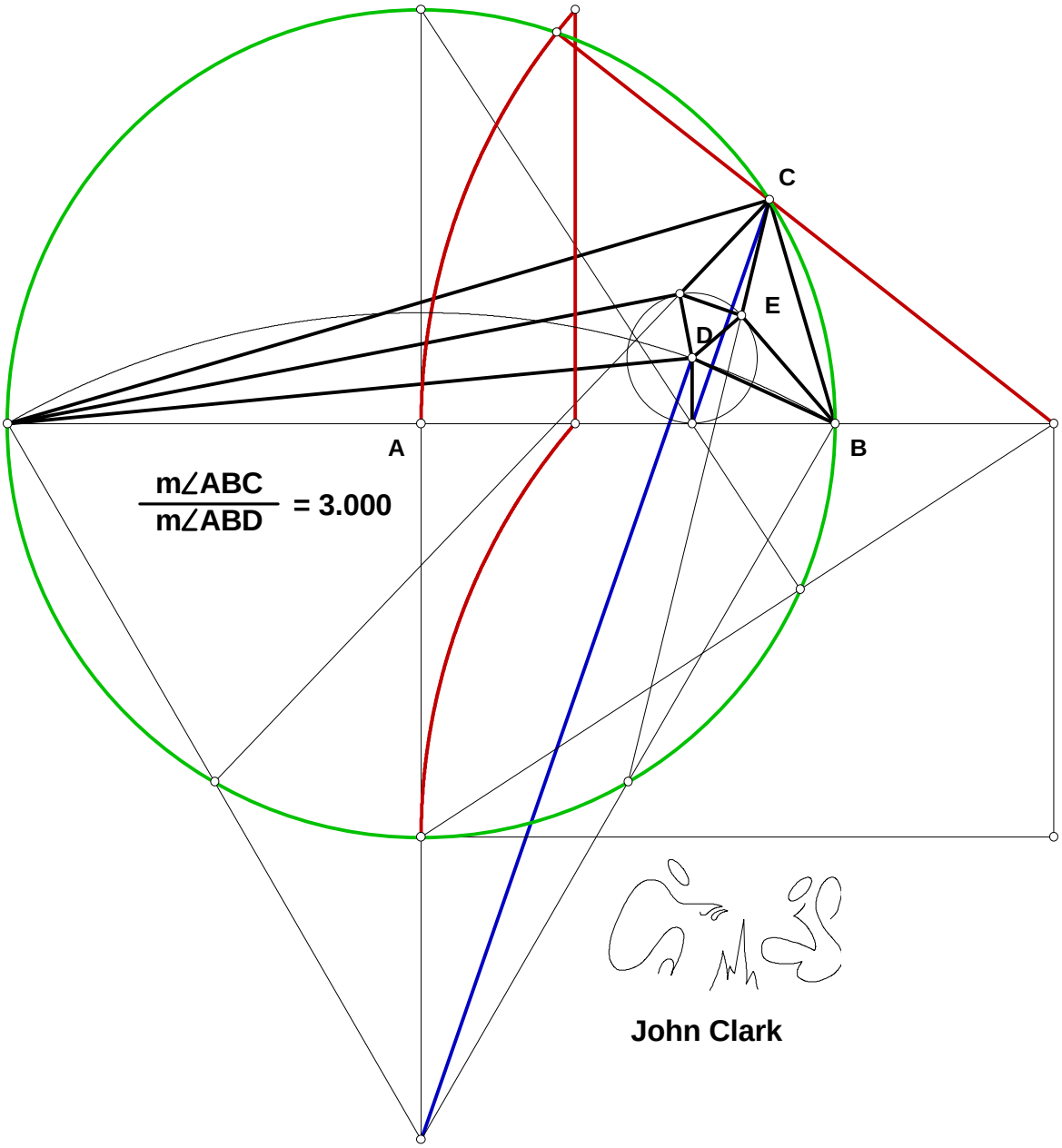
Three Pieces Of Paper



John Clark

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Introduction

Is a novel about the search for a method to divide angles into at least three equal parts. The novel is written in the relatiologic of Geometry and the tautologic of basic Algebra.

I suspect that someday I might have something of significance to write here. J. C.

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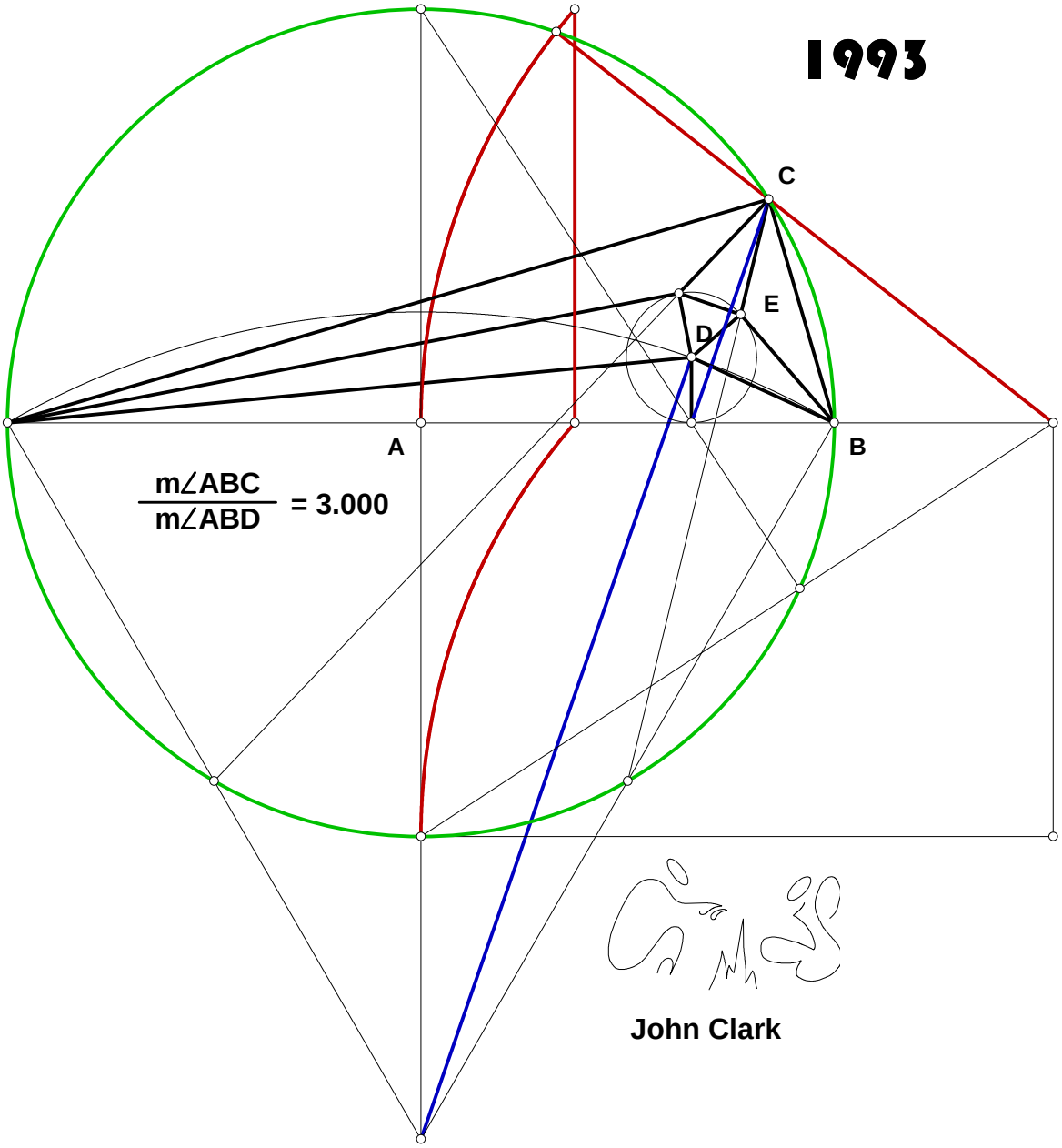
1993



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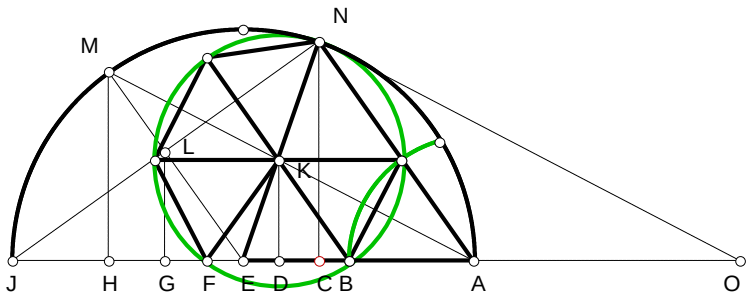
11/11/93 The Archamedian Paper Trisector

If one accepts the facts of the original figure, one only need prove that BK = AB.
If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?



$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN}{2}$ $GJ := \frac{CJ}{2}$

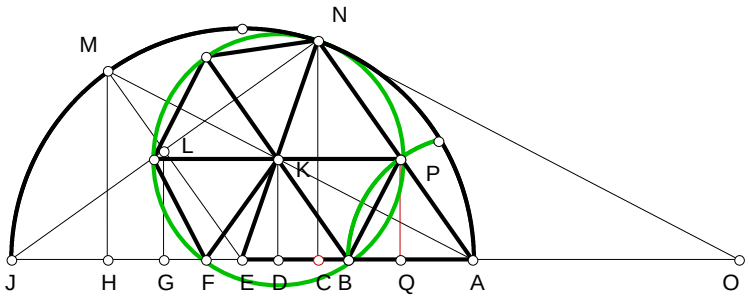
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

$AB - BK = 0$ $AB = 0.25$

If PK is parallel to AJ, then . . . .



$AN := 2 \cdot EL$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

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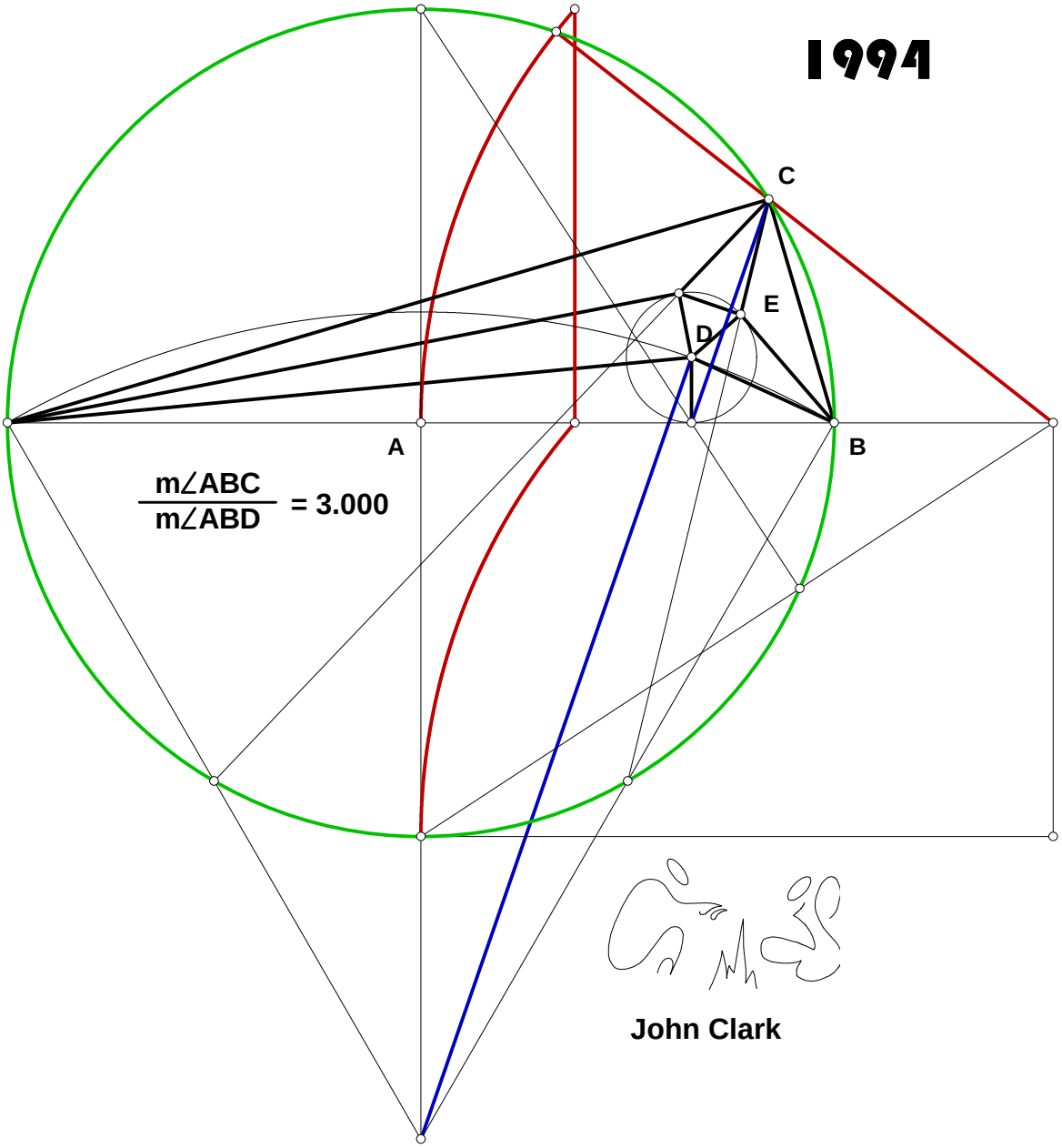
1994



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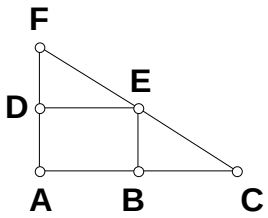
042994

Given AB, AF, BE, what is EF?



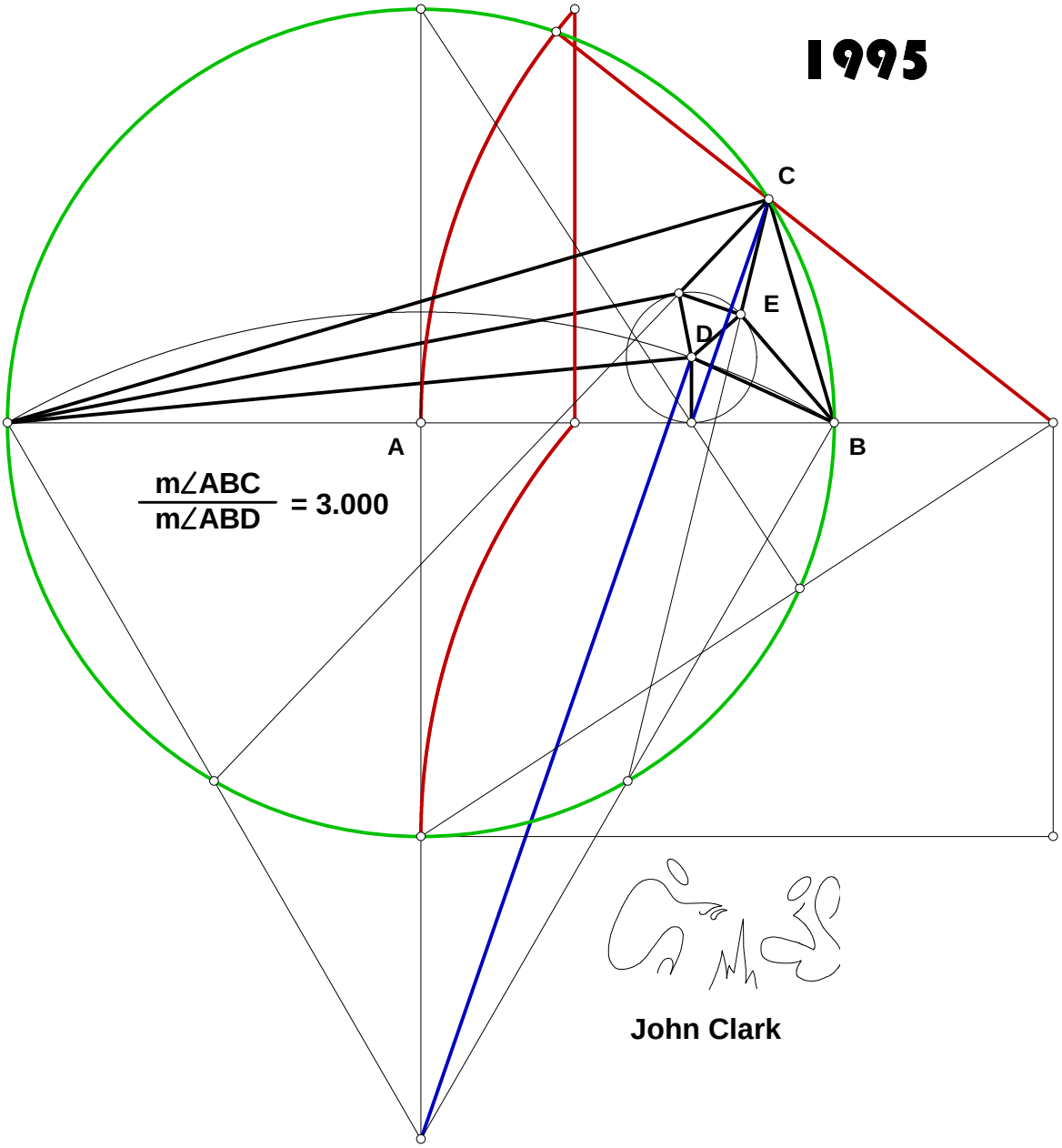
$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$

Three Pieces Of Paper

1995



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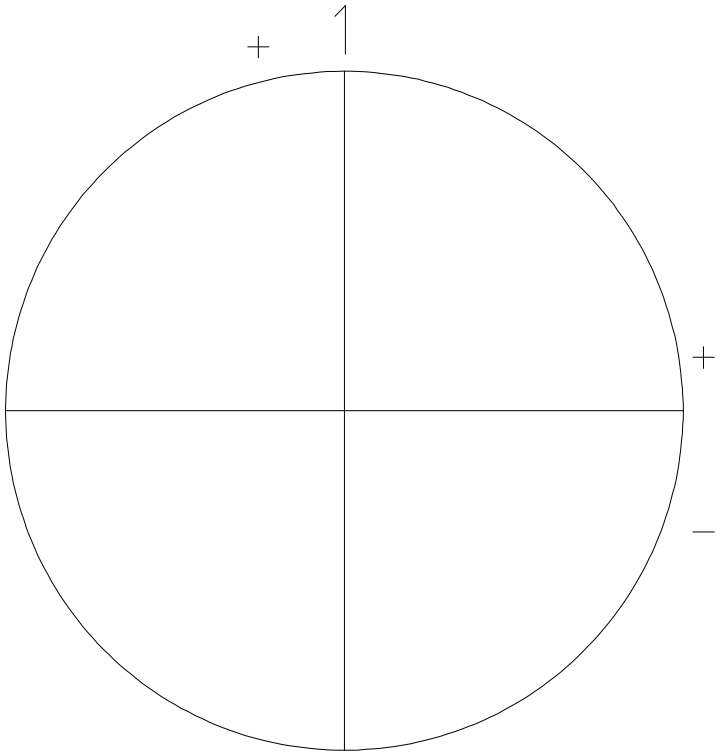
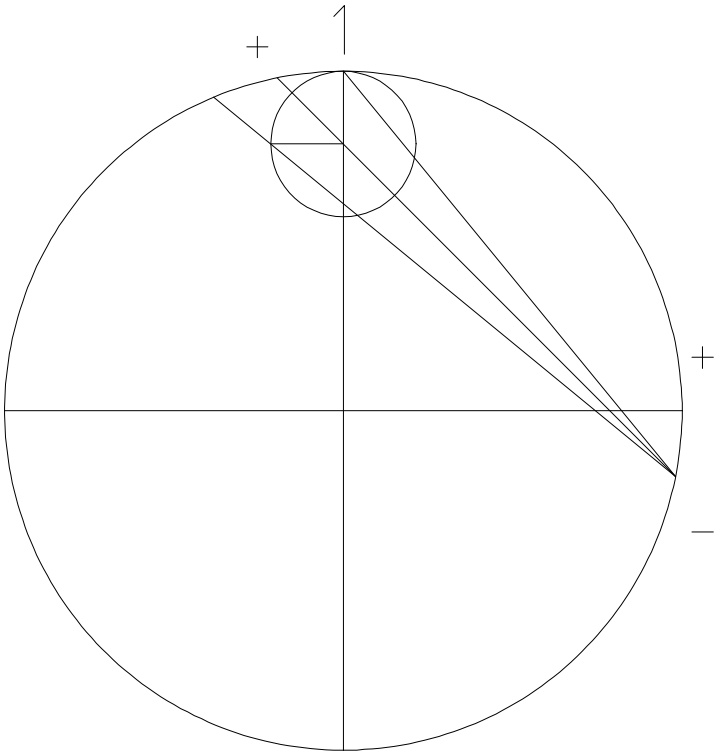


01_12_95.MCD

Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

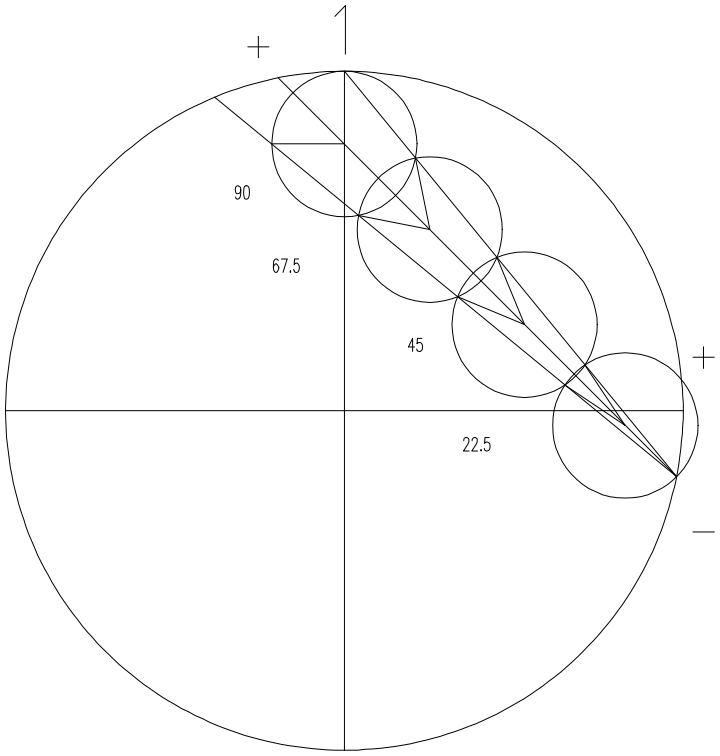
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.



The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$



$$B := 1 + \frac{1}{8} - \frac{1}{8} \qquad B = 1$$

$$\frac{B \cdot 4}{4} \cdot 90 = 90 \qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$$

$$\frac{B \cdot 2}{4} \cdot 90 = 45 \qquad \frac{B}{4} \cdot 90 = 22.5$$

$8 + 1 - 1 = 8$	$8 \cdot 11.25 = 90$
$8 + 1 - 1 - 2 = 6$	$6 \cdot 11.25 = 67.5$
$8 + 1 - 1 - 2 - 2 = 4$	$4 \cdot 11.25 = 45$
$8 + 1 - 1 - 2 - 2 - 2 = 2$	$2 \cdot 11.25 = 22.5$
$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$	

$$\mathrm{mod}(8 + 1 - 1, 2) = 0$$

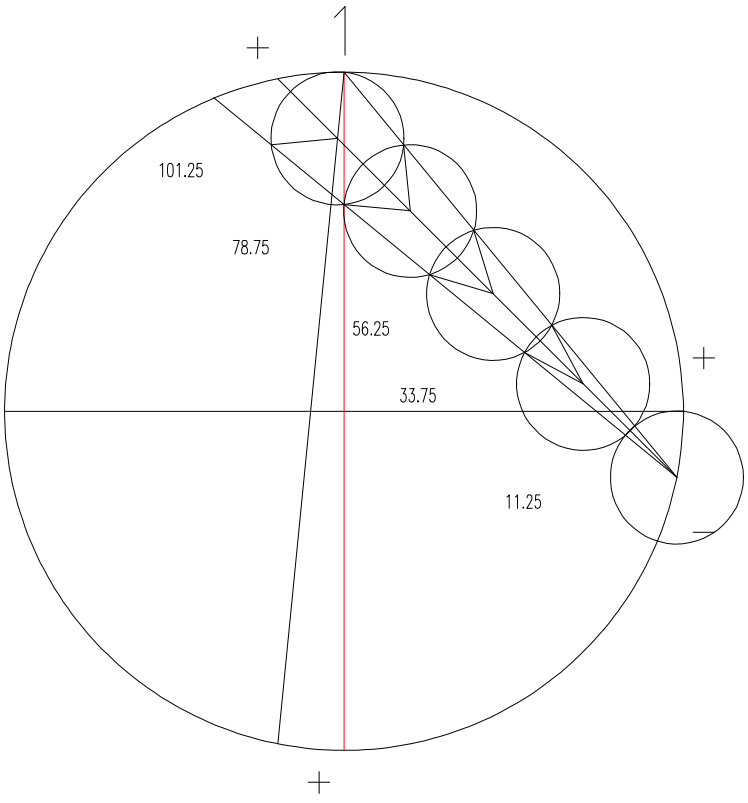
I have added another plus to a quadrant at the bottom of the figure.

$$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

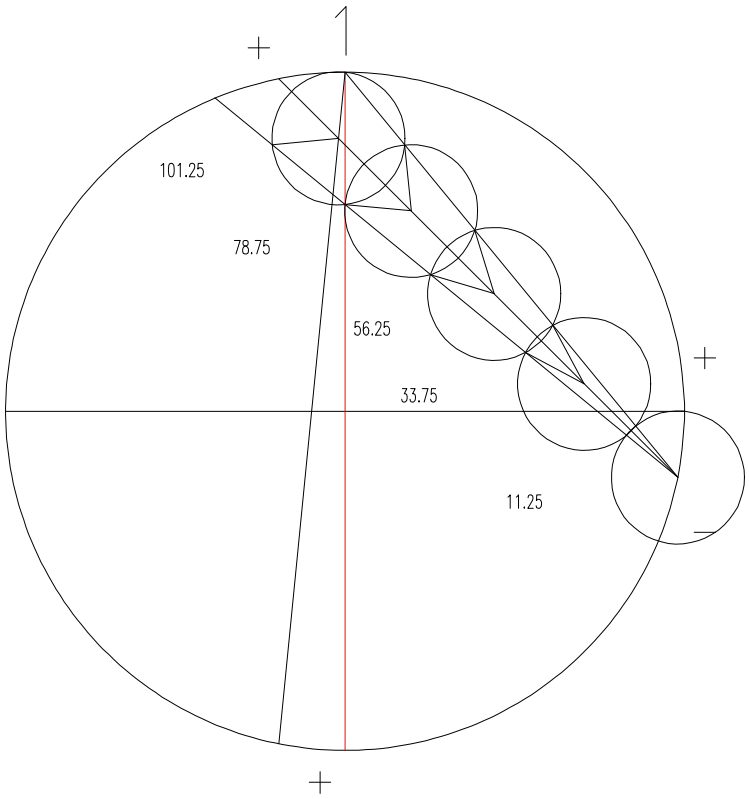
$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$



$8 + 1 + 1 - 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 + 1 - 1 - 2 = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 + 1 - 1 - 2 - 2 = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$	$1 \cdot 11.25 = 11.25$

$$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$$



$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

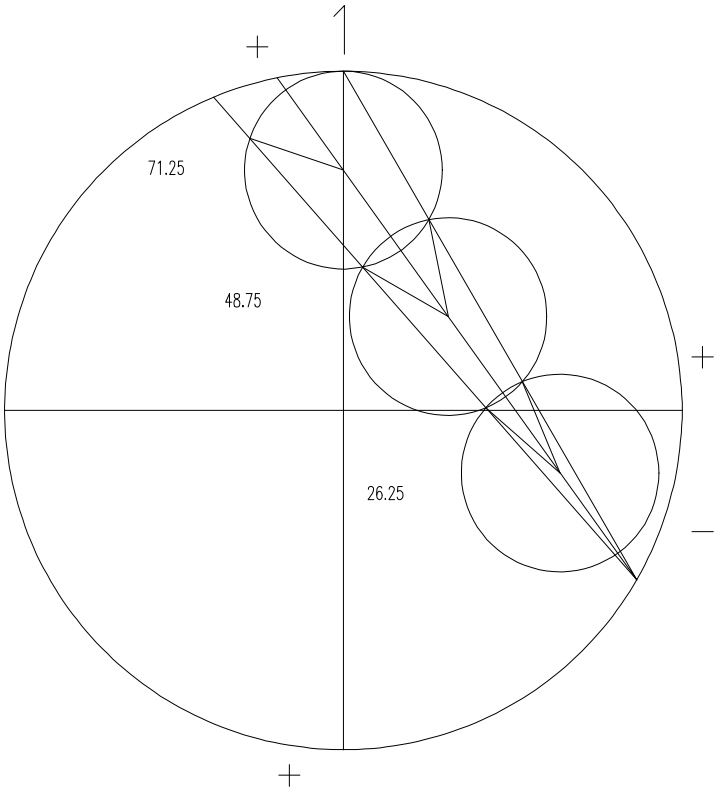
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

$8 + 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 - (1 \cdot 2) = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 - (2 \cdot 2) = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 - (3 \cdot 2) = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 - (4 \cdot 2) = 1$	$1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1, 2) = 1$



$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

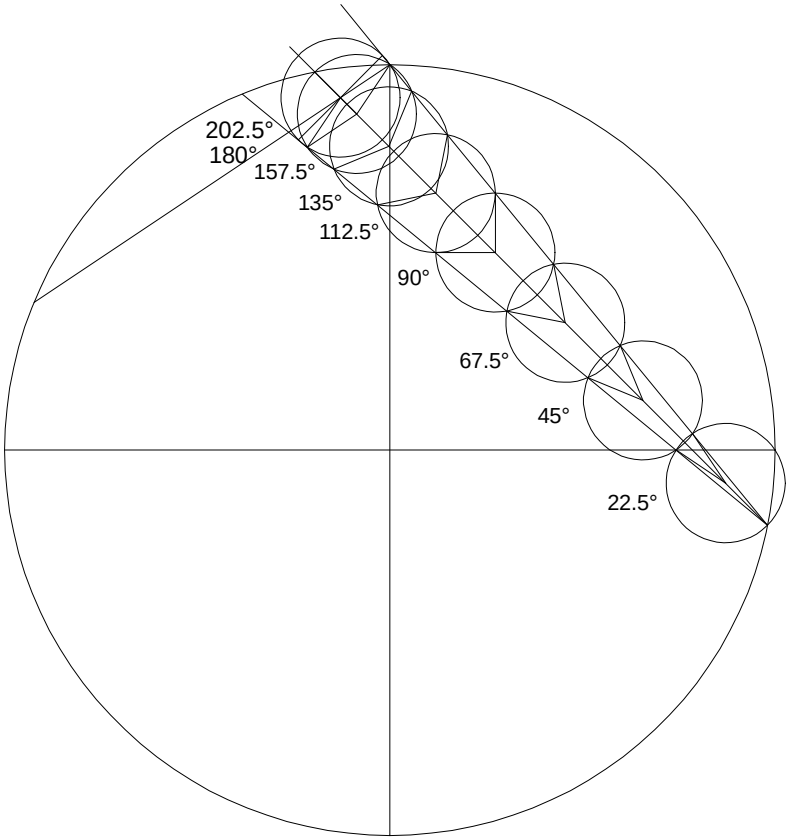
$(24 + 3) - 8 = 19$	$19 \cdot 3.75 = 71.25$
$(24 + 3) - 8 - (1 \cdot 6) = 13$	$13 \cdot 3.75 = 48.75$
$(24 + 3) - 8 - (2 \cdot 6) = 7$	$7 \cdot 3.75 = 26.25$
$(24 + 3) - 8 - (3 \cdot 6) = 1$	$1 \cdot 3.75 = 3.75$

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

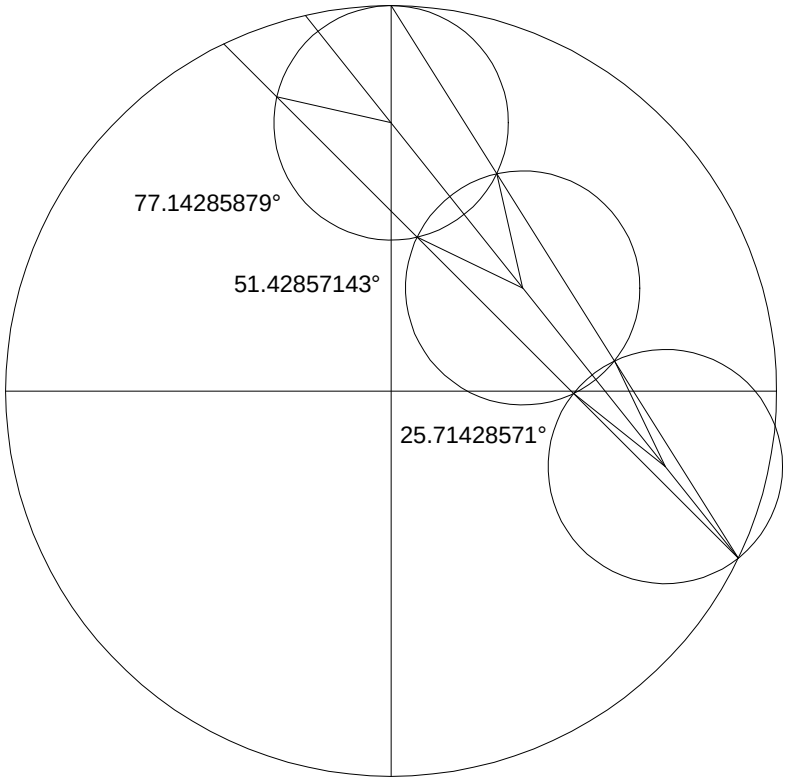
$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$



$8 + 1 - 1 + 10 = 18$	$18 \cdot 11.25 = 202.5$
$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$	$16 \cdot 11.25 = 180$
$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$	$14 \cdot 11.25 = 157.5$
$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$	$12 \cdot 11.25 = 135$
$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$	$10 \cdot 11.25 = 112.5$
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$	$8 \cdot 11.25 = 90$
$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$	$6 \cdot 11.25 = 67.5$
$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$	$4 \cdot 11.25 = 45$
$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$	$2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$



$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

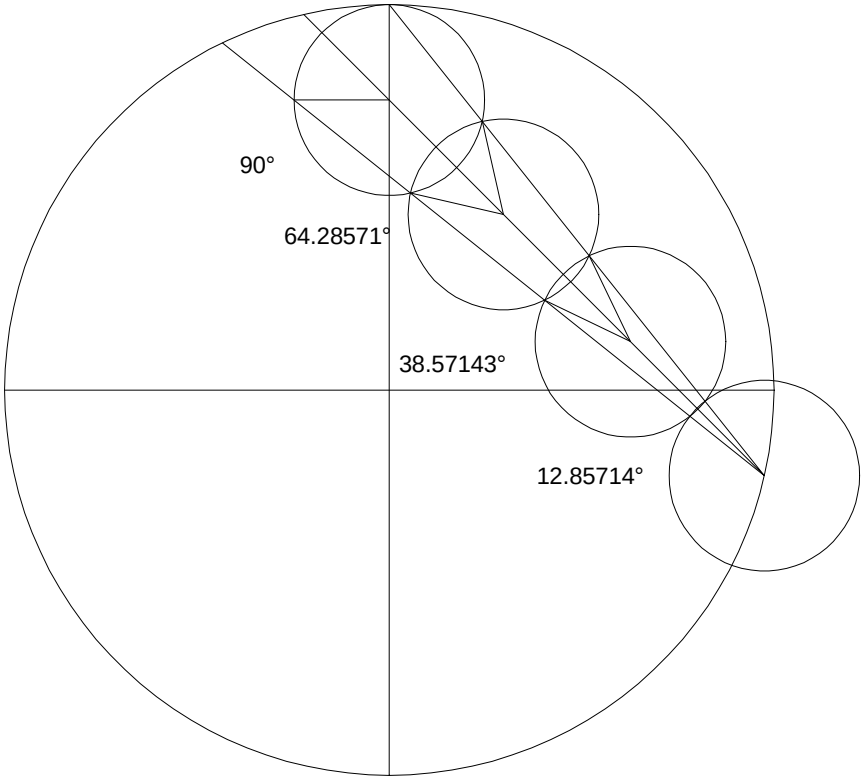
$7 + 1 - (1 \cdot 2) = 6$	$6 \cdot c = 77.1429$
$7 + 1 - (2 \cdot 2) = 4$	$4 \cdot c = 51.4286$
$7 + 1 - (3 \cdot 2) = 2$	$2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$



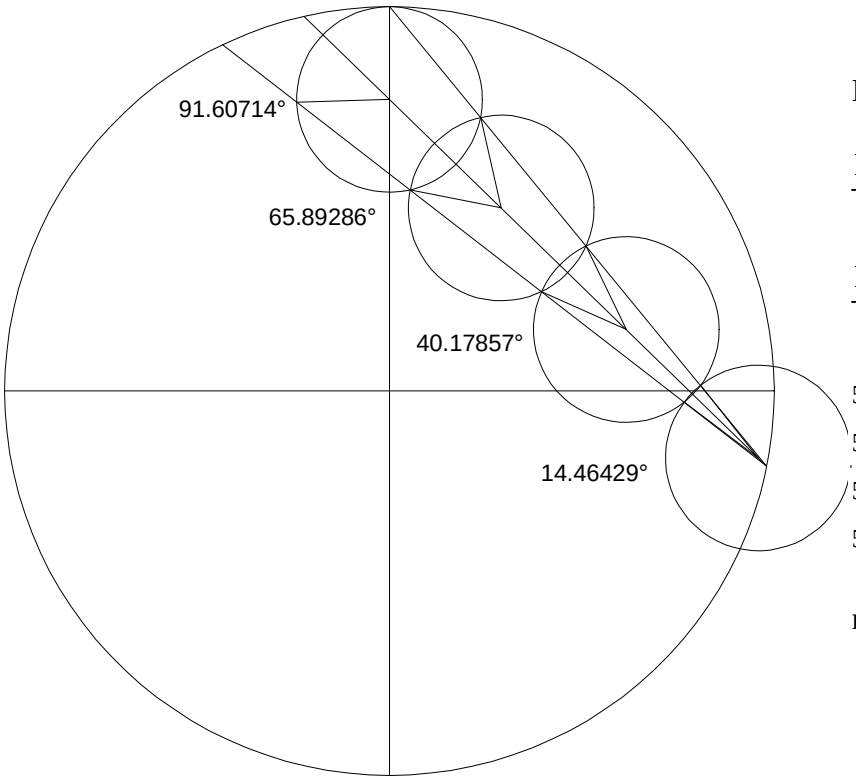
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$



$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

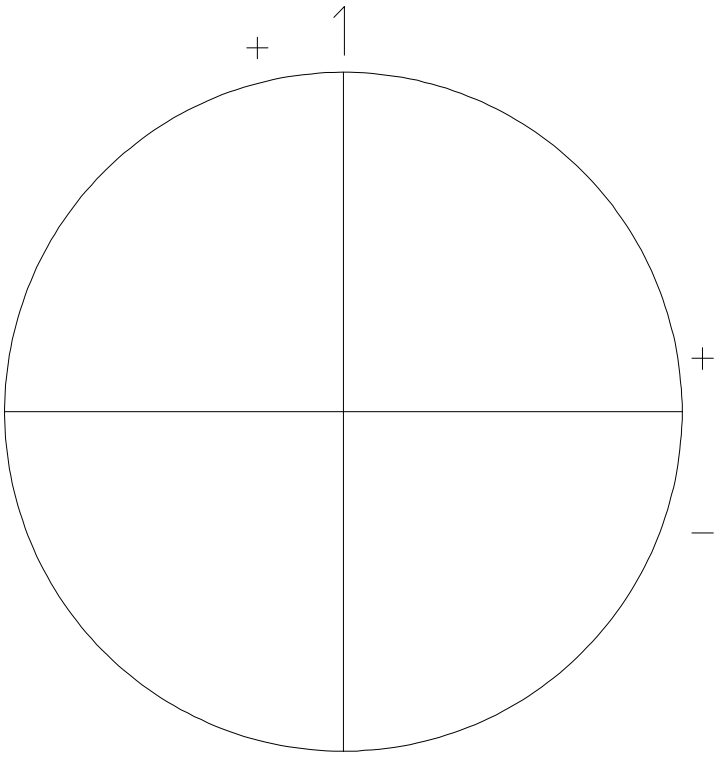
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

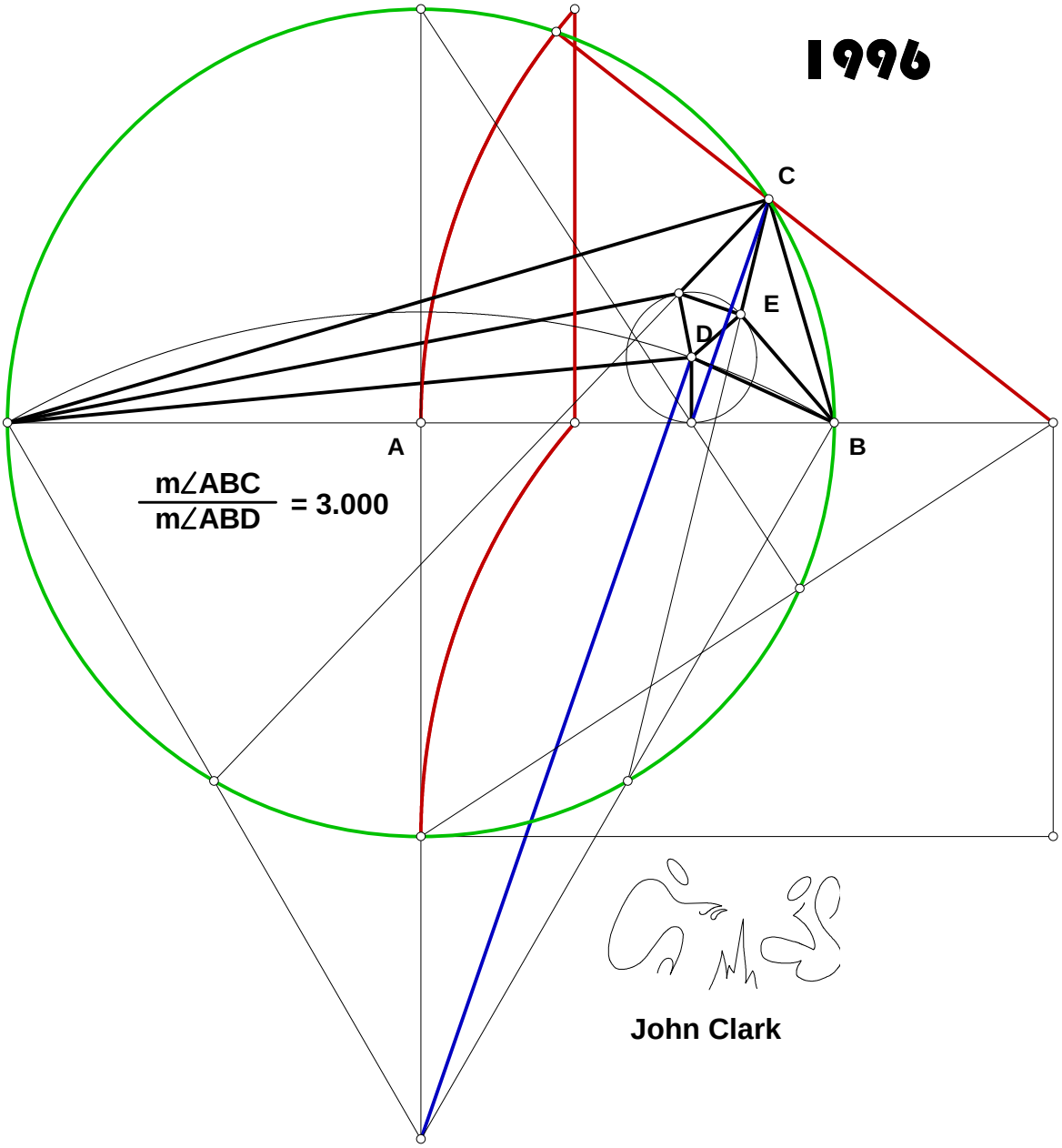
$\mathrm{mod}(7 + 1 - 1, 2) = 1$



Work in progress.

Three Pieces Of Paper

1996



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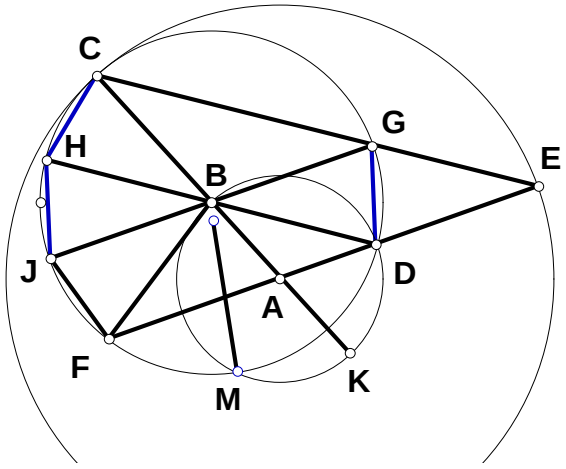
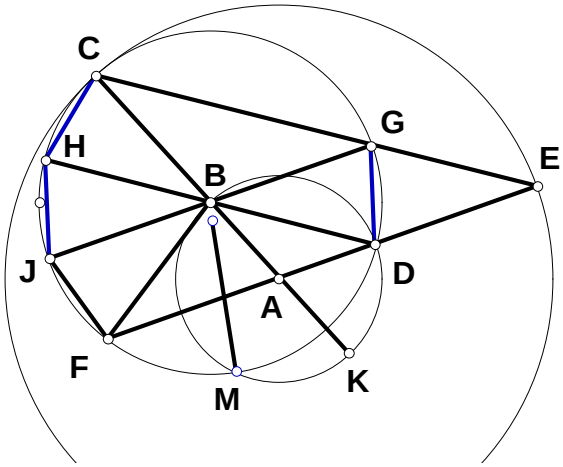
010496 The Archamedian Paper Trisector- Without the Numbers.



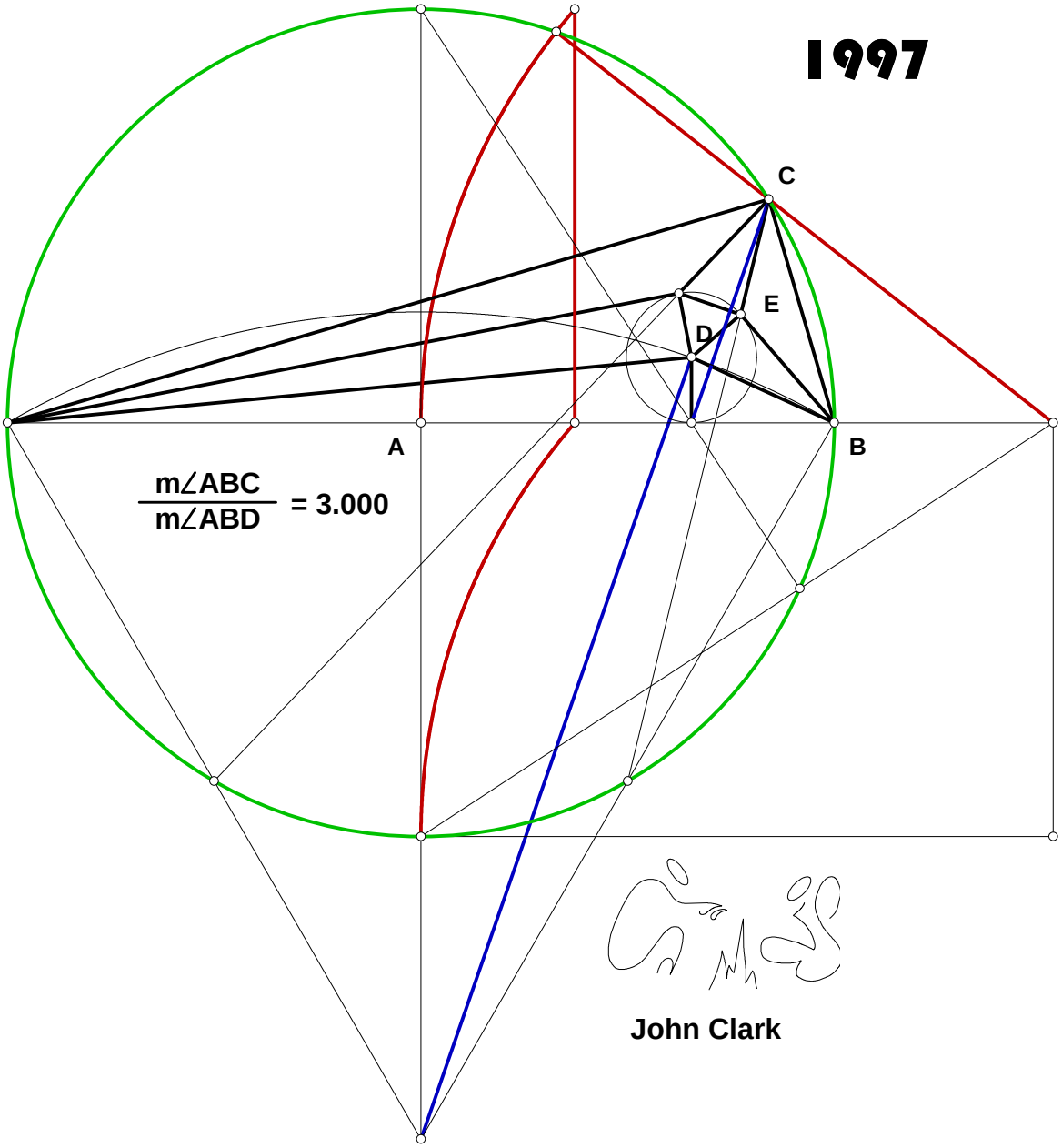
Given any circle AB.
Given any circle BC such that BC ≤ 2AB.
Construct AE such that AE = AC.
As AC = AB + BC
and AD = AB so too DE = BC.
Construct DH parallel to BD.
Construct CE.
As AB = AD and AC = AE,
Δ ABD is proportional to Δ ACE,
therefore CE is parallel to BD.
From here one can take two paths.
Construct GJ parallel to EF.
As CE is parallel to DH,
DG = CH.
As GJ is parallel to EF,
DG = FJ.

As ∠ HBJ is opposite
and equal to ∠ GBD,
DG = HJ,
therefore ∠ DG is $\frac{1}{3}$ CF.
As CE is parallel to DH
and EF is parallel to GJ,
EGBD is equilateral.
By construction DK = KM.
As DH is parallel to CE,
CH = DG.
As DK is equal and opposite CH,
MK + DK + DG is $\frac{1}{3}$ DG.



Three Pieces Of Paper

1997



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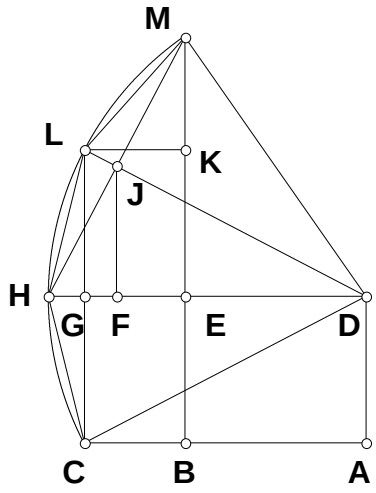
042897



$N := 1.458 \quad AD := 1 \quad AC := AD \cdot N \quad N := N$

$CD := \sqrt{AD^2 + AC^2} \quad DH := CD$

$CG := AD \quad DG := AC \quad GH := DH - DG \quad CH := \sqrt{GH^2 + CG^2}$

$HJ := CG \quad DJ := DG$

$FH := \frac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH} \quad EF := FH \quad DE := DH - (EF + FH)$

$AB := DE \quad EG := DG - DE \quad LM := CH \quad LK := EG$

$KM := \sqrt{LM^2 - LK^2} \quad BE := AD \quad BK := 2 \cdot BE \quad BM := BK + KM$

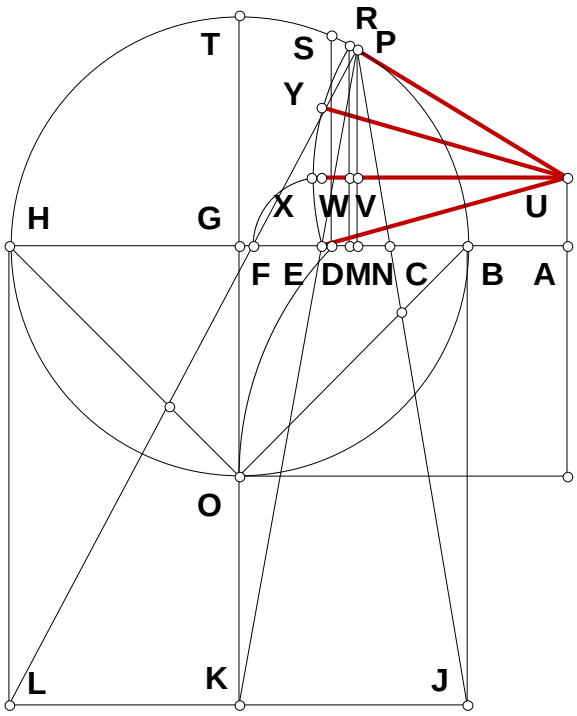
Some Algebraic Names:

$\sqrt{N^2 + 1} - CD = 0 \quad N - AC = 0 \quad \sqrt{N^2 + 1} - N - GH = 0 \quad \sqrt{2 \cdot N^2 - 2 \cdot \sqrt{N^2 + 1} \cdot N + 2} - CH = 0$

$\frac{1}{\sqrt{N^2 + 1}} - FH = 0 \quad \frac{(N - 1) \cdot (N + 1)}{\sqrt{N^2 + 1}} - AB = 0 \quad 2 + \sqrt{\frac{5 \cdot N^2}{N^2 + 1} - \frac{4 \cdot N}{\sqrt{N^2 + 1}} + \frac{1}{N^2 + 1}} - BM = 0$

Trisection and the Cube Roots 042997

If trisection can be placed at RUE, then PV is proportional to RW.



$N := 5 \qquad AB := 1 \qquad AH := AB \cdot N$

$BH := AH - AB \quad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$

$CF := AF - AC \qquad CE := \frac{CF}{2}$

$AE := AC + CE$

$AU := CE \quad NV := AU \quad MW := AU$

(For the next two equations see 042897.)

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

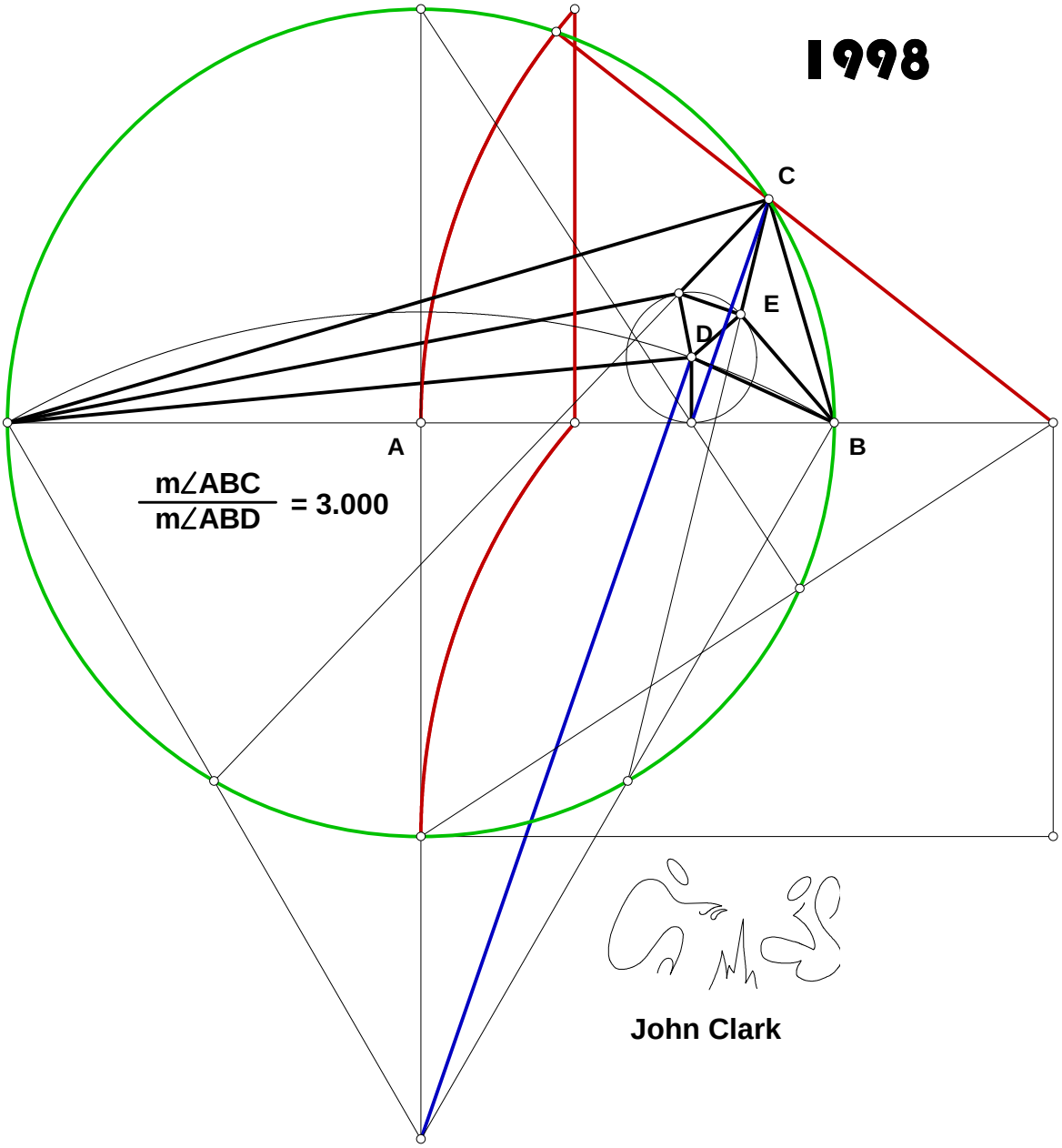
$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{5 \cdot \left(\frac{AE}{AU}\right)^2}{\left(\frac{AE}{AU}\right)^2 + 1} - \frac{4 \cdot \frac{AE}{AU}}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} + \frac{1}{\left(\frac{AE}{AU}\right)^2 + 1}} \qquad RW := MR - MW$$

$BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH} \qquad NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV$

$AN := AC + CN \qquad UV := AN \quad UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$

Three Pieces Of Paper

1998



John Clark

m∠OKL = 30.872°

m∠HAQ = 20.581°

m∠OTK = 15.436°

m∠IJV = 10.291°

m∠IBJ = 5.145°

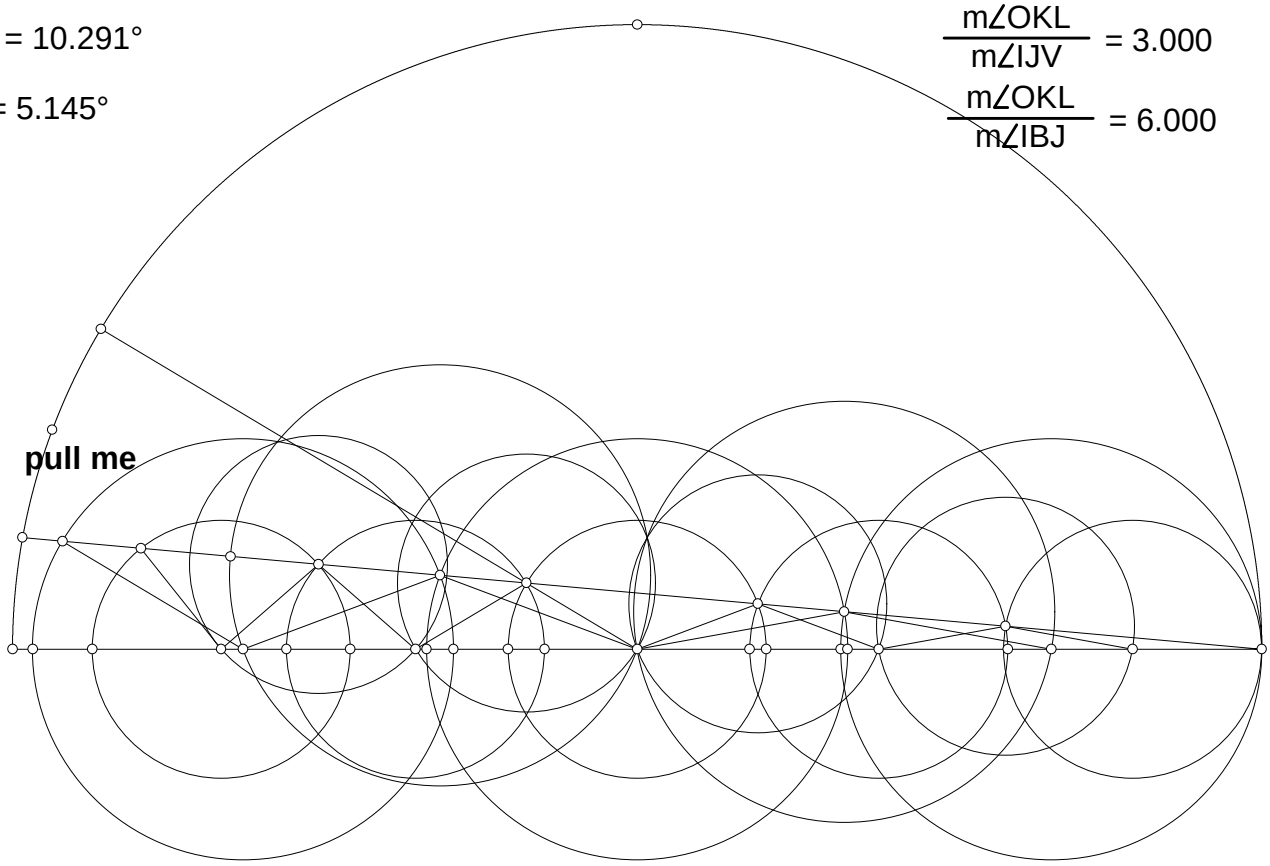
▲ **Show** △ **Hide**

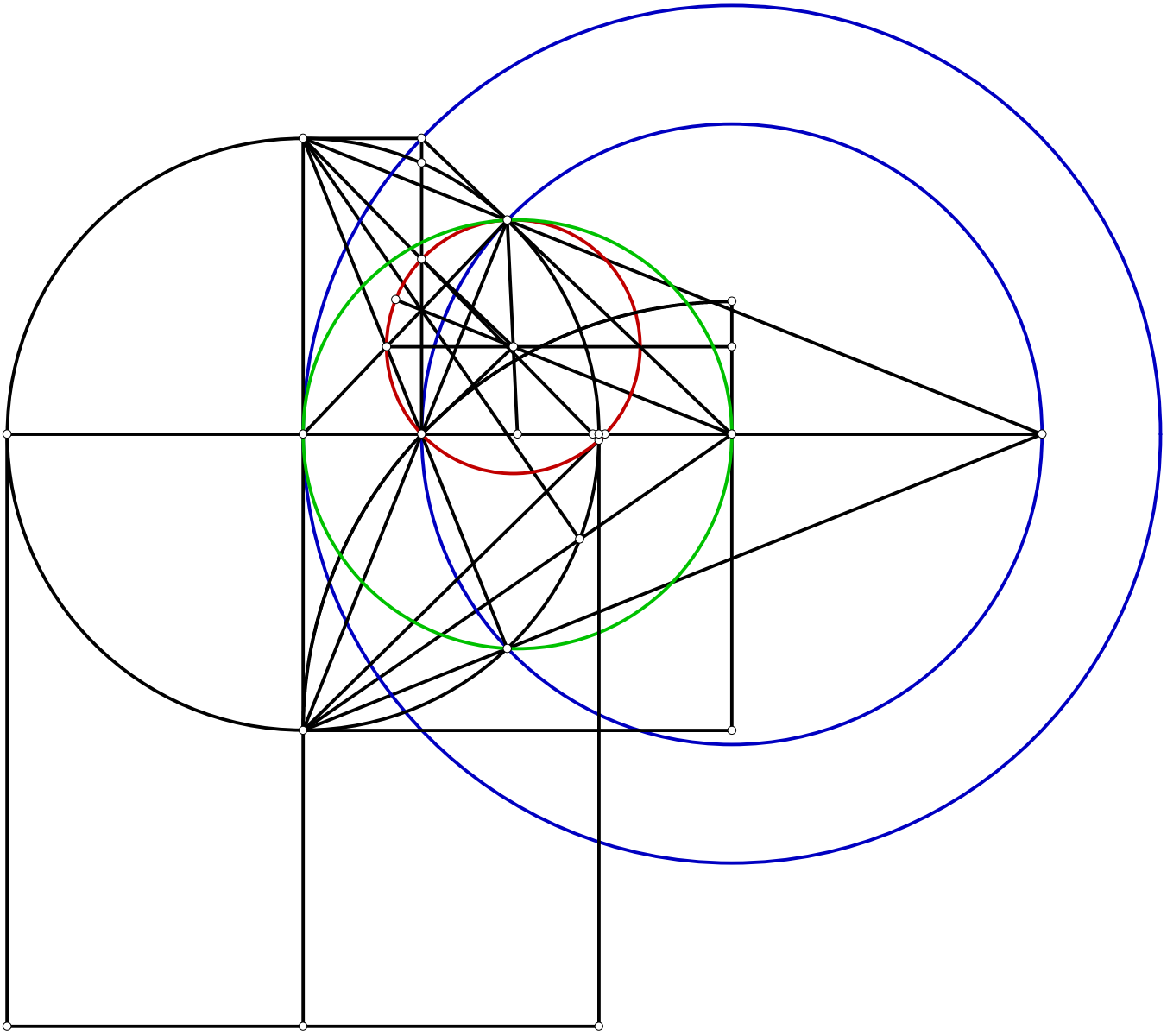
$\frac{m\angle OKL}{m\angle HAQ} = 1.500$

$\frac{m\angle OKL}{m\angle OTK} = 2.000$

$\frac{m\angle OKL}{m\angle IJV} = 3.000$

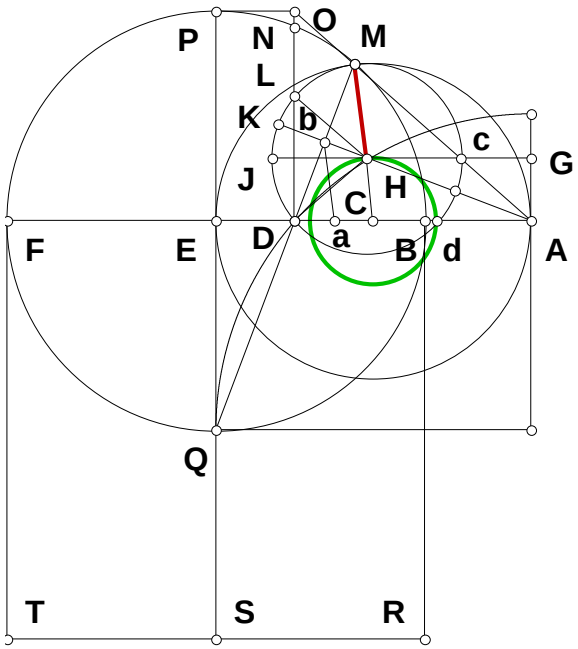
$\frac{m\angle OKL}{m\angle IBJ} = 6.000$





A Square Root Figure And Triseciton 042398

In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?



$N := 5 \quad AB := 1 \quad AF := AB \cdot N \quad BF := AF - AB$

$AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2} \quad BD := AD - AB$

$DE := BE - BD \quad EQ := BE \quad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}}$

$PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ} \quad DM := QM - DQ$

$AE := AB + BE \quad AC := \frac{AE}{2} \quad Db := \frac{DM}{2}$

$CM := AC \quad ab := \frac{CM \cdot Db}{DM} \quad CD := AD - AC$

$Ca := \frac{CD}{2} \quad Aa := AC + Ca \quad CH := \frac{ab \cdot AC}{Aa}$

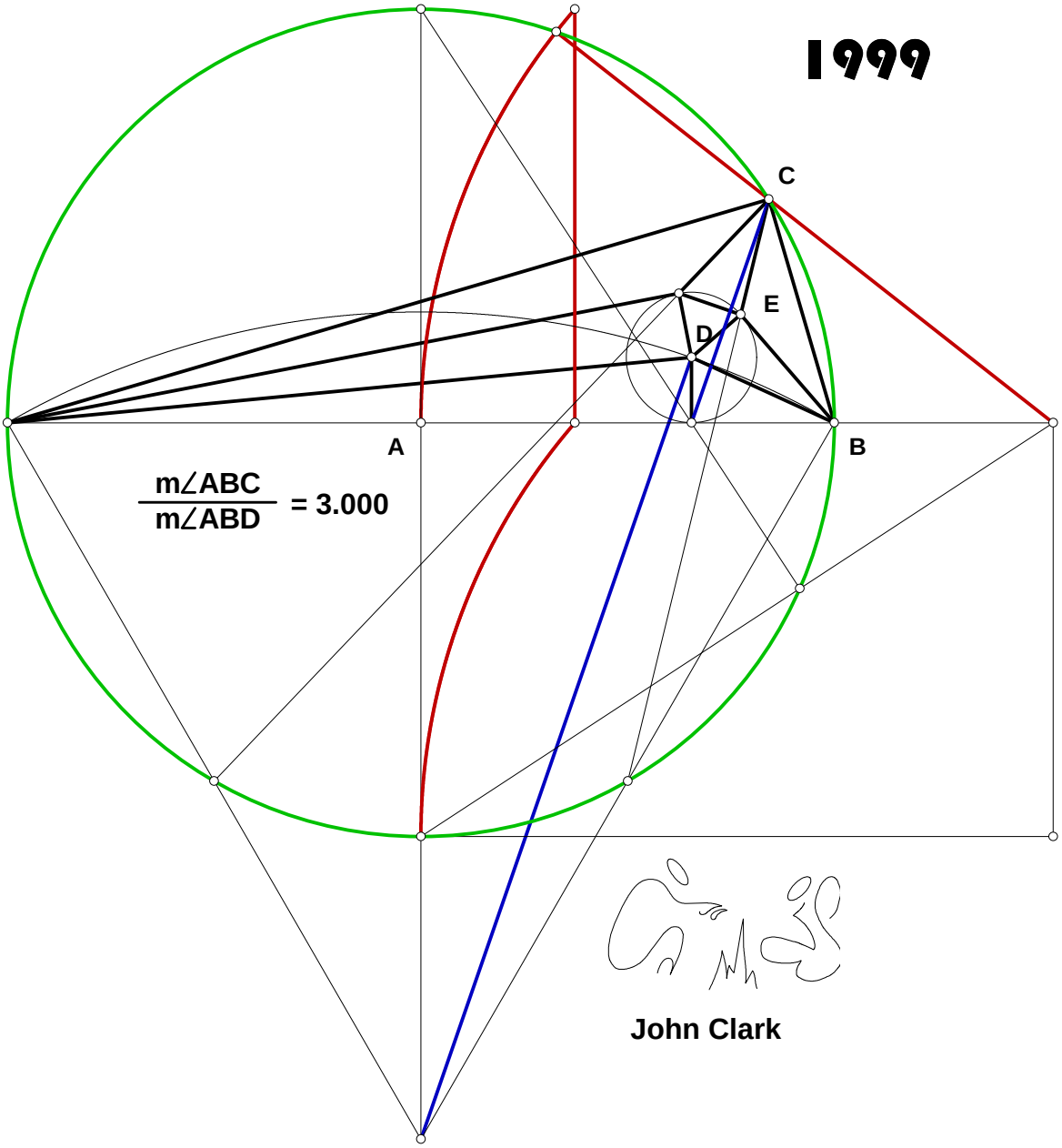
$AM := AD \quad Ac := \frac{AM \cdot CH}{CM} \quad HM := CM - CH$

$HM - Ac = 0$

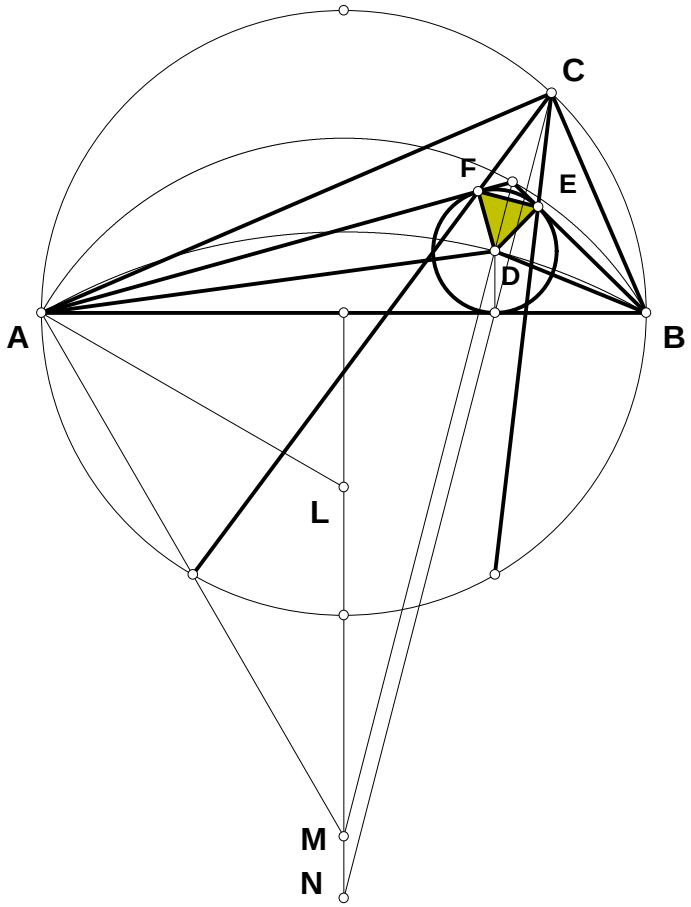
$$HM - AB \cdot \frac{\sqrt{N} \cdot (N + 1)}{N + 4 \cdot \sqrt{N} + 1} = 0 \qquad CH - AB \cdot \frac{(N + 1)^2}{4 \cdot N + 16 \cdot \sqrt{N} + 4} = 0$$

Three Pieces Of Paper

1999



John Clark


C
F
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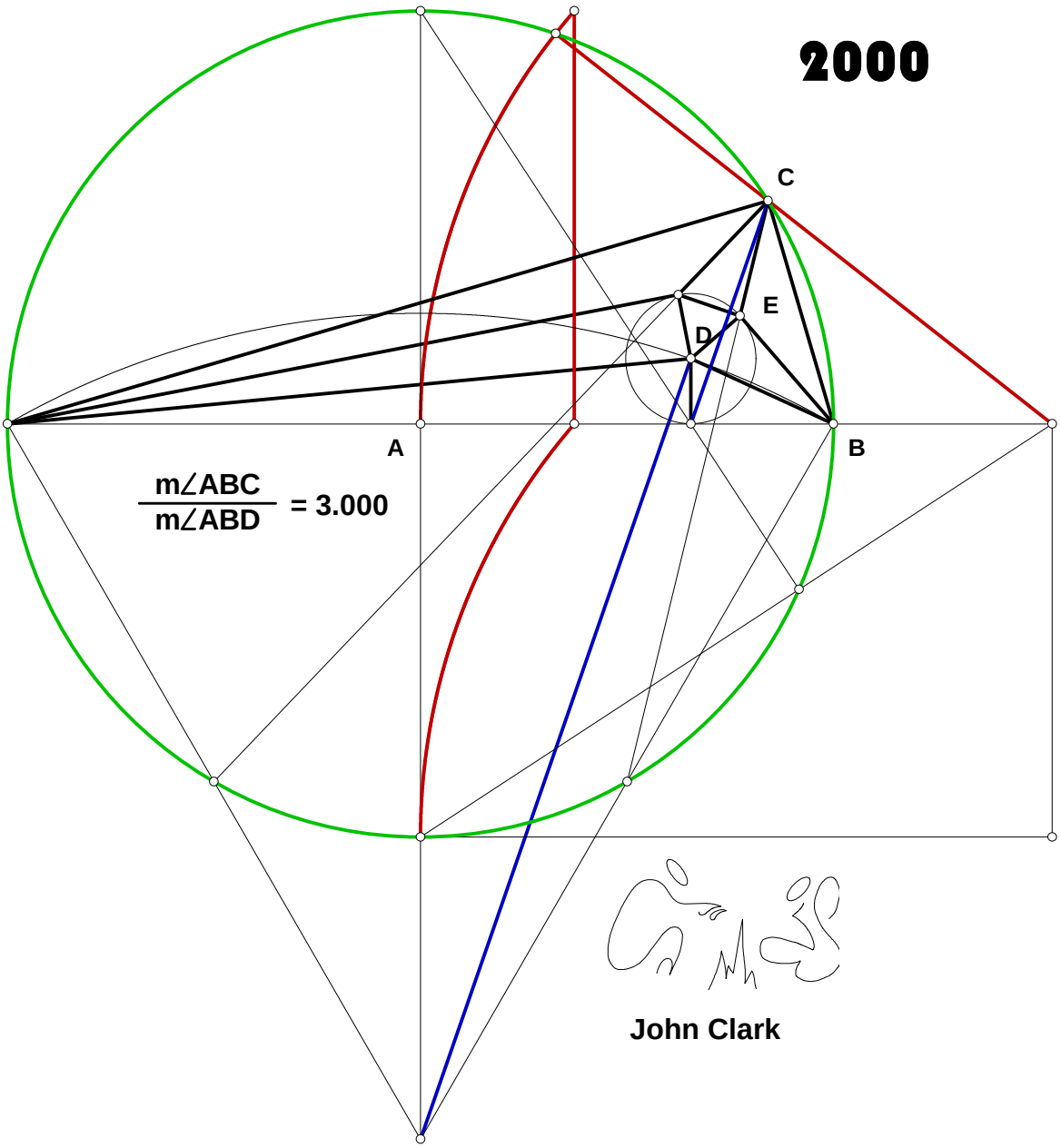
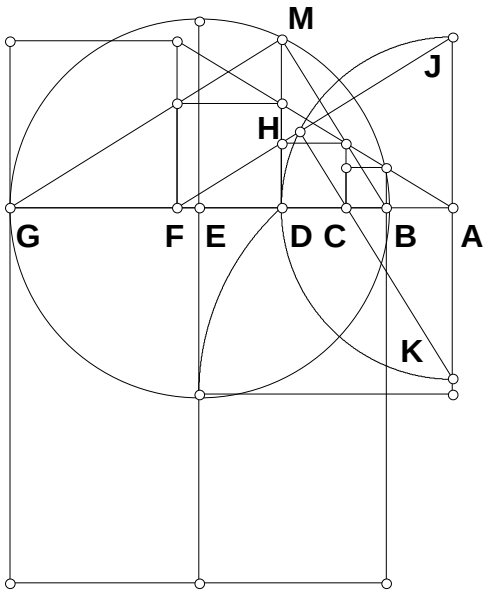
Three Pieces Of Paper



John Clark

07/09/00 Alternate Method Quad Roots



$N := 3 \quad AB := 1 \quad AG := AB \cdot N \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

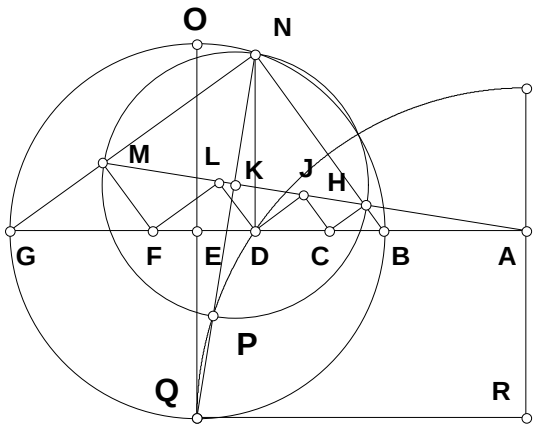
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \quad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

08/01/00 Alternate Method Quad Roots



$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \qquad DE := AE - AD \qquad NY := DE$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

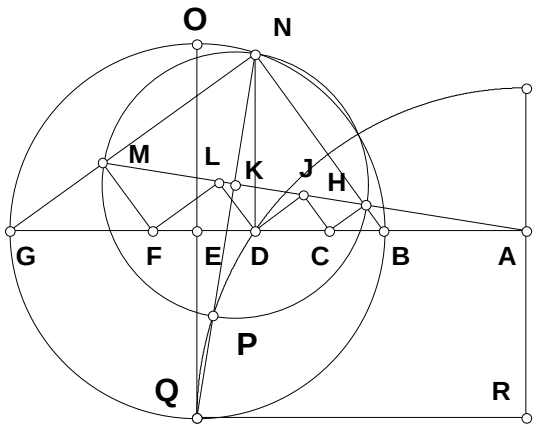
$DN := \sqrt{BD \cdot DG} \qquad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \qquad OQ := BG \qquad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \qquad NP := NQ - PQ \qquad MN := \sqrt{\frac{NP^2}{2}} \qquad BN := \sqrt{BD^2 + DN^2} \qquad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \qquad FG := \frac{BG \cdot GM}{GN} \qquad AF := AG - FG \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

08/01/00B Alternate Method Quad Roots



$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad BN := \sqrt{BD^2 + DN^2}$

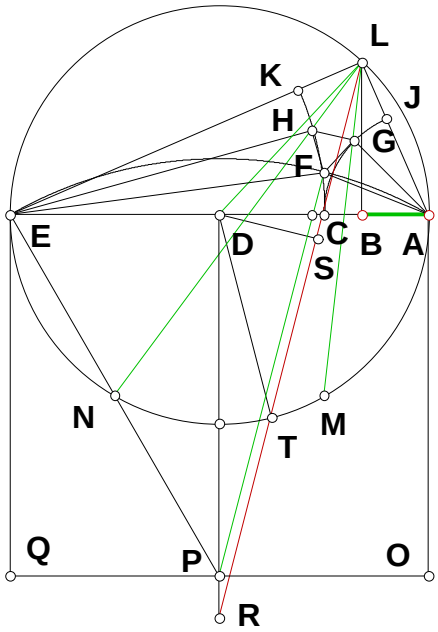
$GN := \sqrt{DG^2 + DN^2} \qquad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \qquad AN := \sqrt{AD^2 + DN^2} \qquad AE := AB + BE \qquad AQ := \sqrt{AE^2 + EQ^2}$

$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \qquad KM := KN \qquad MN := \sqrt{KN^2 + KM^2} \qquad GM := GN - MN \qquad GF := \frac{BG \cdot GM}{GN}$

$AF := AG - GF \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

In Trisection What Is AB? 08/02/00



In the trisection figure given and given AC as the Unit what is AB?

$AC := 1 \quad N := 5 \quad AE := AC \cdot N \quad AD := \frac{AE}{2}$

$EP := AE \quad DE := AD \quad DP := \sqrt{EP^2 - DE^2} \quad FP := EP$

$CE := AE - AC \quad CD := CE - DE \quad CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF \quad DR := DP + PR \quad CR := \sqrt{CD^2 + DR^2}$

$CS := \frac{CD^2}{CR} \quad DS := \sqrt{CD^2 - CS^2} \quad DL := AD$

$LS := \sqrt{DL^2 - DS^2} \quad RS := CR - CS \quad LR := RS + LS$

$BD := \frac{CD \cdot LR}{CR} \quad AB := AD - BD \quad ST := LS \quad RT := RS - ST$

In trisection the length RT to the similarity point is equal to the radius of the circle.

$RT - \frac{N}{2} = 0 \quad AE := N \quad AD := \frac{N}{2}$

$DP := \frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{N^2} \quad CE := N - 1 \quad CD := \frac{1}{2} \cdot N - 1 \quad CF := \frac{1}{2} \cdot \sqrt{(3 \cdot N - 2) \cdot (N + 2)} - \frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{N^2}$

$DR := \frac{1}{2} \cdot \sqrt{(3 \cdot N - 2) \cdot (N + 2)} \quad N - CR = 0 \quad CS := \frac{1}{4} \cdot \frac{(N - 2)^2}{N} \quad LS := \frac{1}{4} \cdot \frac{\left(N^2 + 4 \cdot N - 4\right)}{N}$

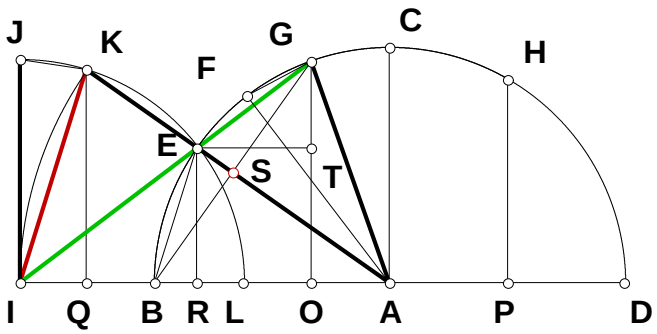
$DS := \frac{1}{4} \cdot \frac{\sqrt{(3 \cdot N - 2) \cdot (N - 2)^2 \cdot (N + 2)}}{N} \quad RS := \frac{1}{4} \cdot (3 \cdot N - 2) \cdot \frac{(N + 2)}{N} \quad LR := \frac{1}{N} \cdot \left(N^2 + 2 \cdot N - 2\right)$

$BD := \frac{1}{2} \cdot (N - 2) \cdot \frac{\left(N^2 + 2 \cdot N - 2\right)}{N^2} \quad \frac{(3 \cdot N - 2)}{N^2} - AB = 0 \quad AB \cdot N^2 - 3 \cdot N + 2 = 0$

If the ratio of AB to N is given, such as N/AB = X, then the Unit AC can be found from a cubic equation.

$X := 4 \quad \frac{\left[\left[-X + \sqrt{-X^2 \cdot (X - 1)}\right]^{\left(\frac{2}{3}\right)} + X\right]}{\left[-X + \sqrt{-X^2 \cdot (X - 1)}\right]^{\left(\frac{1}{3}\right)}} = 3.064$

080300 Trisection



If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yeild a trisected figure JKL.

$N := 3 \qquad BD := 1$

$AB := \frac{BD}{2} \qquad AD := AB \qquad AP := \frac{AD}{2}$

$BP := AB + AP \qquad BO := \frac{BP}{N} \qquad AE := AB$

$DO := BD - BO \qquad GO := \sqrt{BO \cdot DO}$

$BG := \sqrt{GO^2 + BO^2} \qquad BS := \frac{BG}{2} \qquad ER := BS \qquad TO := ER \qquad GT := GO - TO \qquad AS := \sqrt{AB^2 - BS^2}$

$ES := AE - AS \qquad BR := ES \qquad OR := BO - BR \qquad ET := OR \qquad IO := \frac{ET \cdot GO}{GT} \qquad BI := IO - BO$

$AI := BI + AB \qquad BE := \sqrt{ER^2 + BR^2} \qquad GE := BE \qquad GI := \frac{GE \cdot GO}{GT} \qquad EI := GI - GE \qquad AK := AI \qquad IK := EI$

$IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI} \qquad EK := AK - AE \qquad \frac{EK}{IQ} = 2$

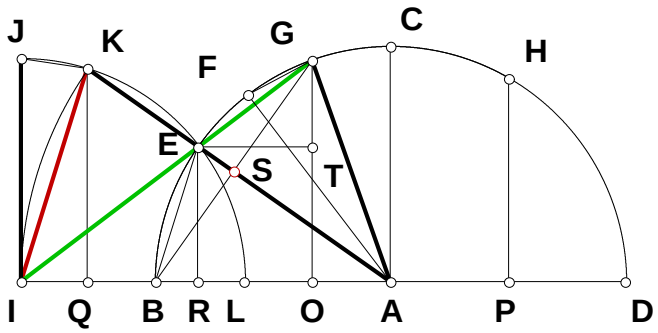
Some Algebraic Names:

$\frac{3}{4 \cdot N} - BO = 0 \qquad 1 - \frac{3}{4 \cdot N} - DO = 0 \qquad \frac{1}{(4 \cdot N)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N - 3} - GO = 0$

$\frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BG = 0 \qquad \frac{1}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BS = 0 \qquad \frac{1}{4} \cdot \frac{\sqrt{3}}{N} \cdot \sqrt{4 \cdot N - 3} - \frac{1}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - GT = 0$

$\frac{1}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - AS = 0 \qquad \frac{1}{2} - \frac{1}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - ES = 0 \qquad \frac{3}{(4 \cdot N)} - \frac{1}{2} + \frac{1}{4} \cdot \sqrt{4 - \frac{3}{N}} - OR = 0$

$\frac{-1}{4} \cdot \frac{\left[3 \cdot \sqrt{4 \cdot N - 3} \cdot \sqrt{N} - 2 \cdot \sqrt{4 \cdot N - 3} \cdot N^{\left(\frac{3}{2}\right)} + 4 \cdot N^2 - 3 \cdot N\right]}{\left[N^{\left(\frac{3}{2}\right)} \cdot \left(-\sqrt{4 \cdot N - 3} + \sqrt{N}\right)\right]} - IO = 0 \qquad \frac{-1}{2} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - 2 \cdot \sqrt{N}\right)}{\left(\sqrt{4 \cdot N - 3} - \sqrt{N}\right)} - BI = 0$



$$\frac{-1}{\left(-2\cdot\sqrt{4\cdot N-3}+2\cdot\sqrt{N}\right)}\cdot\sqrt{N}-AI=0$$

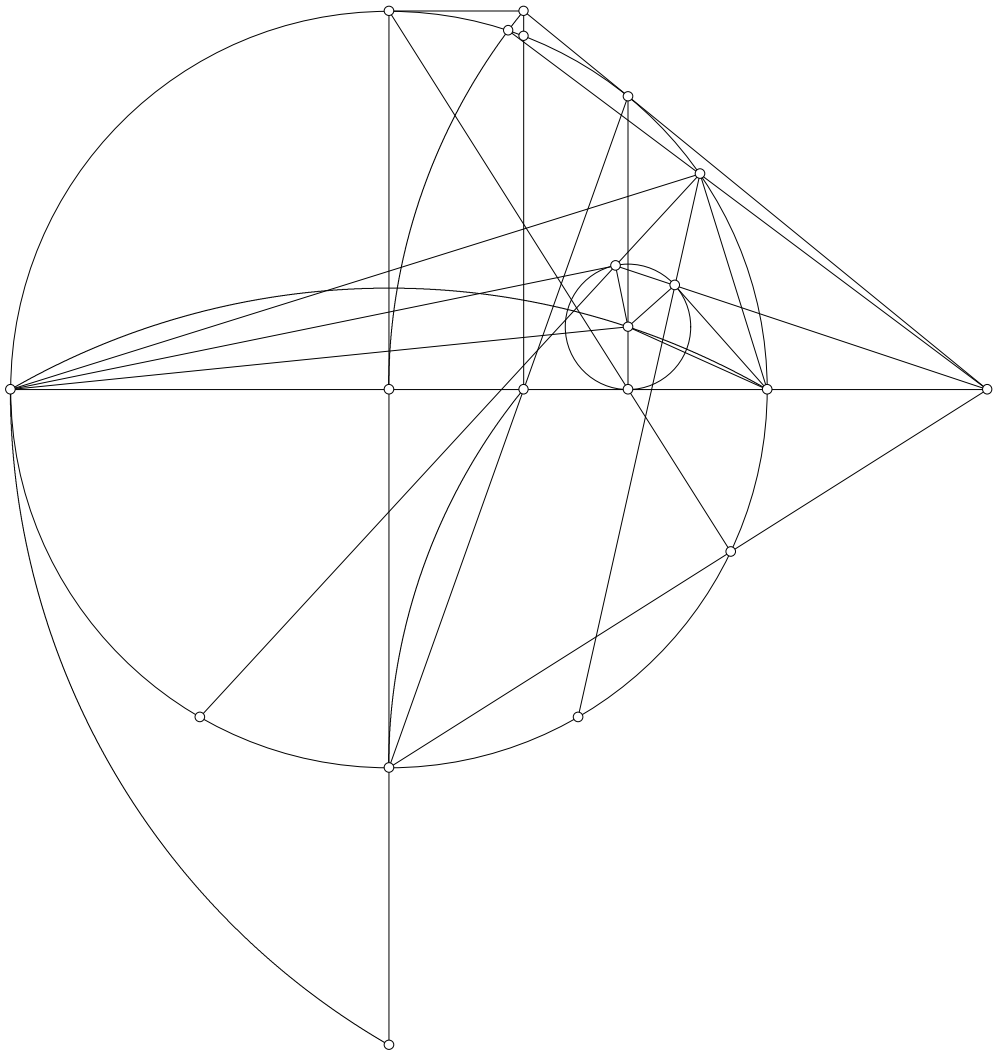
$$\frac{1}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N}}\cdot\sqrt{4\cdot N-3}}-BE=0$$

$$\frac{1}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N}+\sqrt{4\cdot N-3}\right)}{\sqrt{N}}}\cdot\frac{\sqrt{4\cdot N-3}}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-GI=0$$

$$\frac{-1}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N-3}}{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)}-EI=0$$

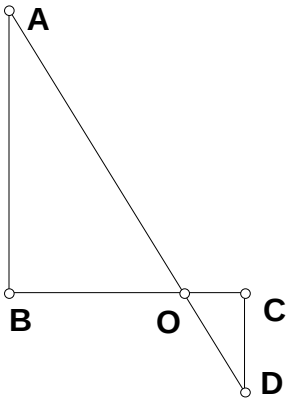
$$\frac{1}{4}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-IQ=0$$

$$\frac{1}{2}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-EK=0$$



08/07/00 Proportion Series II

Two unknowns have the same proportion as two givens and the sum of the unknowns are known. Find the two unknowns.



$AB := 9 \qquad CD := 3 \qquad BC := 5$

$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$

$BO = 3.75 \qquad CO = 1.25$

$BO + CO - BC = 0$

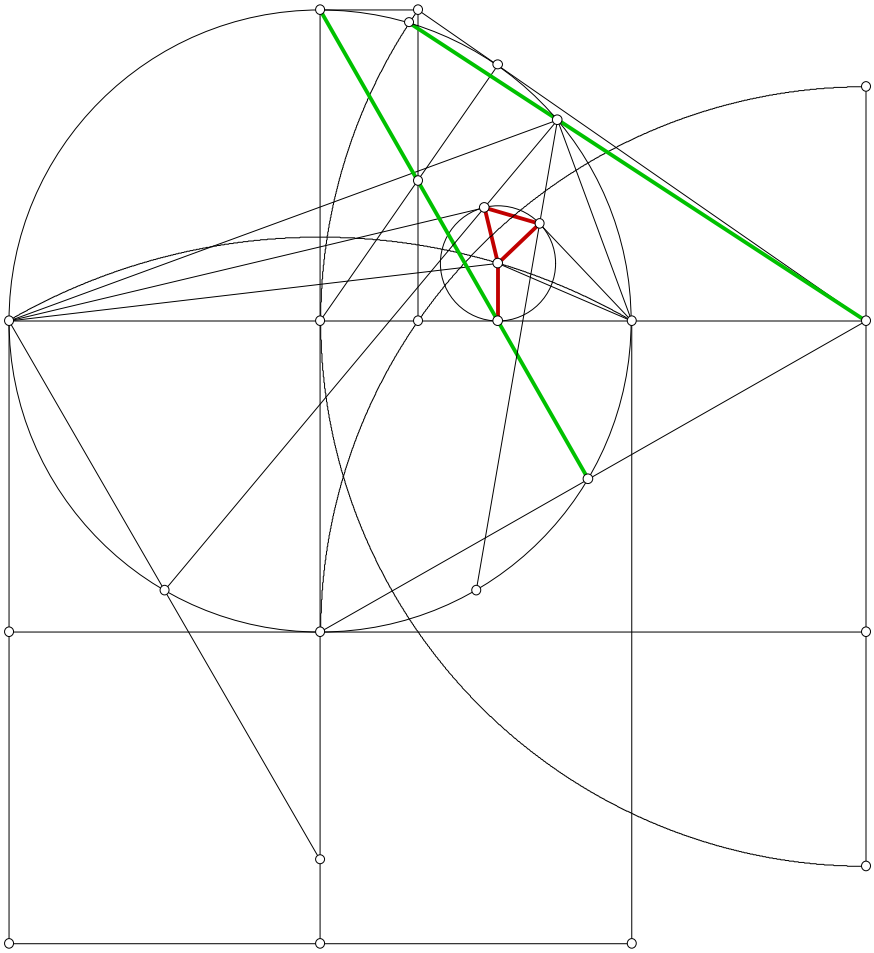
$\frac{AB}{CD} - \frac{BO}{CO} = 0$

m∠UDG = 69.867°

m∠UDCI = 23.289°

$$\frac{m\angle UDG}{m\angle UDCI} = 3.000$$

Trisection and the square root figure.

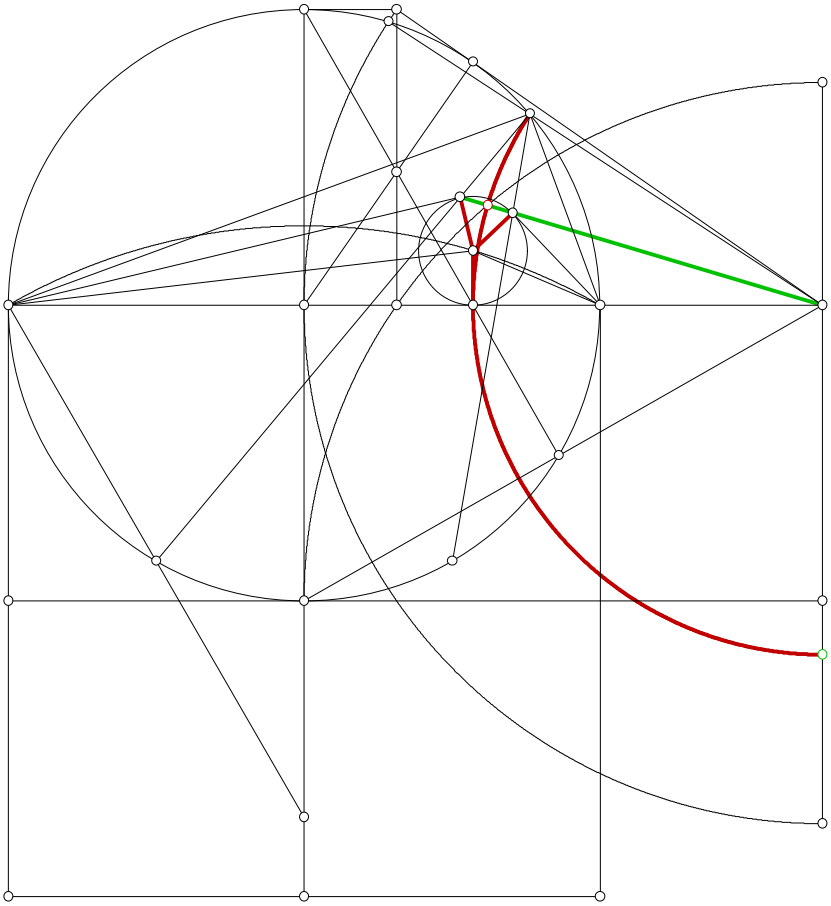


m∠UDG = 69.867°

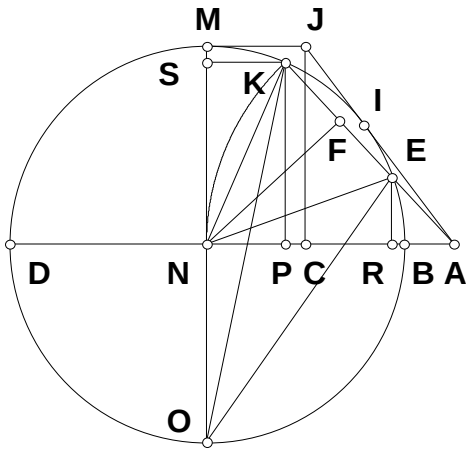
m∠UDCI = 23.289°

$$\frac{m\angle UDG}{m\angle UDCI} = 3.000$$

Trisection and the square root figure.



08/23/00 Trisection In A Square Root Figure



Given the square root figure drawn for trisection, what is AR given AB and AD?

$N := 3 \qquad AB := 1 \qquad AD := AB \cdot N$

$AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB$

$BN := \frac{BD}{2} \qquad KN := BN \qquad CJ := BN$

$AJ := \sqrt{AC^2 + CJ^2} \qquad AK := AJ$

$AN := AB + BN \qquad AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN}$

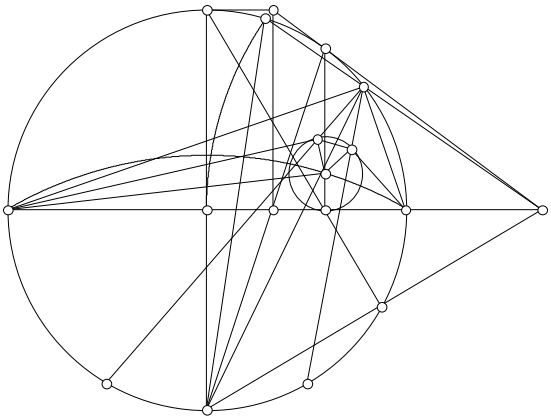
$AF := \frac{AP \cdot AN}{AK} \qquad FK := AK - AF \qquad EF := FK$

$AE := AK - 2 \cdot (EF) \qquad AR := \frac{AP \cdot AE}{AK} \qquad N \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} - AR = 0 \qquad BR := AR - AB$

$(3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3} - BR = 0$

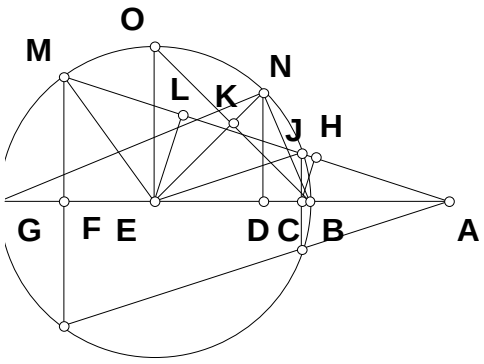
Does KS = FK?

$BP := AP - AB \qquad DP := BD - BP \qquad NP := BN - BP \qquad KS := NP \qquad KS - FK = 0$



09/03/00 Ratios In Trisection

How does BF vary with BC? How does DF vary with BC?



$N_1 := 4 \quad N_2 := 8$

$BG := 1 \quad BE := \frac{BG}{2} \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2}$

$EN := BE \quad EK := \frac{BE \cdot BE}{BO} \quad KN := EN - EK \quad BK := \frac{BO}{2}$

$BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE$

$AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC \quad AE := AB + BE$

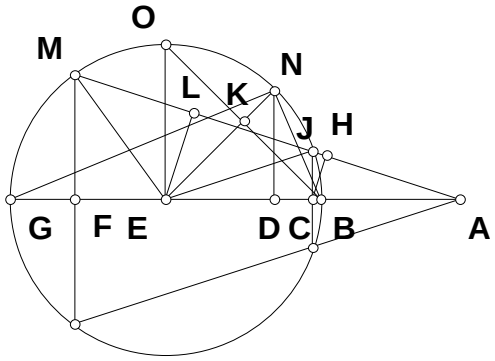
$JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB$

$$\frac{-1}{4} \cdot \left(-2 + \sqrt{2}\right) \cdot \frac{N_1}{N_2} - BC = 0$$

$$BF - \frac{1}{8} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{N_2^3} = 0$$

$$\frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2 \cdot \left[\left(2 \cdot \sqrt{2} - 3\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right)\right]}{\left(N_2^2 \cdot N_1\right)} - \frac{BF}{BC} = 0$$

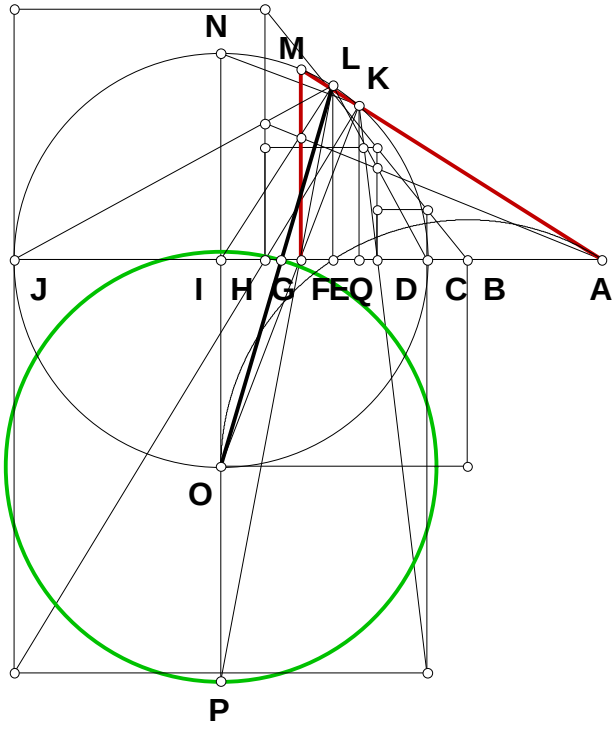


$$DF := BF - BD$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(7\cdot\sqrt{2} - 10\right)\cdot\left(12\cdot\sqrt{2}\cdot N_2^{\,2} + 17\cdot N_2^{\,2} + 2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2\right)\right]}{4\,N_2^{\,3}} - DF = 0$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(2\cdot\sqrt{2} - 3\right)\cdot\left(2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2 + 17\cdot N_2^{\,2} + 12\cdot\sqrt{2}\cdot N_2^{\,2}\right)\right]}{N_1\cdot N_2^{\,2}} - \frac{DF}{BC} = 0$$

Goshdarn Good Pencil 09/16/00



$N := 2$

$BC := 1 \quad BJ := BC \cdot N \quad BE := \sqrt{BC \cdot BJ}$

$CJ := BJ - BC \quad CI := \frac{CJ}{2} \quad IO := CI$

$NO := CJ \quad CR := CJ$

$CE := BE - BC \quad EI := CI - CE \quad EJ := CJ - CE$

$EL := \sqrt{CE \cdot EJ} \quad EG := \frac{EI \cdot EL}{EL + IO} \quad GI := EI - EG$

$GO := \sqrt{GI^2 + IO^2} \quad OP := GO$

$IP := IO + OP \quad EF := \frac{EI \cdot EL}{EL + IP} \quad FI := EI - EF$

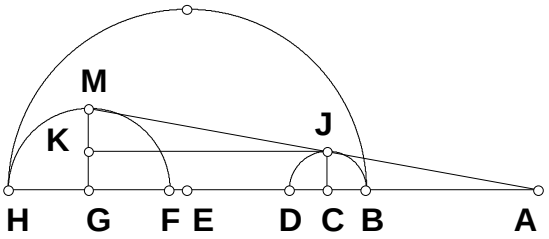
$FO := \sqrt{FI^2 + IO^2} \quad OK := \frac{IO \cdot NO}{FO}$

$FK := OK - FO \quad FQ := \frac{FI \cdot FK}{FO} \quad QI := FQ + FI \quad CQ := CI - QI \quad QJ := CJ - CQ \quad QK := \sqrt{CQ \cdot QJ}$

$CD := \frac{CQ \cdot CR}{CR + QK} \quad BD := CD + BC \quad \left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 0.000004486957912$

Midpoints and Similarity Points 09/18/00

What is AE given the radius of the two circles and the difference between their centers? (External Unit).



$N_1 := 2 \quad N_2 := 3 \quad N_3 := 8$

$BC := N_1 \quad GH := N_2 \quad CG := N_3$

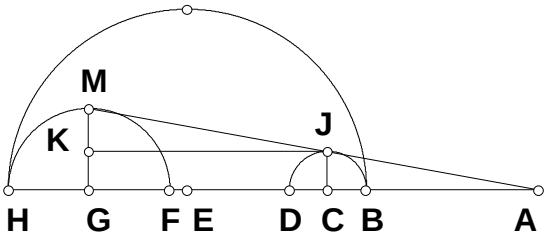
$CJ := BC \quad GM := GH \quad GK := CJ$

$BH := BC + CG + GH \quad BE := \frac{BH}{2}$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{N_3 \cdot N_2 - 2 \cdot N_1 \cdot N_2 + N_1^2 + N_3 \cdot N_1 + N_2^2}{2 \cdot (N_2 - N_1)} = 0$$

What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).



$N_1 := 3 \quad N_2 := 7 \quad N_3 := 4 \quad N_4 := 9$

$BH := 1 \quad BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2}$

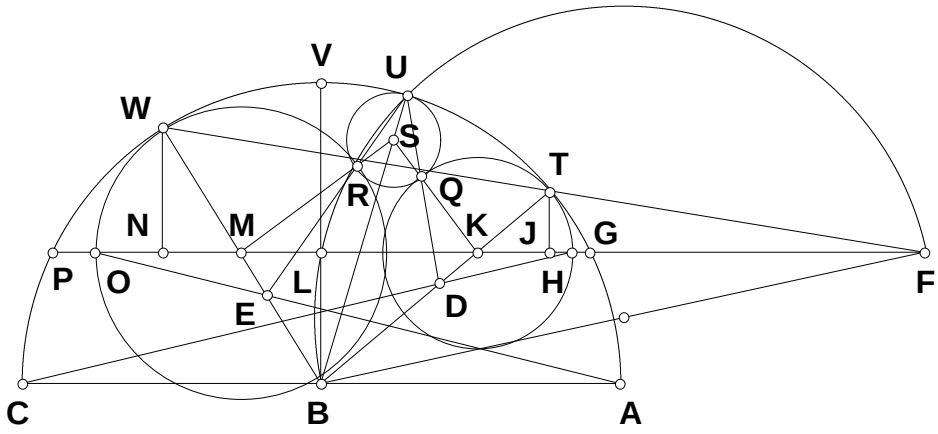
$GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$

$CJ := BC \quad GM := GH \quad GK := CJ$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{(N_3 \cdot N_2 - 4 \cdot N_3 \cdot N_1 + N_1 \cdot N_4)}{2 \cdot (N_3 \cdot N_2 - N_1 \cdot N_4)} = 0$$

100600



$N_1 := 2.286$

$N_2 := 4.805$

$N_3 := 1.5364$

$AC := 3.104$

$AB := \frac{AC}{2}$ $BW := AB$

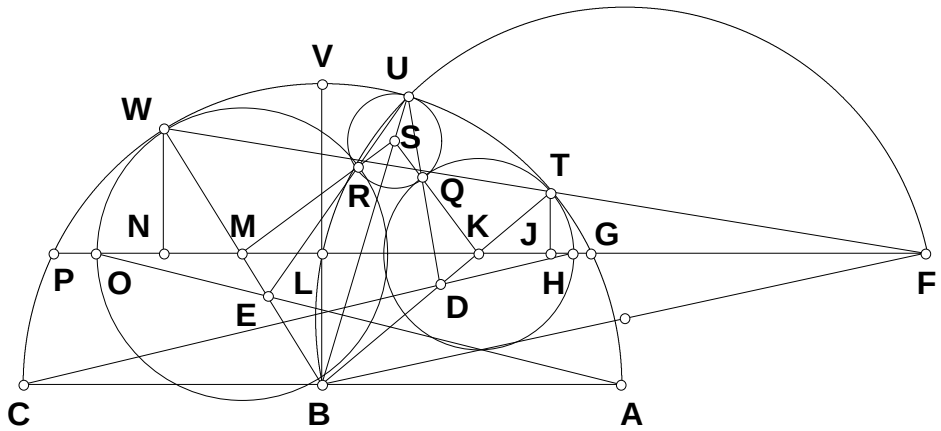
$BT := AB$ $BV := AB$

$BL := \frac{BV}{N_1}$ $LV := BV - BL$ $LP := (LV \cdot (BL + BV))^{\frac{1}{2}}$ $GL := LP$ $GP := GL + LP$ $GK := \frac{GP}{N_2}$

$GM := \frac{GP}{N_3}$ $KL := GL - GK$ $LM := GM - GL$ $BK := \left(BL^2 + KL^2\right)^{\frac{1}{2}}$ $BM := \left(BL^2 + LM^2\right)^{\frac{1}{2}}$

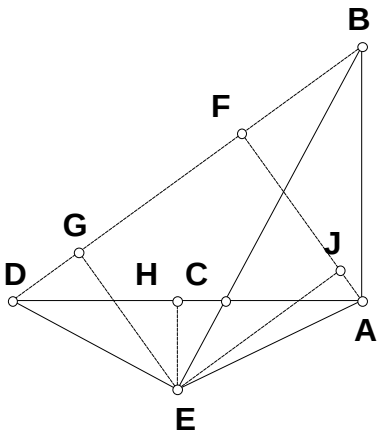
$KT := BT - BK$ $MW := BW - BM$ $KM := GM - GK$ $D := KM$ $R_L := MW$ $R_S := KT$

$FM := \frac{D \cdot R_L}{R_L - R_S}$ $FL := FM - LM$ $BF := \left(BL^2 + FL^2\right)^{\frac{1}{2}}$



11/13/00

Given AB, DE, AD find BE, AC, CD, EC, BC.



$N_1 := 2 \qquad N_2 := 4 \qquad N_3 := 1$

$AB := N_1 \qquad AD := N_2 \qquad DE := N_3$

$BD := \sqrt{AB^2 + AD^2} \qquad BF := \frac{AB^2}{BD} \qquad DG := \frac{DE^2}{BD} \qquad BE := \sqrt{AD^2 - DE^2}$

$AF := \sqrt{AB^2 - BF^2} \qquad EG := \sqrt{DE^2 - DG^2}$

$FG := BD - (BF + DG) \qquad EJ := FG \qquad FJ := EG \qquad AJ := AF - FJ \qquad AE := \sqrt{EJ^2 + AJ^2} \qquad S_1 := AD \qquad S_2 := DE \qquad S_3 := AE$

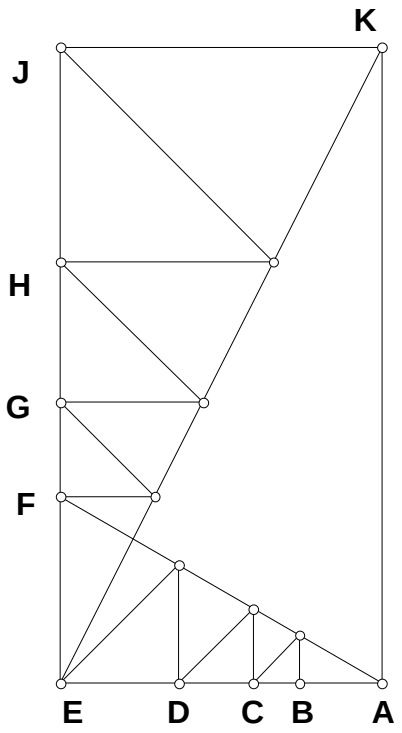
$AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \qquad EH := \sqrt{AE^2 - AH^2} \qquad CH := \frac{EH \cdot AH}{AB + EH} \qquad AC := AH - CH \qquad CE := \frac{AC \cdot DE}{AB} \qquad CD := AD - AC$

$$N_1 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_1^2 \cdot N_2 - N_1 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - AC = 0$$

$$N_3 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_2 \cdot N_1^2 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - CE = 0$$

Means On Means 11/28/00

Modify 02/28/98 for Mean proportionals between E and J.



$AE := 1 \qquad N := 3 \qquad EJ := AE \cdot N \qquad JK := AE$

$HJ := \frac{JK \cdot EJ}{JK + EJ} \qquad EH := EJ - HJ \qquad GH := \frac{EH \cdot HJ}{EH + HJ}$

$EG := EH - GH \qquad FG := \frac{EG \cdot GH}{EG + GH} \qquad EF := EG - FG$

$DE := \frac{EF \cdot AE}{EF + AE} \qquad AD := AE - DE \qquad CD := \frac{AD \cdot DE}{AD + DE}$

$AC := AD - CD \qquad BC := \frac{AC \cdot CD}{AC + CD} \qquad AB := AC - BC$

$M := 0 .. 3 \qquad P := 0 .. 3 \qquad AEAB_{M,P} := \left[\frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

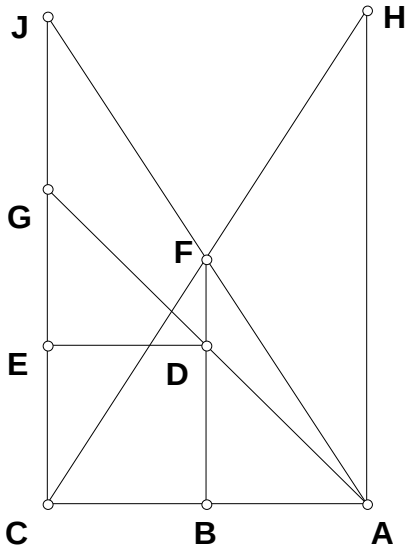
$$AEAB = \begin{bmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.563 & 34.328 \\ 1 & 2.688 & 7.223 & 19.411 \\ 1 & 2.266 & 5.133 & 11.63 \end{bmatrix}$$

$AEAB_{3,3} - \frac{AE}{AB} = 0 \qquad AEAB_{3,2} - \frac{AE}{AC} = 0 \qquad AEAB_{3,1} - \frac{AE}{AD} = 0 \qquad AEAB_{3,0} - \frac{AE}{AE} = 0$

$AEAB_{3,3} - \frac{AE}{AB} = 0$

Multiplication and Division-Line By A Line 11/29/00

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



$AC := 1 \qquad N_1 := 3 \qquad N_2 := 12 \qquad AH := N_1$

$CJ := N_2 \qquad AB := \frac{AH}{(CJ + AH)} \cdot AC$

$BC := AC - AB \quad BD := BC \quad CG := \frac{BD \cdot AC}{AB}$

$CG - \frac{N_2}{N_1} = 0 \qquad CG = 4$

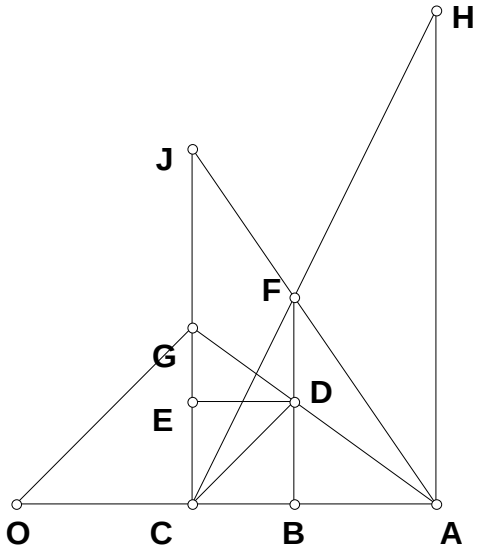
For Multiplication:

$AC := 1 \qquad N_1 := 5 \qquad N_2 := 7 \qquad AH := N_1$

$CG := N_2 \qquad CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$

$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$

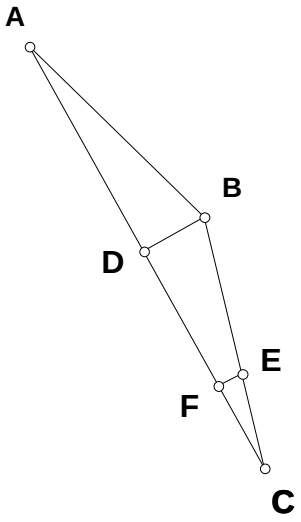
$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$



120500

From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.

Since three points determine a plane, can AB ever be rotated such that a different answer would result?



$N_1 := 3.013 \quad N_2 := 4.059 \quad N_3 := 1.136 \quad N_4 := .947$

$BC := N_1 \quad AC := N_2 \quad CE := N_3 \quad EF := N_4$

$BD := \frac{EF \cdot BC}{CE} \quad BD = 2.512$

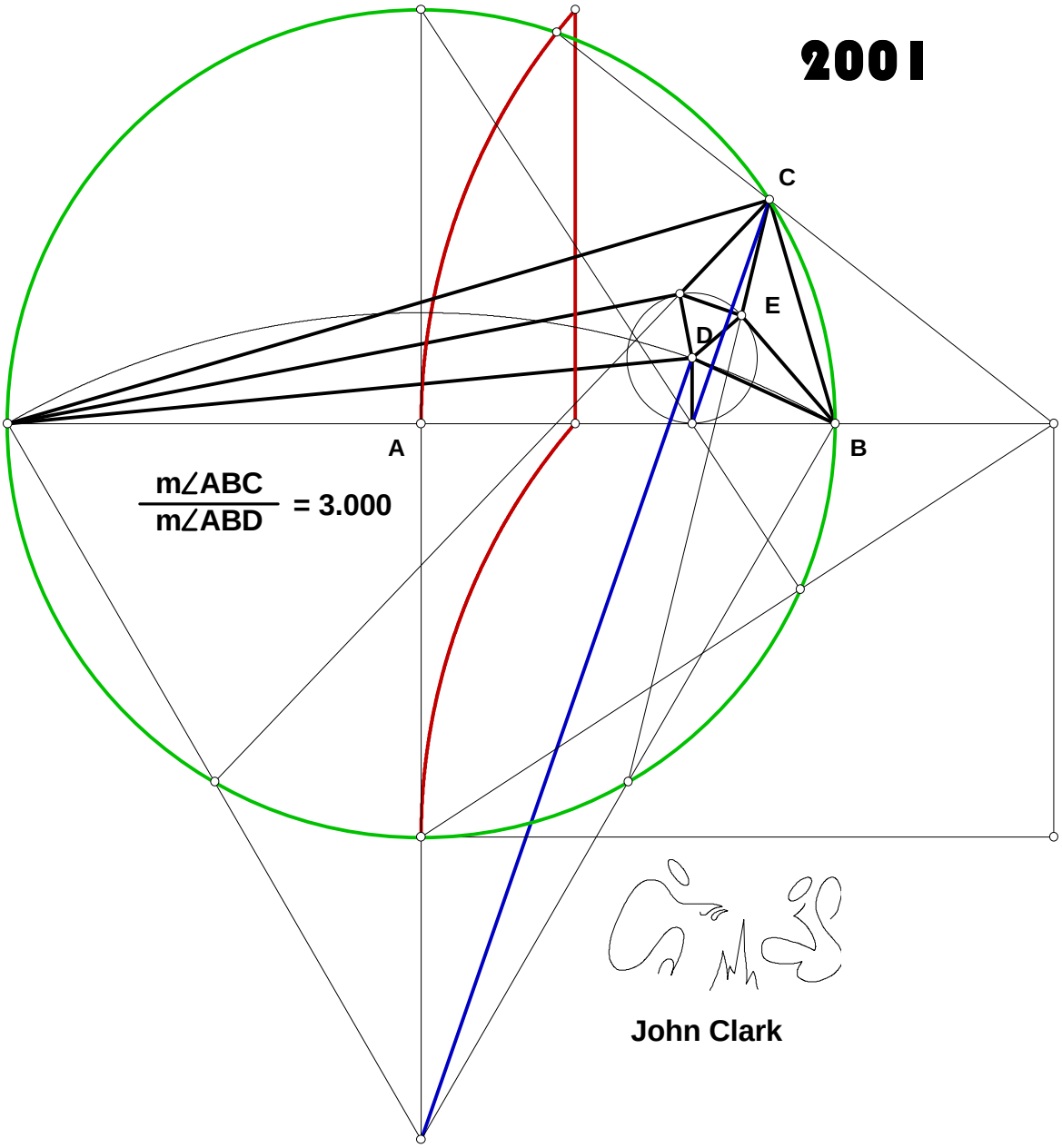
$CF := \sqrt{CE^2 - EF^2} \quad CD := \frac{CF \cdot BC}{CE} \quad CD = 1.664$

$AD := AC - CD \quad AD = 2.395$

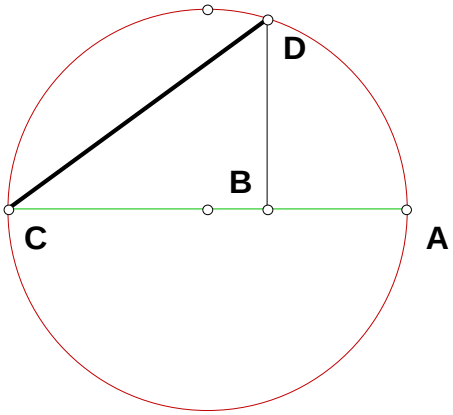
$AB := \sqrt{BD^2 + AD^2} \quad AB = 3.47$

$$AB - \sqrt{N_2^2 - \frac{2}{N_3} \cdot N_2 \cdot \sqrt{-N_4^2 + N_3^2} \cdot N_1 + N_1^2} = 0$$

Three Pieces Of Paper



John Clark



Alternate method for common segment common endpoint square root. $\sqrt{AC \cdot BC} = CD$

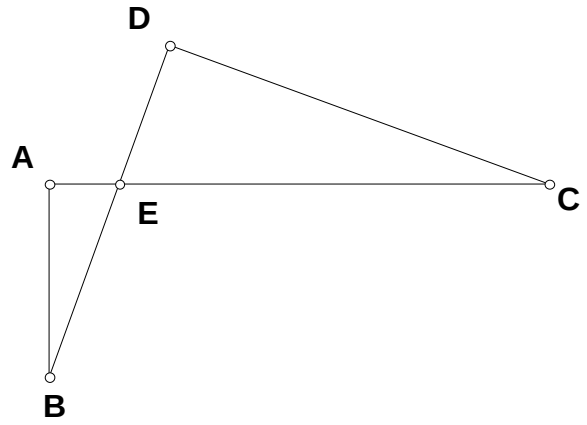
$N_1 := 5 \quad N_2 := 3 \quad AC := N_1 \quad BC := N_2$

$AB := AC - BC \quad BD := \sqrt{AB \cdot BC} \quad CD := \sqrt{BD^2 + BC^2}$

$\sqrt{N_2 \cdot N_1} - CD = 0$

Three Given Five Taken 042101

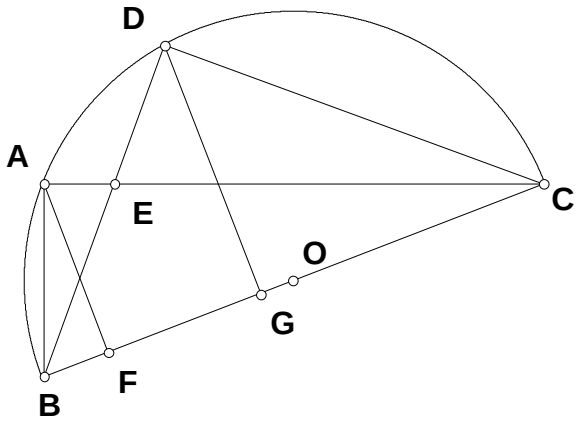
Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?



$N_1 := 2 \qquad N_2 := 3 \qquad N_3 := 4$

$AB := N_1 \quad CD := N_2 \quad AC := N_3$

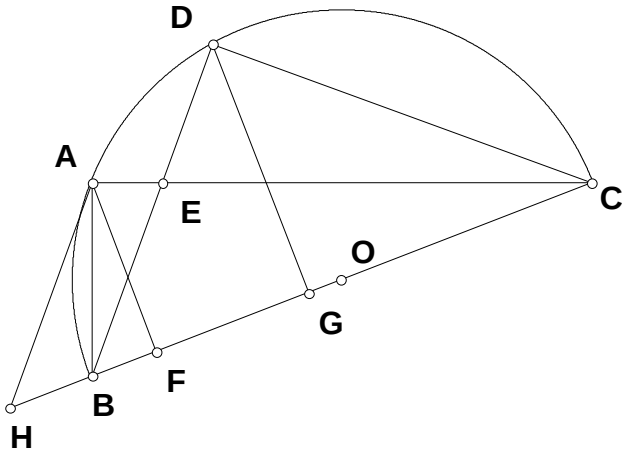
$BC := \sqrt{AB^2 + AC^2} \qquad CG := \frac{CD^2}{BC} \qquad BF := \frac{AB^2}{BC}$



$BG := BC - CG \qquad CF := BC - BF$

$AF := \sqrt{AB^2 - BF^2} \qquad DG := \sqrt{CD^2 - CG^2}$

$FH := \frac{BG \cdot AF}{DG} \qquad CH := CF + FH$



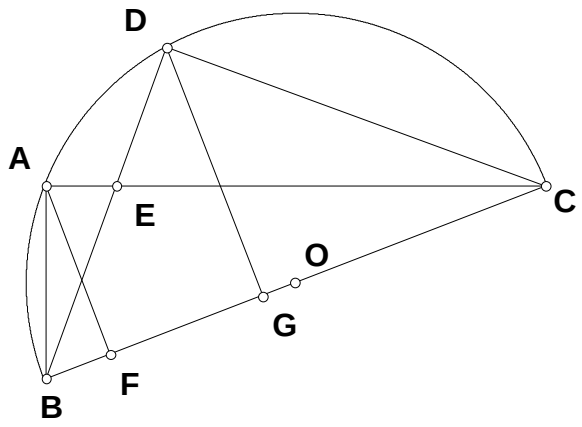
$BD := \sqrt{BG^2 + DG^2}$

$AH := \frac{BD \cdot FH}{BG} \qquad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \qquad CE := \frac{AC \cdot BC}{CH}$

$AE := AC - CE$

Idea about process progression: Given two differences one renders a set number of processes upon them, then one progresses to three?



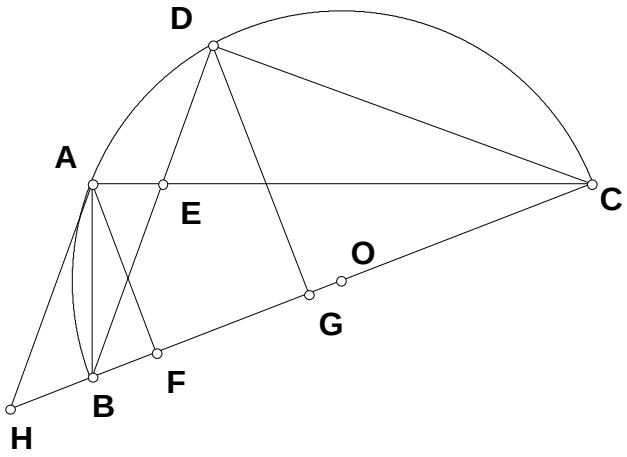
Some Algebraic Names:

$$\sqrt{N_1^2 + N_3^2} - BC = 0 \qquad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$$

$$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0$$

$$\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0$$

$$\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0$$



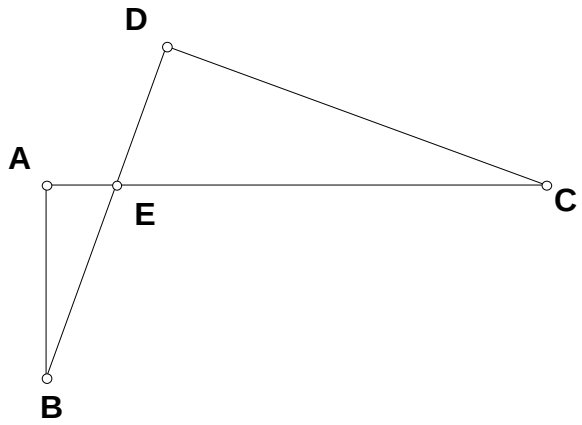
$$\frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$$

$$N_2 \cdot \sqrt{\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\left(N_1^2 + N_3^2\right)}} - DG = 0$$

$$\frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0$$

$$\left[\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} + \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}}\right] - CH = 0 \qquad N_3 \cdot \frac{N_1}{N_2} - AH = 0$$

The Five Sought:



$$\sqrt{N_1^2 + N_3^2 - N_2^2} - BD = 0$$

$$N_2 \cdot \frac{\left(\sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - DE = 0$$

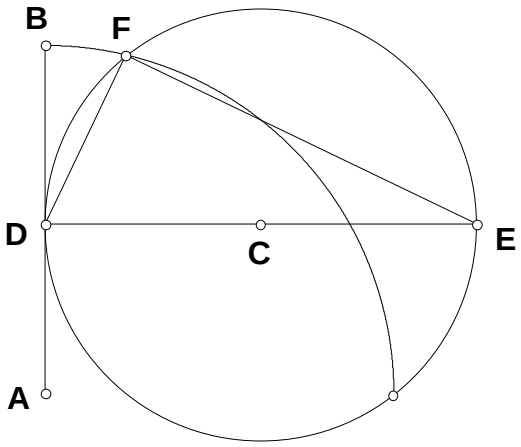
$$N_1 \cdot \frac{\left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - BE = 0$$

$$\frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1} - CE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - AE = 0$$

042201

Given AB as unit, AD and DC, what is EF and DF?



$N_1 := 2.052 \quad N_2 := .62 \quad AB := 1.802$

$AD := \frac{AB}{N_1} \quad CD := AB \cdot N_2 \quad DE := 2\ CD$

$AF := AB \quad CF := CD$

$AC := \sqrt{AD^2 + CD^2} \quad S_1 := AF \quad S_2 := AC \quad S_3 := CF$

$AG := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad CG := \sqrt{AC^2 - AG^2}$

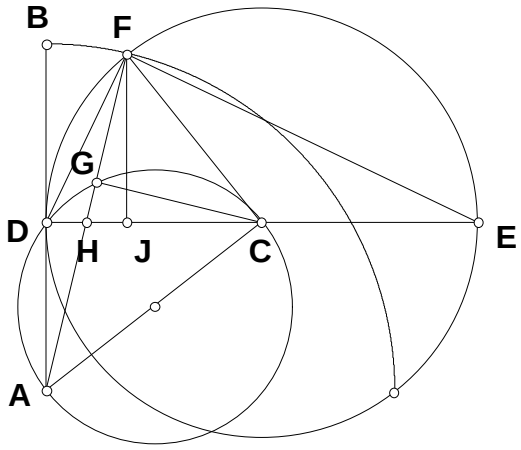
$L_1 := AD \quad L_2 := CG \quad L_3 := CD$

$DH := L_3 - \frac{L_2 \cdot \left(L_1^2 + L_3^2\right)}{L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1}$

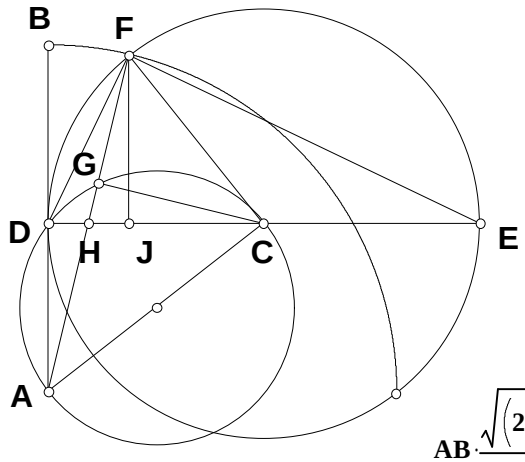
$AH := L_1 \cdot \frac{\left(L_1^2 + L_3^2\right)}{\left(L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1\right)}$

$FH := AF - AH \quad HJ := \frac{DH \cdot FH}{AH}$

$DJ := DH + HJ \quad EJ := DE - DJ \quad FJ := \frac{AD \cdot FH}{AH} \quad EF := \sqrt{FJ^2 + EJ^2} \quad DF := \sqrt{DJ^2 + FJ^2}$



Some Algebraic Names:



$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0$$

$$(2\,AB) \cdot N_2 - DE = 0 \qquad AB \cdot \sqrt{\frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{N_1^{\,2}}} - AC = 0$$

$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^{\,2}\right)}{N_1^{\,2}} - AG = 0$$

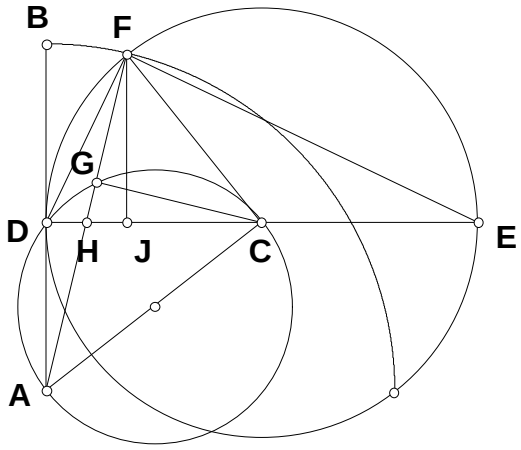
$$AB \cdot \frac{\sqrt{\left(2 \cdot N_1^{\,2} \cdot N_2 + N_1^{\,2} - 1\right) \cdot \left(2 \cdot N_1^{\,2} \cdot N_2 - N_1^{\,2} + 1\right)}}{2 \cdot N_1^{\,2}} - CG = 0$$

$$AB \cdot \frac{N_2 \cdot N_1^{\,3} + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}}}{N_1^{\,3} + N_1^{\,2} \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + N_1} - DH = 0$$

$$2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - FH = 0$$

$$\frac{-1}{2} \cdot AB \cdot \frac{\begin{bmatrix} -N_1^{\,4} \cdot N_2^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ldots \\ + 2 \cdot N_1^{\,3} \cdot N_2 - 2 \cdot N_1^{\,5} \cdot N_2 \ldots \\ + 6 \cdot N_1^{\,5} \cdot N_2^{\,3} - 3 \cdot N_2^{\,2} \cdot N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ldots \\ + 2 \cdot N_2^{\,3} \cdot N_1^{\,3} - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ldots \\ + N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \end{bmatrix}}{\left[N_1 \cdot \left[\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right) \cdot \left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)\right]\right]} - HJ = 0$$



$$\frac{1}{2}\cdot AB\cdot\frac{\left(N_1{}^3\cdot N_2+N_2\cdot N_1-\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)}{\left[\left(1+N_2{}^2\cdot N_1{}^2\right)\cdot N_1\right]}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N_2\cdot N_1+4\cdot N_2{}^3\cdot N_1{}^3-N_1{}^3\cdot N_2+\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)}{\left[\left(1+N_2{}^2\cdot N_1{}^2\right)\cdot N_1\right]}-EJ=0$$

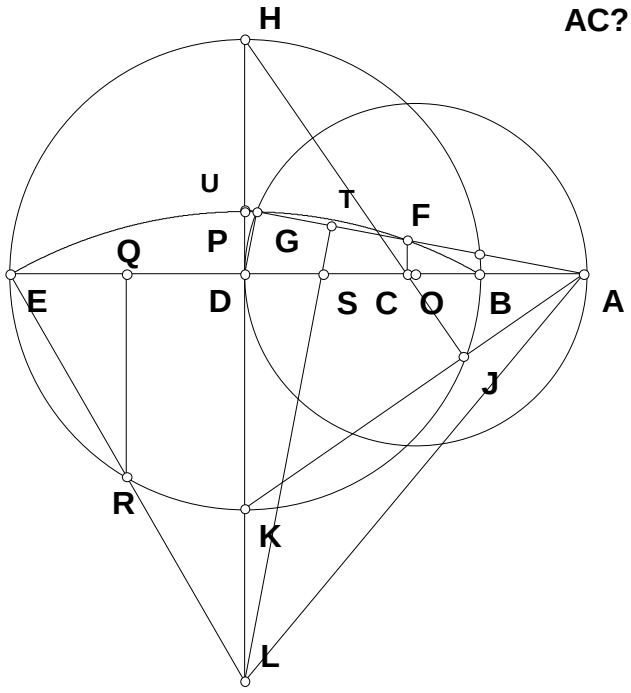
$$\frac{-1}{2}\cdot AB\,\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}+1-N_1{}^2+2\cdot N_2{}^2\cdot N_1{}^2\right)}{\left[N_1\cdot\left(1+N_2{}^2\cdot N_1{}^2\right)\right]}-FJ=0$$

$$\sqrt{AB^2\cdot N_2\cdot\frac{\left(3\cdot N_1\cdot N_2+4\cdot N_2{}^3\cdot N_1{}^3-N_1{}^3\cdot N_2+\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)}{\left[\left(1+N_2{}^2\cdot N_1{}^2\right)\cdot N_1\right]}}-EF=0$$

$$\sqrt{\left(N_1{}^3\cdot N_2+N_1\cdot N_2-\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)\cdot AB^2\cdot\frac{N_2}{\left[N_1\cdot\left(1+N_2{}^2\cdot N_1{}^2\right)\right]}}-DF=0$$

Counterpoint 042301

CF is the radius of a circle expressed in trisection. Is AF the counterpart of AC?



$N := 5 \qquad AB := 1$

$AE := AB \cdot N \qquad BE := AE - AB$

$BD := \frac{BE}{2} \qquad AD := AB + BD$

$AO := \frac{AD}{2} \qquad DE := BD \qquad DQ := \frac{DE}{2}$

$EQ := DQ \qquad BQ := BD + DQ$

$QR := \sqrt{BQ \cdot EQ} \qquad ER := \sqrt{QR^2 + EQ^2}$

$EL := 2 \cdot ER \qquad LP := EL \qquad DL := \frac{QR \cdot EL}{ER}$

$L_1 := \frac{LP}{DL} \qquad L_2 := \frac{AO}{LP}$

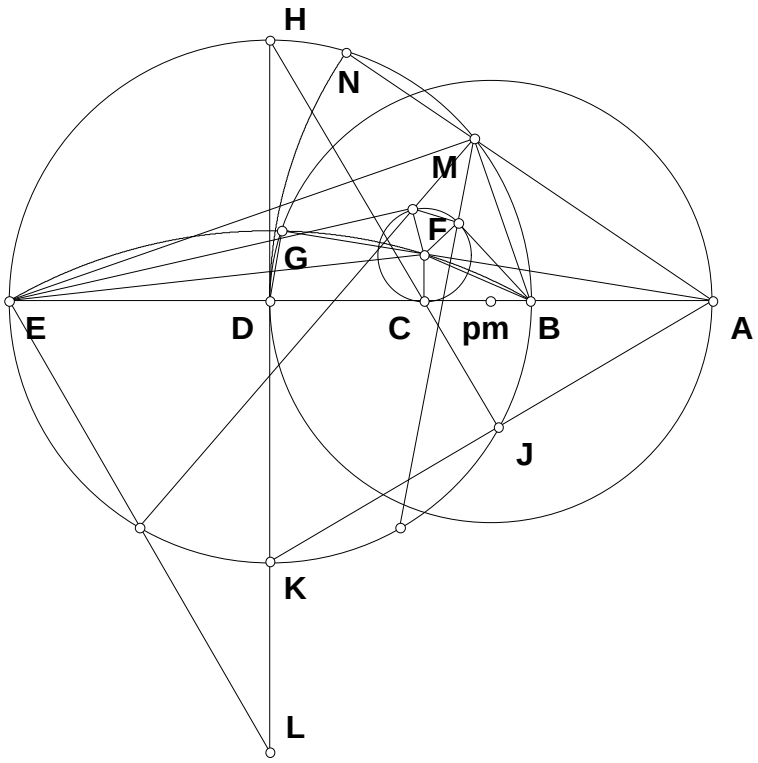
$$AG := \sqrt{LP^2 \cdot L_2 \cdot \frac{\left(3 \cdot L_1 \cdot L_2 + 4 \cdot L_2^3 \cdot L_1^3 - L_1^3 \cdot L_2 + \sqrt{2 \cdot L_1^2 + 4 \cdot L_1^4 \cdot L_2^2 - 1 - L_1^4}\right)}{\left[\left(1 + L_2^2 \cdot L_1^2\right) \cdot L_1\right]}}$$

$DG := \sqrt{AD^2 - AG^2} \qquad AU := \frac{AD \cdot AD}{AG} \qquad DU := \frac{DG \cdot AD}{AG} \qquad LU := DL + DU \qquad TU := \frac{DG \cdot LU}{AD} \qquad GU := AU - AG$

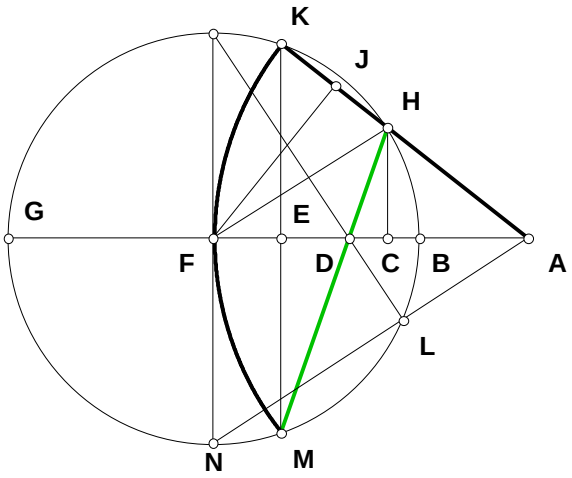
$TG := TU - GU \qquad FT := TG \qquad AF := AG - (FT + TG) \qquad AC := \frac{AD \cdot AF}{AU}$

From 012496 for AC

$$2 \cdot AB \cdot \frac{N}{(1+N)} \qquad AC - 2 \cdot AB \cdot \frac{N}{(1+N)} = 0$$



042401



Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?

$N := 5$ $AB := 1$ $AG := AB \cdot N$

$BG := AG - AB$ $BF := \frac{BG}{2}$ $AF := AB + BF$

$AK := AF$ $FK := BF$ $AE := \frac{2\,AK^2 - FK^2}{2\,AF}$

$AJ := AE$ $JK := AK - AJ$ $HJ := JK$

$AH := AK - (JK + HJ)$ $AC := \frac{AE \cdot AH}{AK}$ $CE := AE - AC$ $BE := AE - AB$ $EG := BG - BE$

$EK := \sqrt{BE \cdot EG}$ $BC := AC - AB$ $CG := BG - BC$ $CH := \sqrt{BC \cdot CG}$ $DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE$ $HM := \sqrt{CE^2 + (EK + CH)^2}$

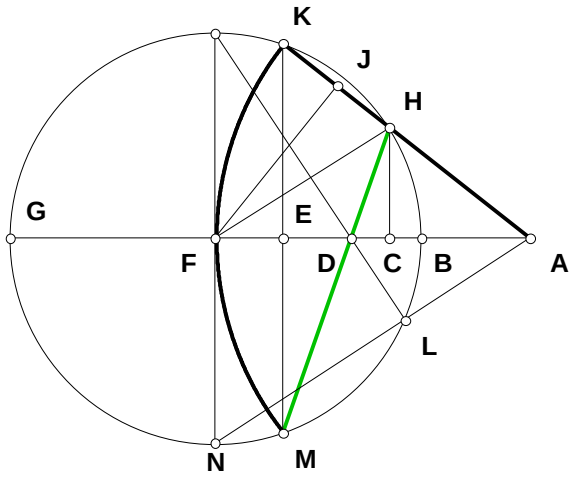
Some Algebraic Names:

$AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$

$\frac{1}{4} \cdot AB \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(1 + N)} - AE = 0$ $\frac{1}{4} \cdot AB \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1 + N)} - JK = 0$ $2 \cdot AB \cdot \frac{N}{(1 + N)} - AH = 0$

$AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - AC = 0$ $\frac{1}{4} \cdot AB \cdot \left(N^2 + 6 \cdot N + 1\right) \cdot \frac{(N - 1)^2}{(1 + N)^3} - CE = 0$

$\frac{1}{4} \cdot AB \cdot (N + 3) \cdot \frac{(N - 1)}{(1 + N)} - BE = 0$ $\frac{1}{4} \cdot AB \cdot (3 \cdot N + 1) \cdot \frac{(N - 1)}{(1 + N)} - EG = 0$



$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-EK=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BC=0$$

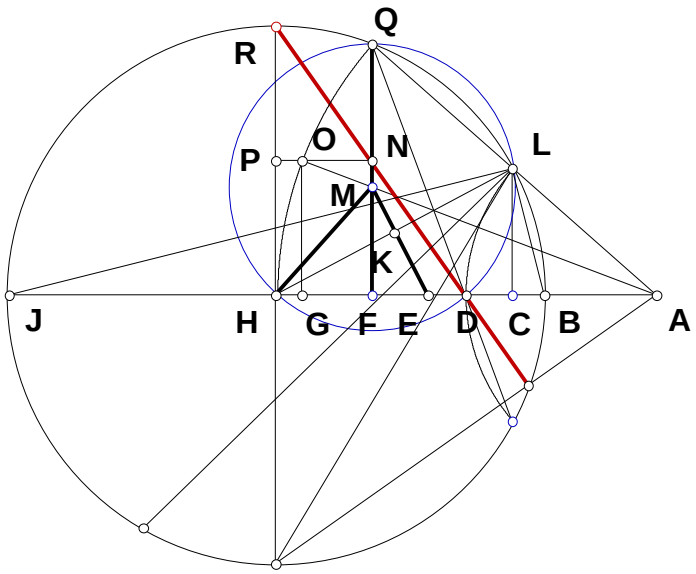
$$AB\cdot N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3}-CG=0$$

$$AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3}-CH=0$$

$$\frac{1}{4}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DE=0 \qquad \frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(N+1)}-DF=0 \qquad \frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{\left(1+6\cdot N+N^2\right)}{(1+N)^2}-HM=0$$

042501

What is the Algebraic name of the circle HM? Does point N divide DR in half?



$N := 5.768$ $AB := .583$ $AJ := AB \cdot N$

$BJ := AJ - AB$ $BH := \frac{BJ}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $AH := AB + BH$

$AO := AH$ $AG := \sqrt{AO^2 - GO^2}$

$HQ := BH$ $AQ := AH$ $FH := \frac{HQ^2}{2 \cdot AH}$

$AF := AH - FH$ $FM := \frac{GO \cdot AF}{AG}$

$HJ := BH$ $FJ := FH + HJ$ $BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ}$ $MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$ $HM - MQ = 0$

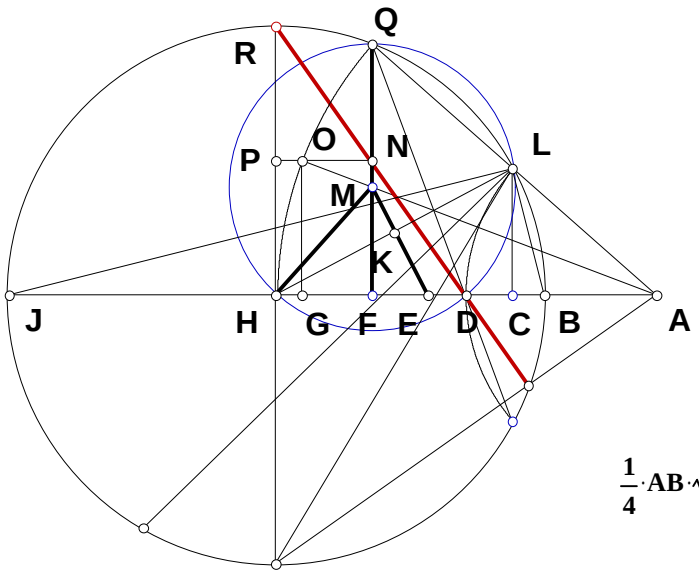
$DH := \frac{HR^2}{AH}$ $\frac{DH}{2} - FH = 0$

Some Algebraic Names:

$AB \cdot N - AB - BJ = 0$ $\frac{1}{2} \cdot AB \cdot (N - 1) - BH = 0$ $\frac{1}{4} \cdot AB \cdot (N - 1) - HP = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AH = 0$ $\frac{1}{4} \cdot AB \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - AG = 0$ $\frac{1}{4} \cdot AB \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$

$\frac{1}{4} \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - AF = 0$ $\frac{1}{4} \cdot (N - 1) \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} - FM = 0$



$$\frac{1}{4}\cdot AB\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)}-FJ=0$$

$$\frac{1}{4}\cdot AB\cdot(N+3)\cdot\frac{(N-1)}{(1+N)}-BF=0$$

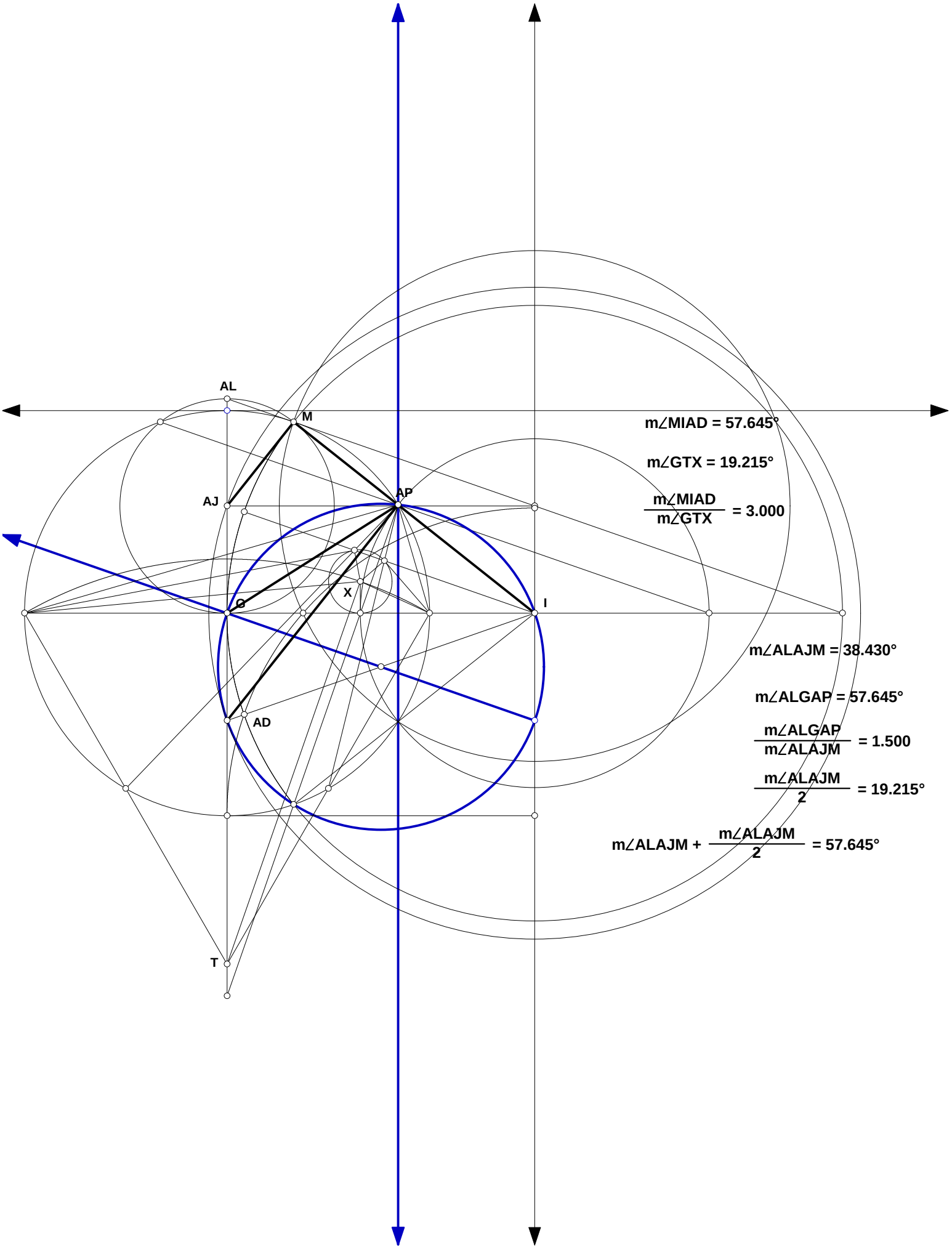
$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-FQ=0$$

$$\frac{1}{2}\cdot(1+N)\cdot AB\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-MQ=0$$

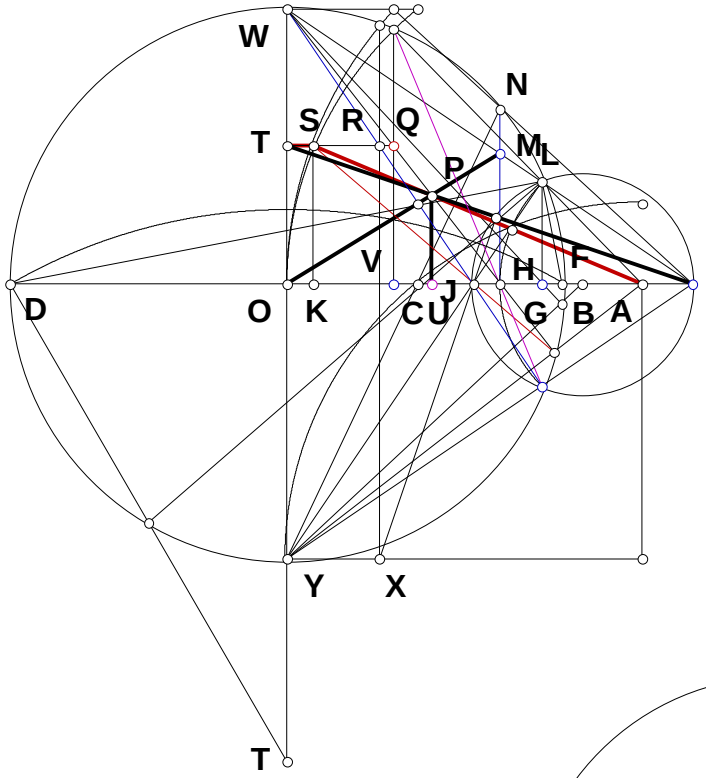
$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-HM=0$$

$$HM-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DH=0 \qquad \frac{DH}{2}-FH=0$$


AL
M
AJ
AP
X
G
I
AD
T
m∠MIAD = 57.645°
m∠GTX = 19.215°
m∠MIAD / m∠GTX = 3.000
m∠ALAJM = 38.430°
m∠ALGAP = 57.645°
m∠ALGAP / m∠ALAJM = 1.500
m∠ALAJM / 2 = 19.215°
m∠ALAJM + m∠ALAJM / 2 = 57.645°

Four Lines To A Point 042901

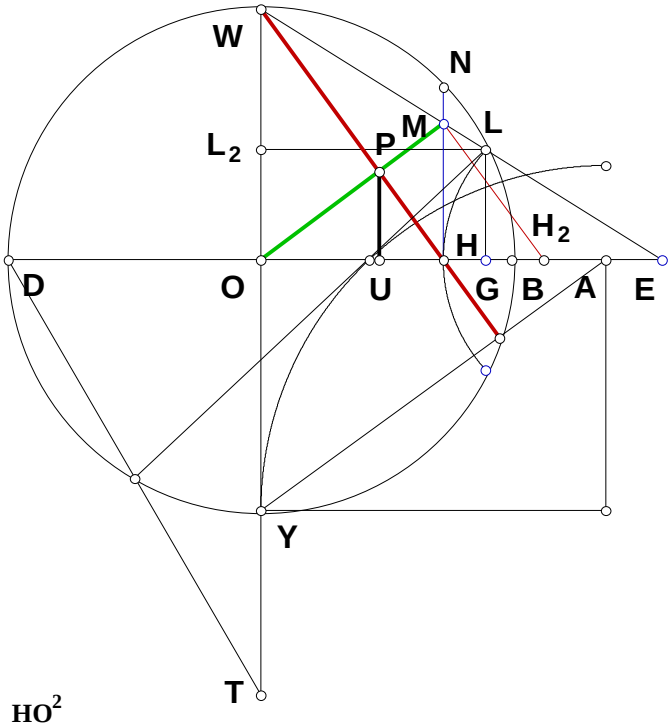


Does the difference OU and PU each have but one Algebraic name?

$N := 5 \qquad AB := 1 \qquad AD := AB \cdot N$

$BD := AD - AB \qquad BO := \frac{BD}{2} \qquad OW := BO$

$OY := BO \qquad DO := BO \qquad AO := AB + BO$



$HO := \frac{OY^2}{AO} \qquad AH := AO - HO \qquad AL := AH$

$OL := BO \qquad GO := \frac{OL^2 + AO^2 - AL^2}{2\,AO}$

$BG := BO - GO \qquad DG := GO + DO$

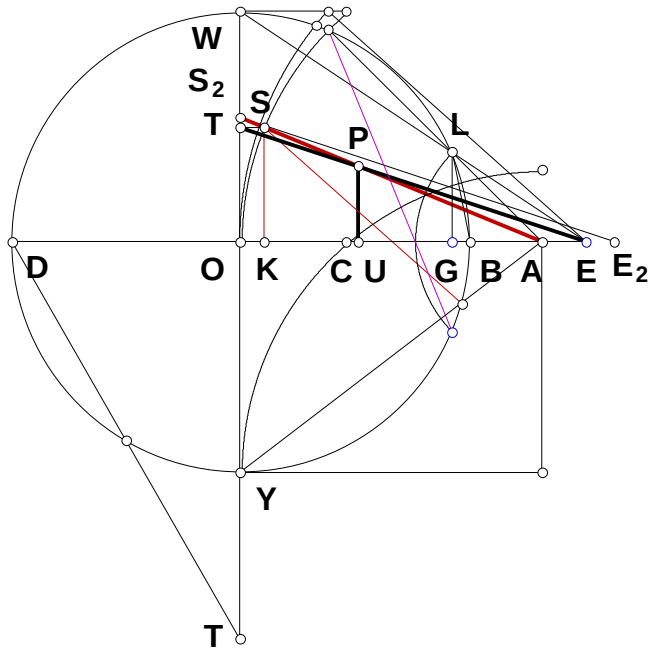
$GL := \sqrt{BG \cdot DG} \qquad EO := \frac{GO \cdot OW}{OW - GL}$

$BE := EO - BO \qquad BH := BO - HO$

$EG := BE + BG \qquad EH := BE + BH$

$HM := \frac{GL \cdot EH}{EG} \qquad HH_2 := \frac{HO \cdot HM}{OW} \qquad OU := \frac{HO^2}{HH_2 + HO}$

$UP := \frac{HM \cdot OU}{HO}$



$$OT := \frac{OW}{2} \qquad KS := OT \qquad AS := AO$$

$$AK := \sqrt{AS^2 - KS^2} \qquad OS_2 := \frac{KS \cdot AO}{AK}$$

$$OE_2 := \frac{EO \cdot OS_2}{OT} \qquad AS_2 := \sqrt{AO^2 + OS_2{}^2}$$

$$AE_2 := OE_2 - AO \qquad AE := EO - AO$$

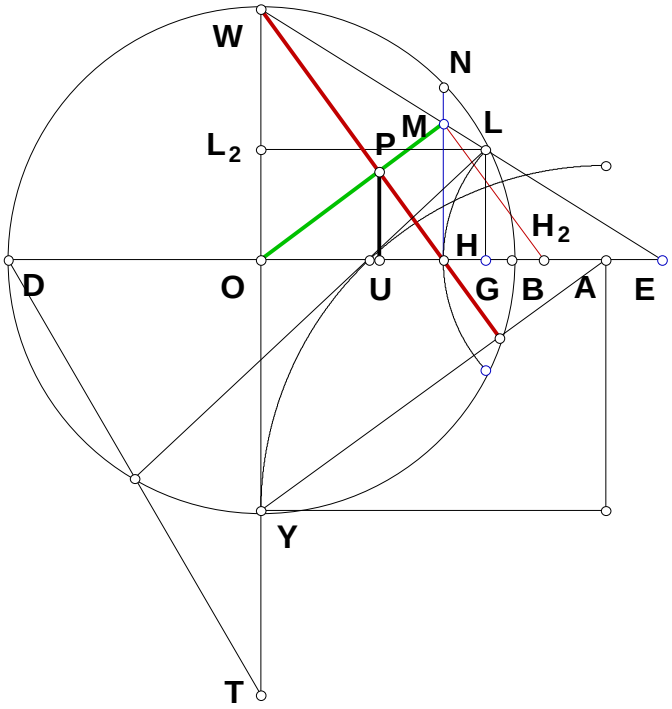
$$AP := \frac{AS_2 \cdot AE}{AE_2} \qquad AU := \frac{AO \cdot AE}{AE_2}$$

$$UO := AO - AU \qquad PU := \frac{OS_2 \cdot AU}{AO}$$

$$UO - OU = 0$$

$$PU - UP = 0$$

Some Algebraic Names:



$$AB\cdot(N-1)-BD=0$$

$$\frac{AB\cdot(N-1)}{2}-BO=0$$

$$\frac{1}{2}\cdot AB\cdot(1+N)-AO=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-HO=0$$

$$2\cdot AB\cdot\frac{N}{(1+N)}-AH=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3}-GO=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BG=0$$

$$AB\cdot(N-1)\cdot N^2\cdot\frac{(N+3)}{(1+N)^3}-DG=0$$

$$\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^3}-GL=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-EO=0$$

$$AB\cdot(N-1)\cdot\frac{\left[\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N-3\cdot N-1\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-BE=0$$

$$AB\cdot\frac{(N-1)}{(1+N)}-BH=0$$

$$AB\cdot\frac{(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\left(N^2+4\cdot N+1\right)}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)^3\right]}-EG=0$$

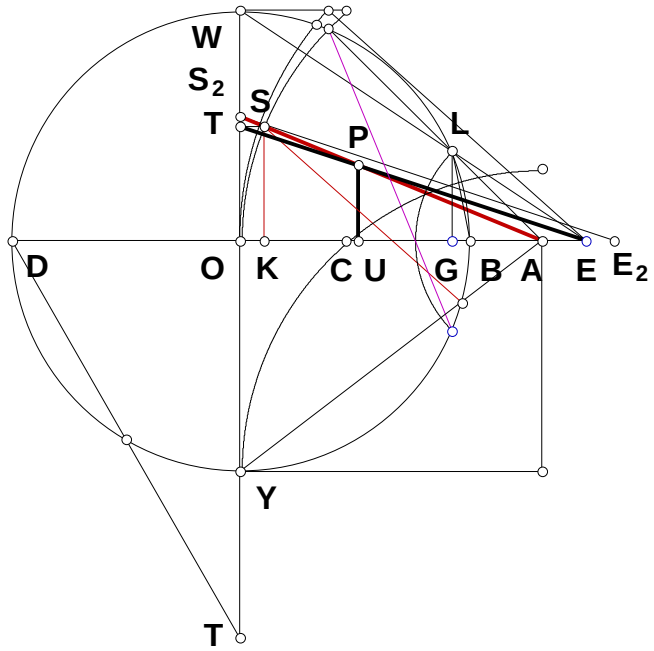
$$AB\cdot(N-1)^2\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)\right]}-EH=0$$

$$N\cdot(N-1)\cdot\frac{AB}{(1+N)}\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HM=0$$

$$AB\cdot\frac{(N-1)^2}{(1+N)^2}\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HH_2=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-OU=0$$

$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UP=0$$



$$\frac{1}{4}\cdot AB\cdot(N-1)-OT=0$$

$$\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-AK=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)\cdot(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OS_2=0$$

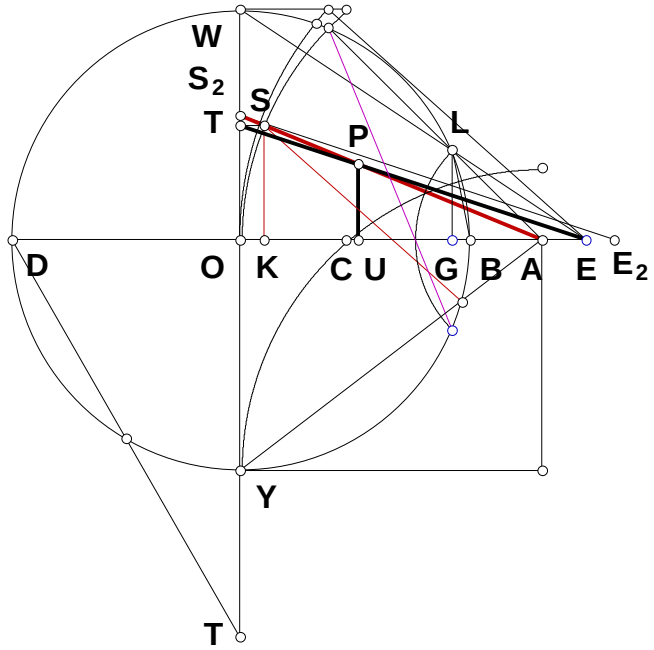
$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OE_2=0$$

$$(1+N)^2\cdot\frac{AB}{\sqrt{3+10\cdot N+3\cdot N^2}}-AS_2=0$$

$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-\frac{1}{2}\cdot AB\cdot(1+N)-AE_2=0$$

$$-AB\cdot N\cdot\frac{\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-AE=0$$

$$\frac{-2\cdot AB\cdot N\cdot(1+N)\cdot\left[N^2+6\cdot N-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AP=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$



$$\frac{-AB\cdot N\cdot\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AU=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UO=0$$

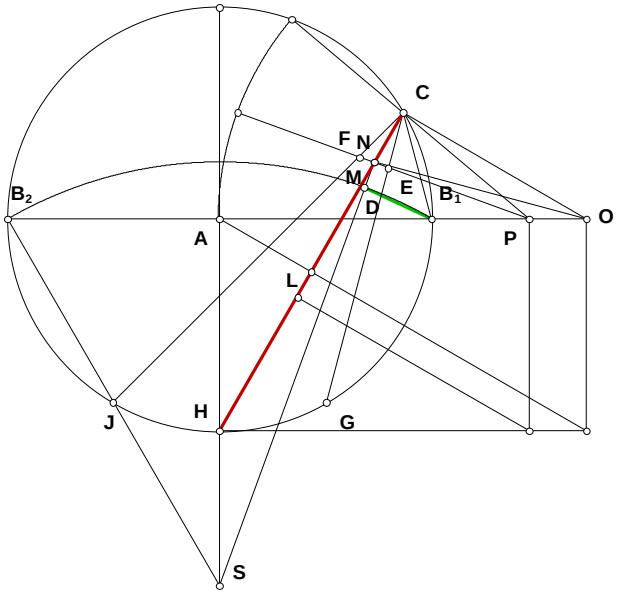
$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-PU=0$$

Compass Construction 043001

$m\angle AB_1C = 75.000°$ $m\angle AB_2C = 15.000°$

$m\angle AB_1D = 25.000°$ $m\angle AB_2D = 5.000°$

$\frac{m\angle AB_1C}{m\angle AB_1D} = 3.000$ $\frac{m\angle AB_2C}{m\angle AB_2D} = 3.000$



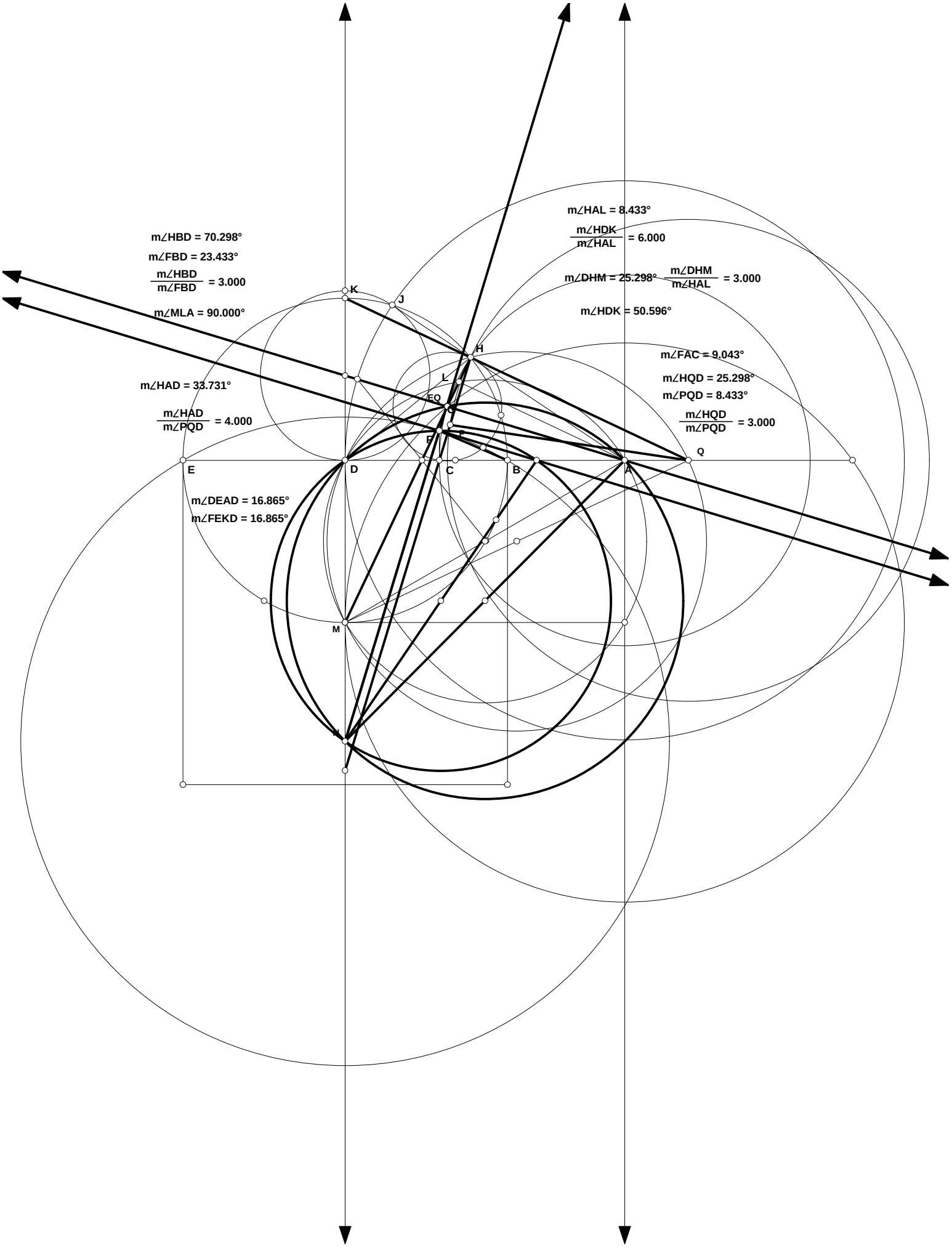
$m\angle ABC - 3 \cdot m\angle ABD = 0.000°$

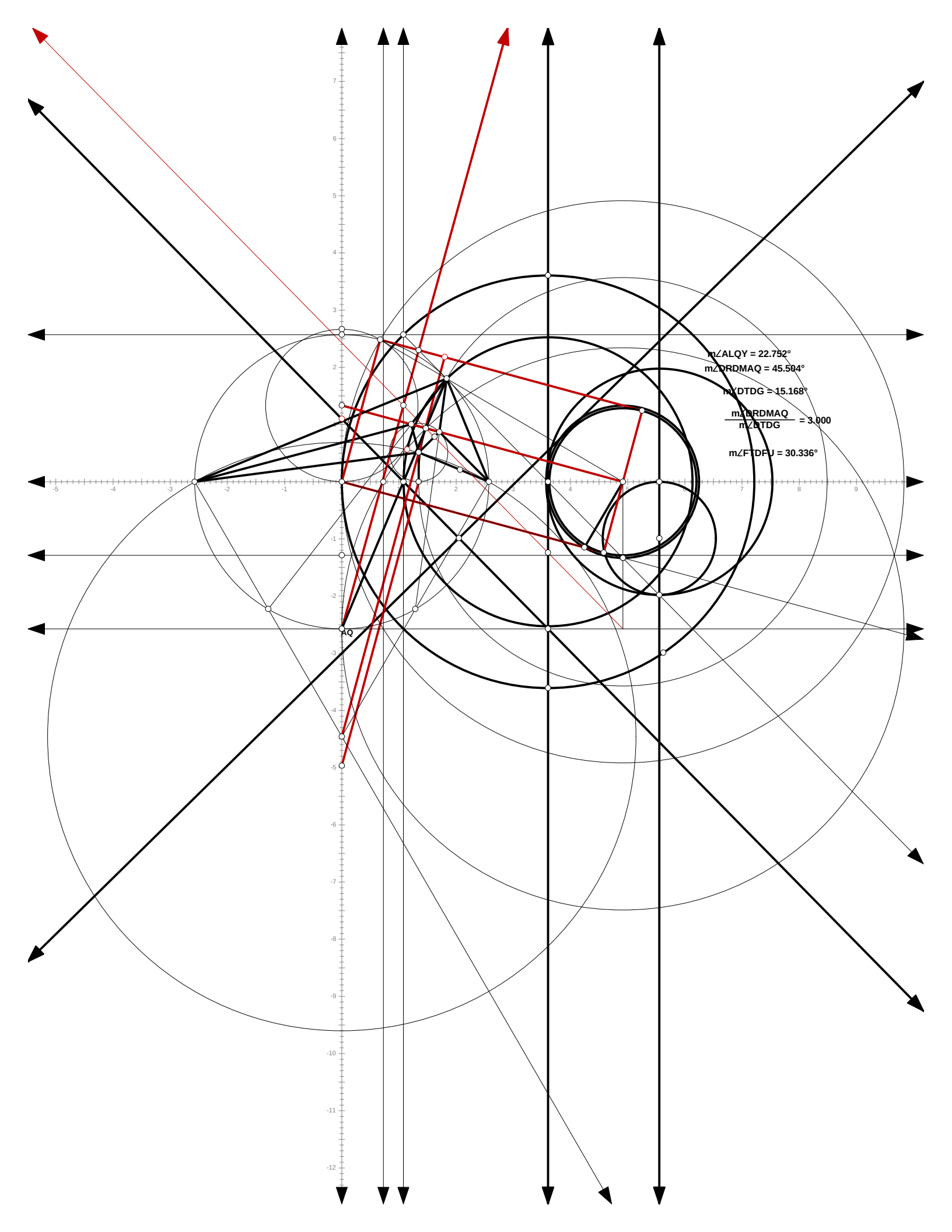
$m\angle AB_2C - 3 \cdot m\angle AB_2D = -0.000°$

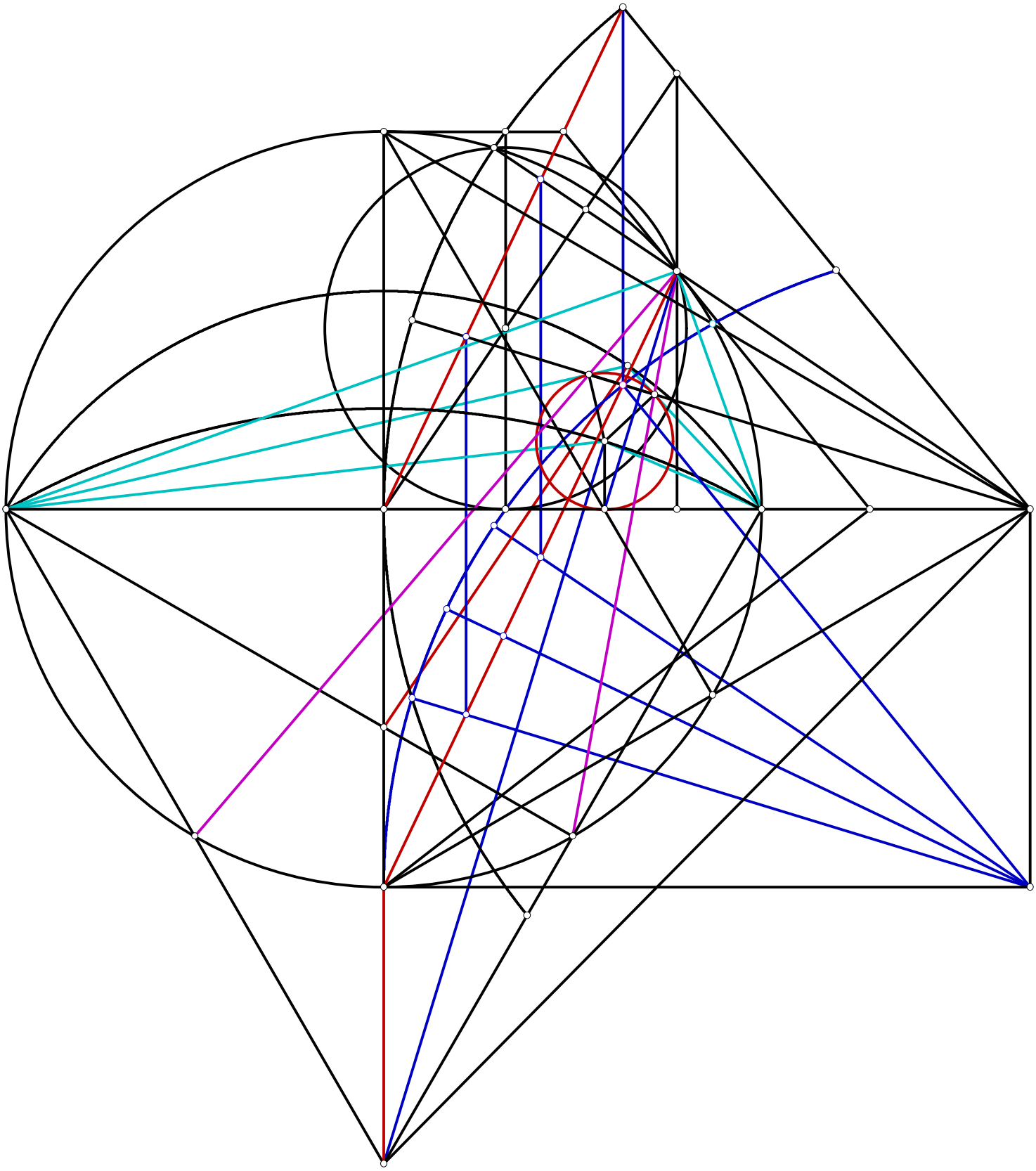
Move C->60 | Move C->75

The process is simple, construct point O and then bisect AOC. From point N construct P from the perpendicular L-which is the midpoint of NS.
Bisect APC constructing E and F which are bisected at point M. Draw S to M to construct point D.

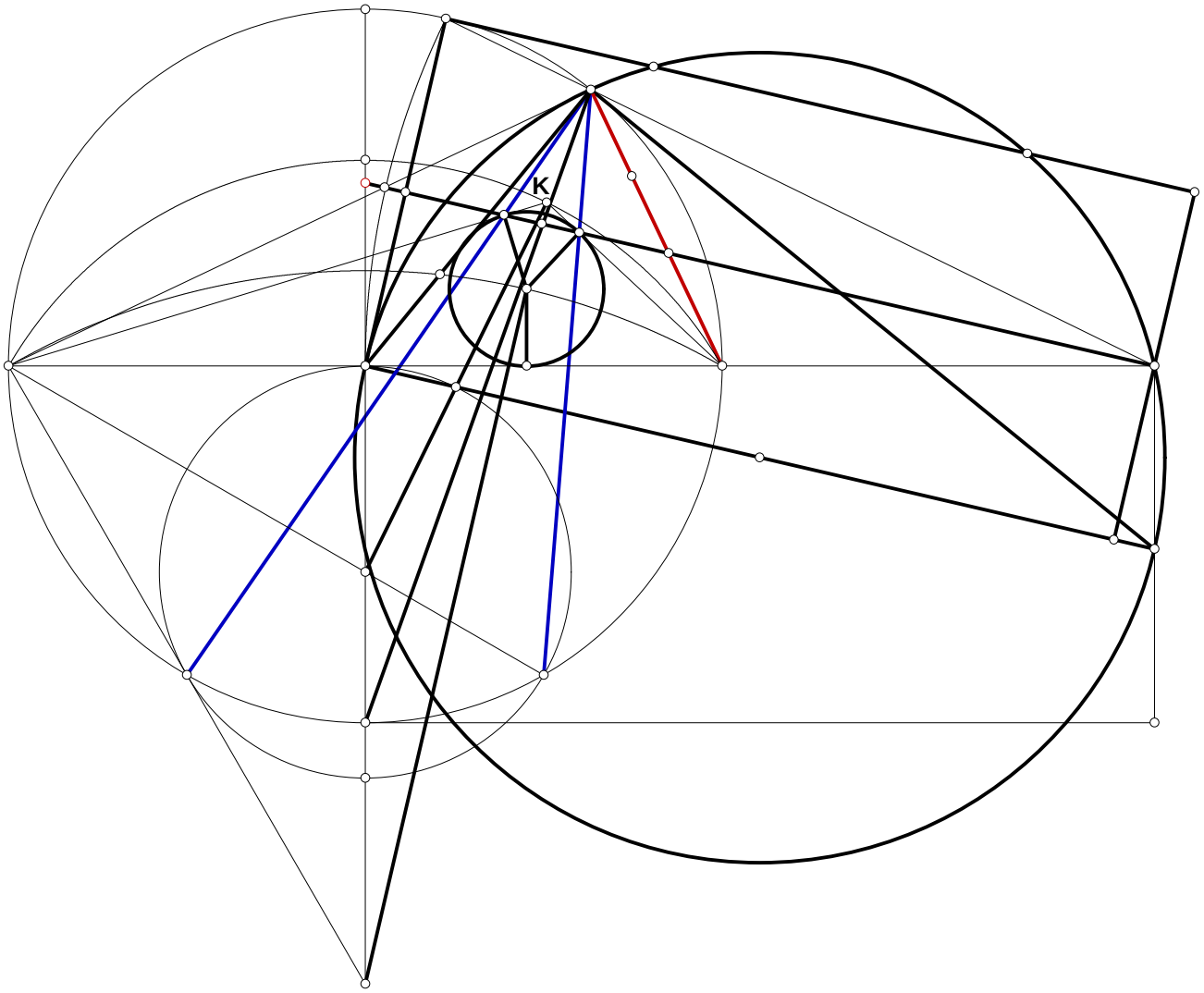
When there is no difference between two things, those things are said to be equal. This is true even of tolerance. If a figure is more accurate than the tools used to construct the figure, then that figure is equal, in that tool, to a solution that is tool independant.

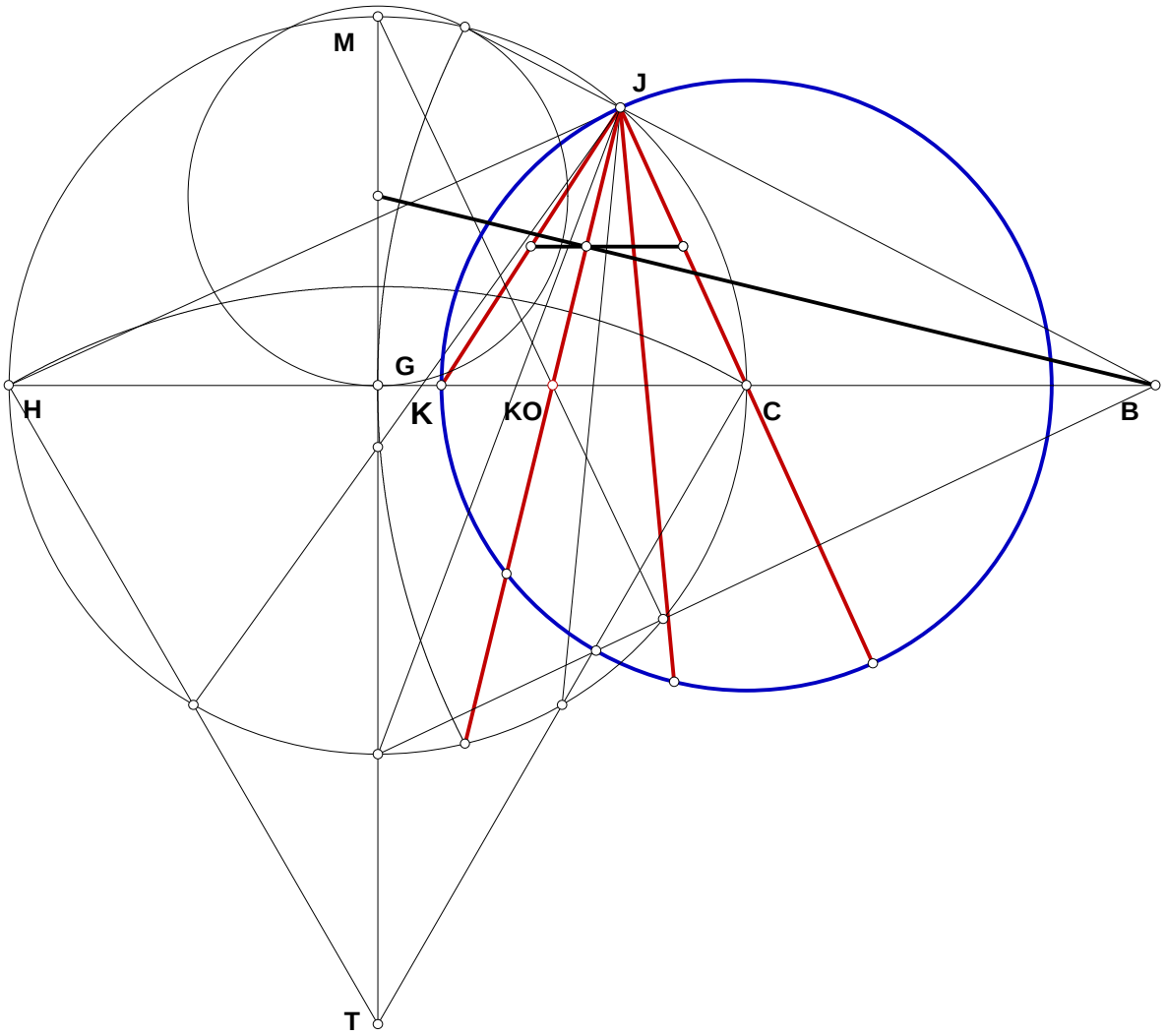

m∠HAL = 8.433°
m∠HDK / m∠HAL = 6.000
m∠HBD = 70.298°
m∠FBD = 23.433°
m∠HBD / m∠FBD = 3.000
m∠DHM = 25.298°
m∠DHM / m∠HAL = 3.000
m∠MLA = 90.000°
m∠HDK = 50.596°
m∠FAC = 9.043°
m∠HAD = 33.731°
m∠HQD = 25.298°
m∠PQD = 8.433°
m∠HAD / m∠PQD = 4.000
m∠HQD / m∠PQD = 3.000
m∠DEAD = 16.865°
m∠FEKD = 16.865°
K
J
H
L
EQ
G
F
I
Q
E
D
C
B
A
M

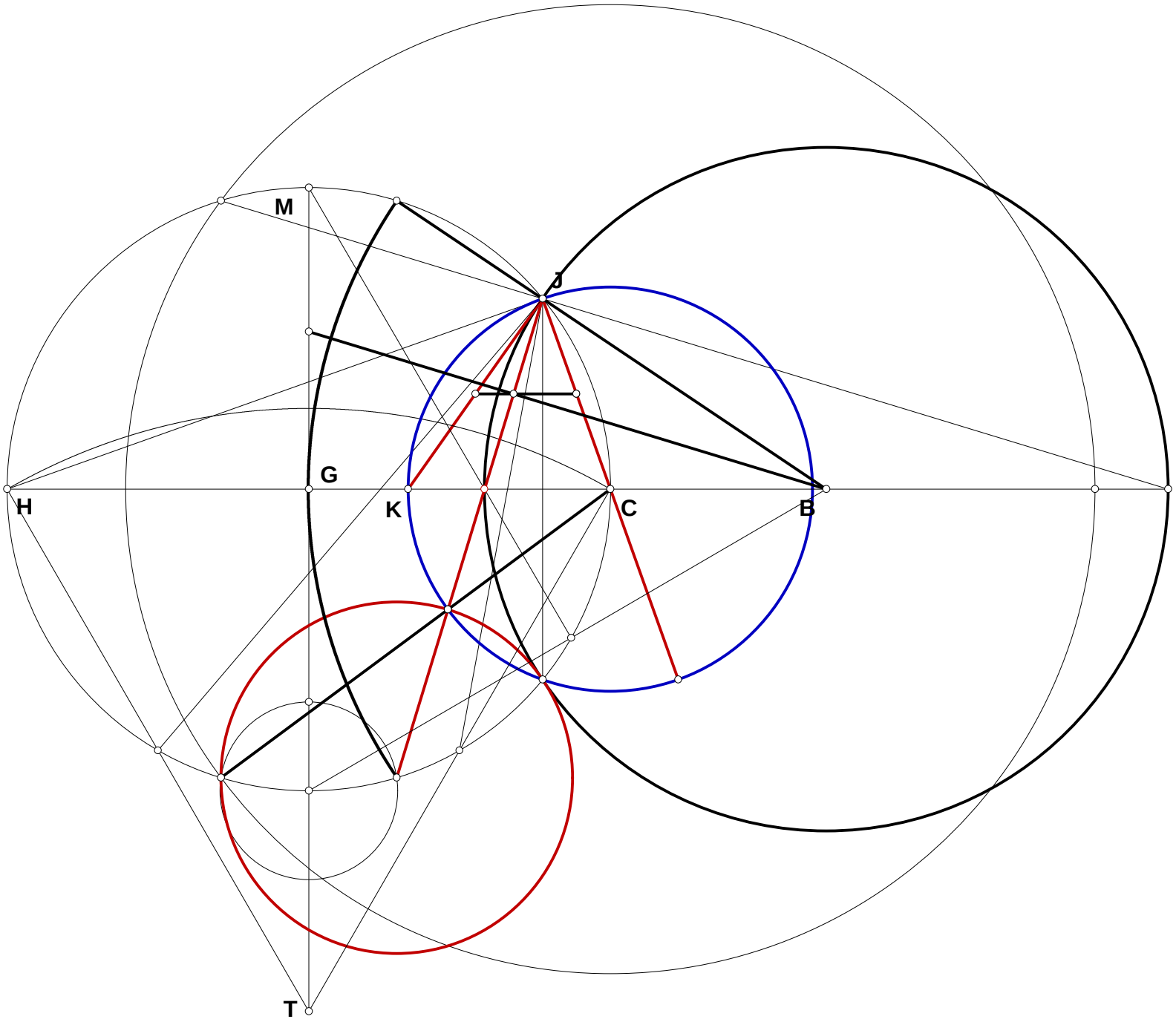

m∠ALQY = 22.752°
m∠DRDMAQ = 45.504°
m∠DTDG = 15.168°
m∠DRDMAQ / m∠DTDG = 3.000
m∠FTDFU = 30.336°
AQ

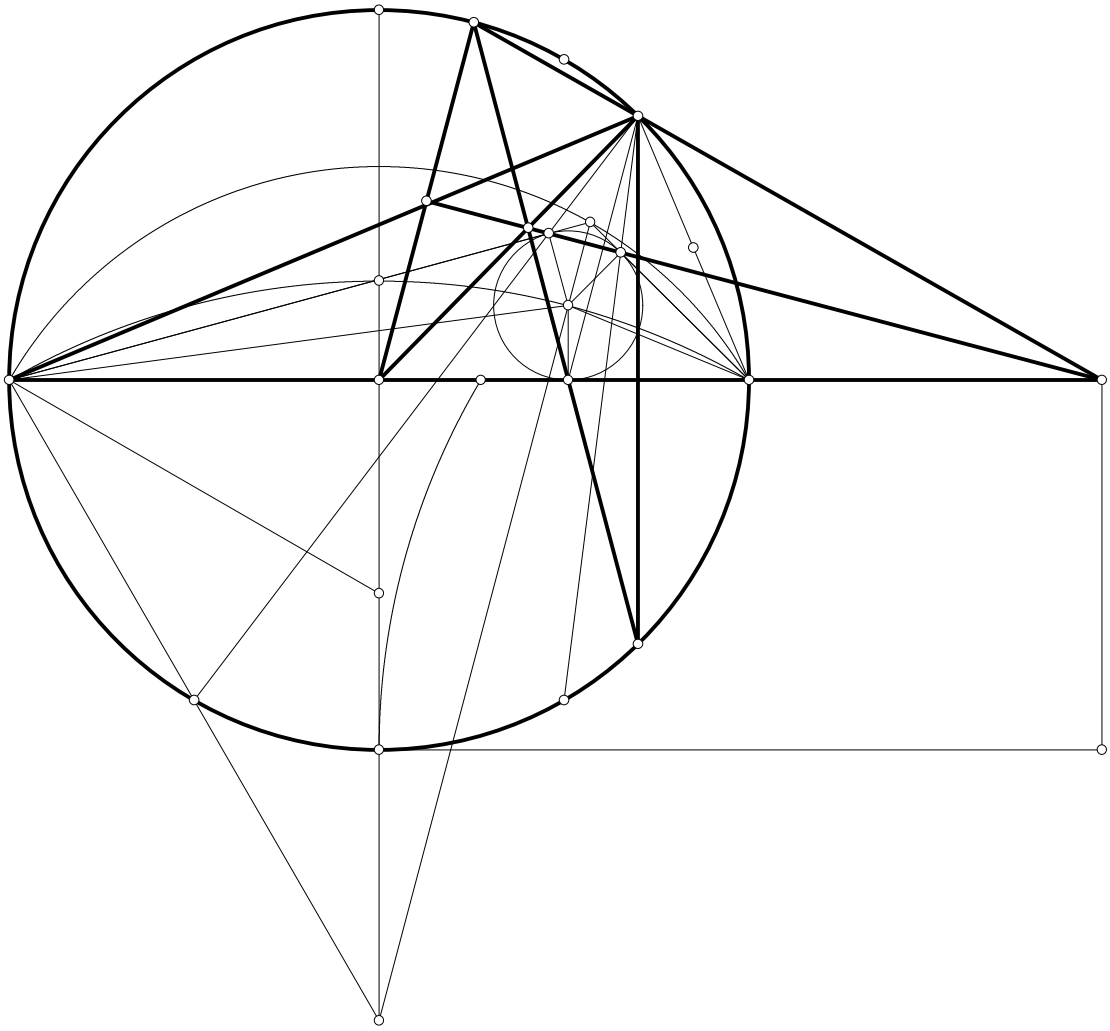


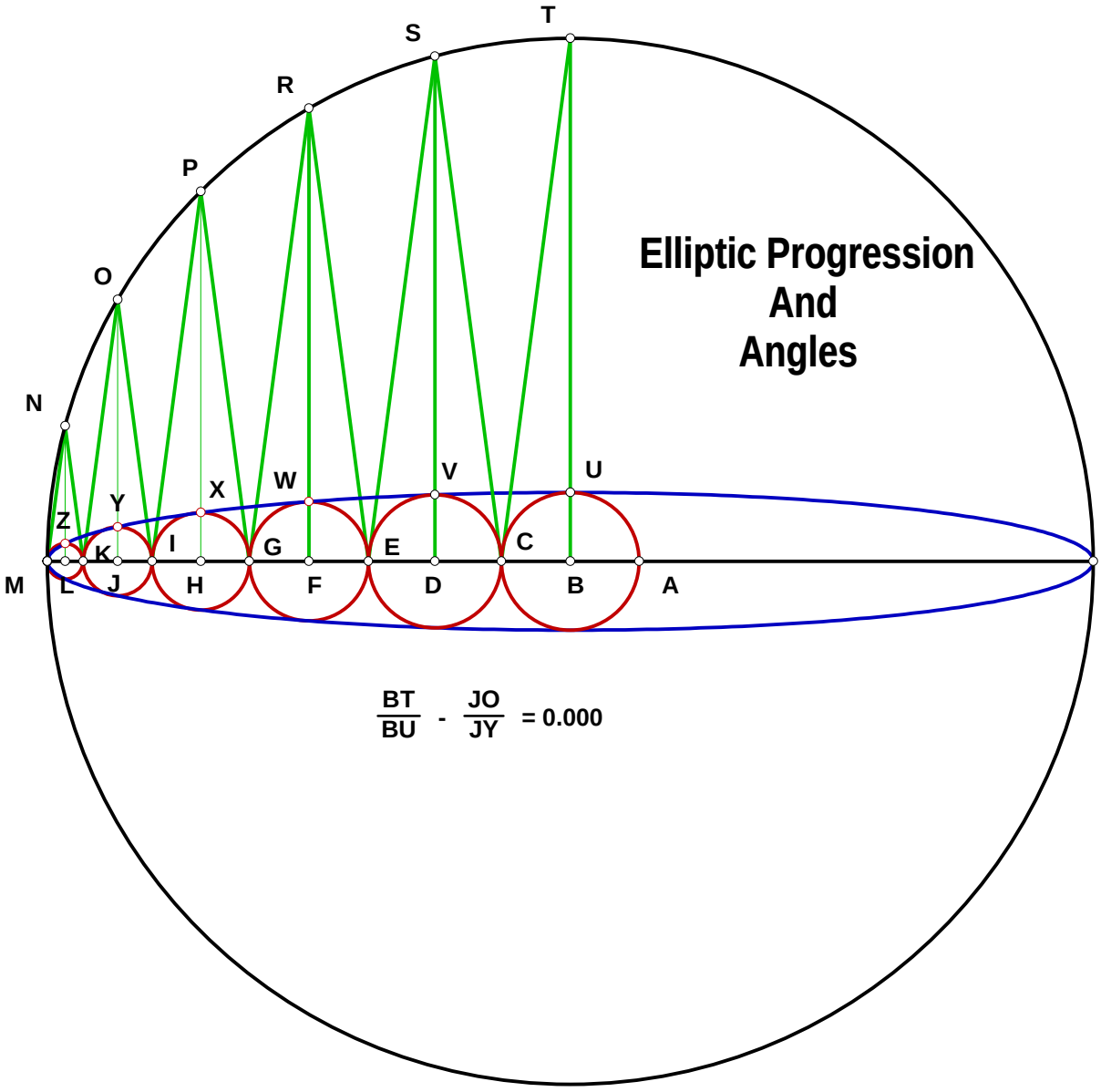
Pull K.

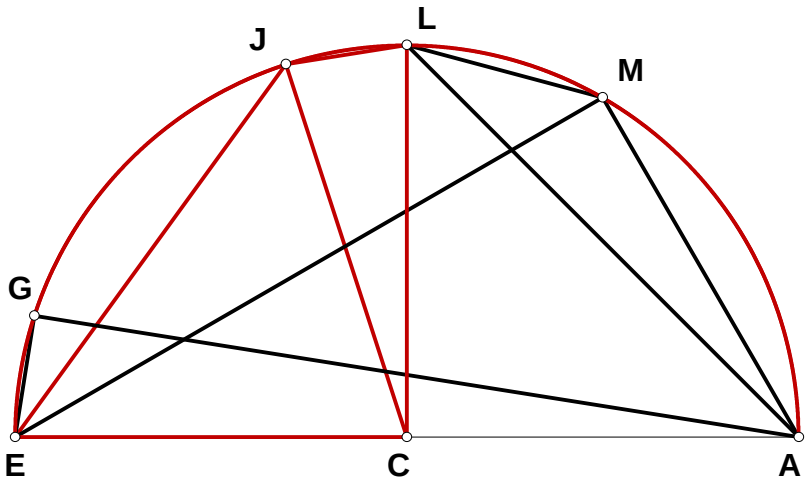



M
J
G
H
K
KO
C
B
T


M
J
G
H
K
C
B
T




T
S
R
P
O
N
M
L
K
J
I
H
G
F
E
D
C
B
A
Z
Y
X
W
V
U
Elliptic Progression
And
Angles
BT/BU - JO/JY = 0.000

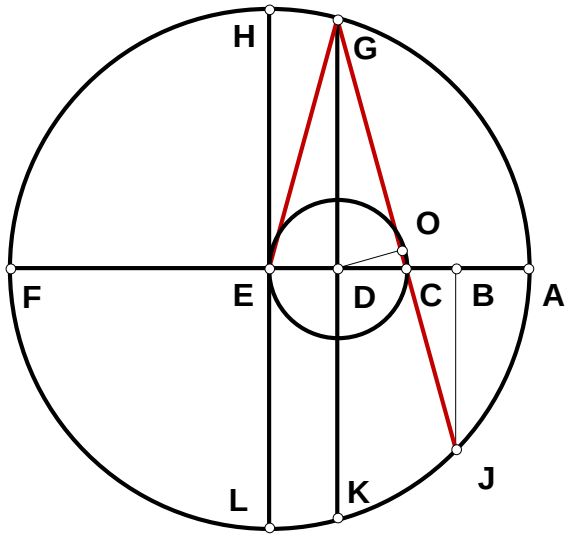


$$EM := \frac{1}{2}\cdot\sqrt{3} \qquad AL := \frac{1}{2}\cdot\sqrt{2} \qquad JL := \frac{1}{4}\cdot\sqrt{8 - 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}} \qquad LM := \frac{1}{4}\cdot\sqrt{6} - \frac{1}{4}\cdot\sqrt{2}$$

$$AM := \frac{1}{2} \qquad EJ := \frac{1}{4}\cdot\sqrt{10 - 2\cdot\sqrt{5}}$$

$$AG := \frac{1}{4}\cdot\sqrt{8 + 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$$

Elliptic Progression Outtake One 0507011



A method of trisection Algebraically.

$N := M$

$N \geq 4 = 1$ $\quad AF := 6$ $\quad AE := \frac{AF}{2}$

$DE := \frac{AF}{N}$ $\quad AD := AE - DE$ $\quad DF := AF - AD$

$DG := \sqrt{AD \cdot DF}$ $\quad CD := DE$ $\quad EG := AE$

$CO := \frac{CD^2}{EG}$ $\quad CG := EG$ $\quad CJ := CG - 4 \cdot CO$

$BC := \frac{CD \cdot CJ}{CG}$ $\quad AB := AE - (2 \cdot DE + BC)$

$BJ := \frac{DG \cdot BC}{CD}$ $\quad BD := BC + CD$

$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2}$ $\quad \frac{JK}{2 \cdot DE} = 1$ **Some Algebraic Names,**

Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.

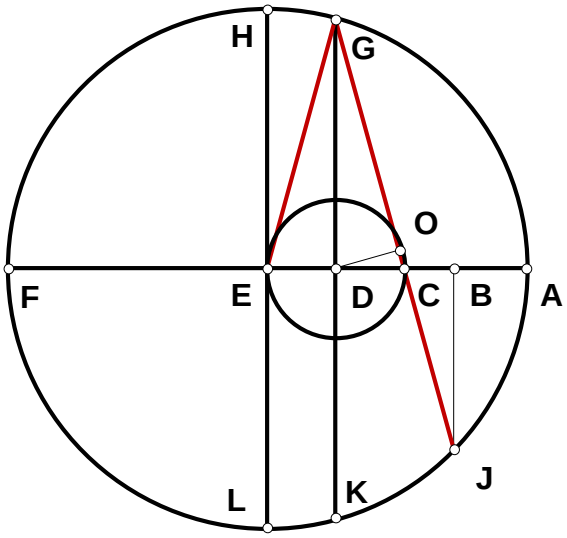
$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0$ $\quad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0$ $\quad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$

$\frac{2\ AF}{N^2} - CO = 0$ $\quad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0$ $\quad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$

$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$ **One of the meanings of trisection is solving for the following equation when given AF and AB.**

$$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0$$

$AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0$ $\quad AF \cdot \frac{2 \cdot (N^2 - 8)}{N^3} - BD = 0$ $\quad \frac{2 \cdot AF}{N} - JK = 0$



$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - \frac{AF}{AB} = 0 \qquad \frac{AF}{AB} = 6$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - 6 = 0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6$$

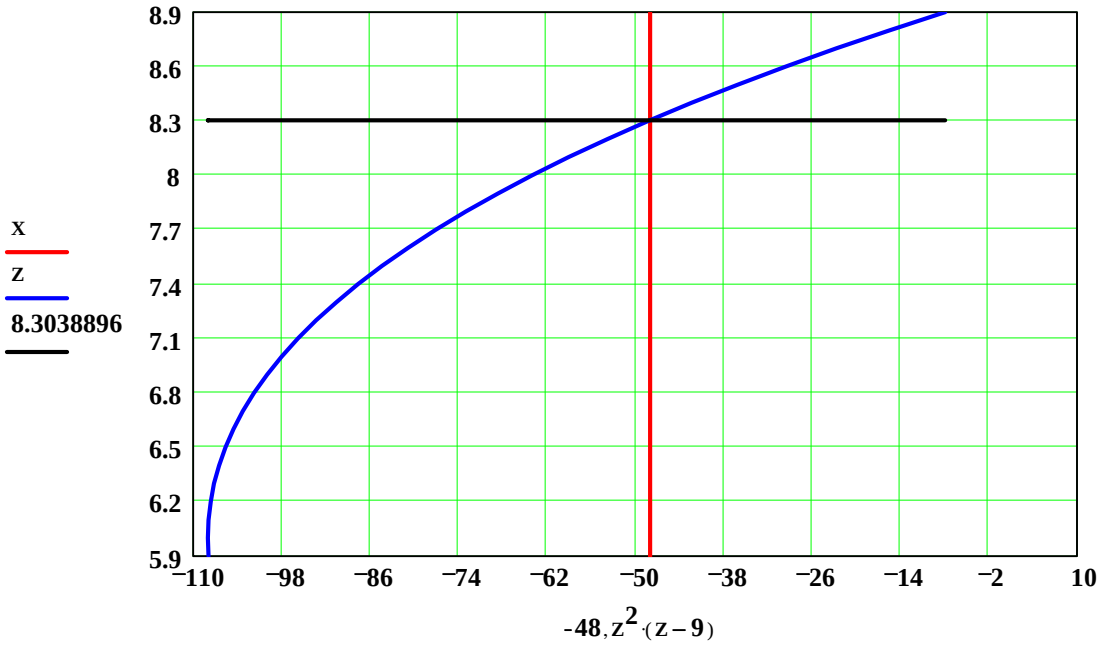
$$2\cdot M^3 - 6\cdot\left[(M+2)\cdot(M-4)^2\right] = 0$$

$$2\cdot M^3 - \left(6\cdot M^3 - 36\cdot M^2 + 192\right) = 0$$

$$4\cdot M^3 - 36\cdot M^2 + 192 = 0$$

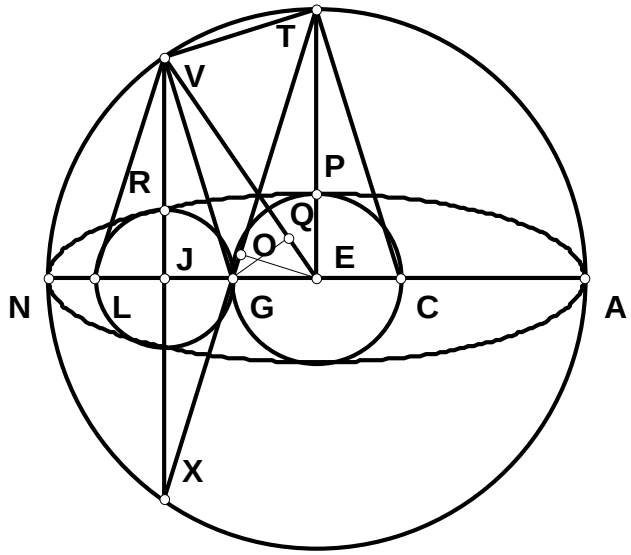
$$M^3 - 9\cdot M^2 + 48 = 0 \qquad M^2\cdot(M-9) + 48 = 0 \qquad M \equiv 8.303889634816388$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6 \qquad DE - \frac{AF}{M} = 0 \qquad Z := 5.9, 6 .. 8.9$$



Elliptic Progression Outtake Two 0507012

Angles TEV and EVJ equals CTG.



$N := 6.381 \quad AN := 2.792$

$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$

$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$

$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$

$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$

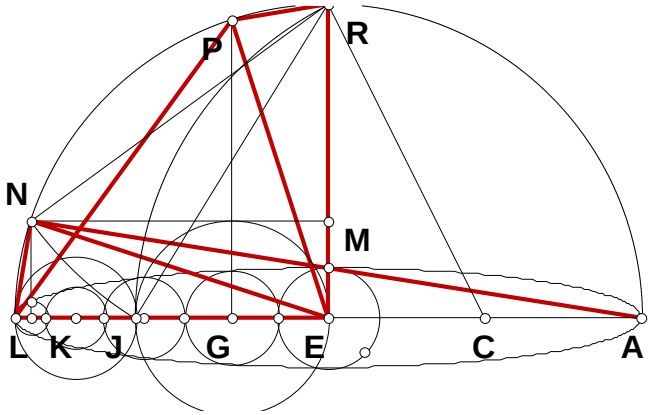
$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \qquad JV := JX$

$EJ := EG + GJ$

$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$

$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \qquad EQ := \frac{EJ \cdot EG}{EV} \quad OQ := \frac{JV \cdot EQ}{EJ} \quad OQ - GJ = 0$

Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013



$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2}$

$CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$

$CJ := CR \qquad EJ := CJ - CE$

$JR := \sqrt{EJ^2 + ER^2} \quad NR := JR$

$EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2} \qquad GL := EL - EG \quad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$

$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$

Irrational **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

$\frac{1}{2} - AE = 0 \qquad \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$

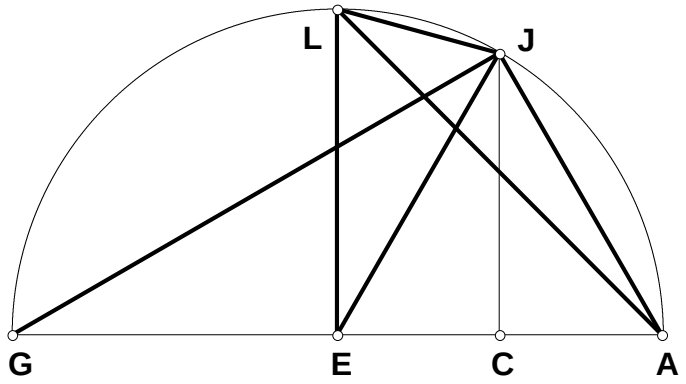
$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$

$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$

$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$

$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$

Outtake Four: Some Names 0507014



$AG := 1 \quad AE := \frac{AG}{2} \quad AC := \frac{AE}{2}$

$CG := AG - AC \quad CJ := \sqrt{AC \cdot CG}$

$EL := AE \quad CE := AC$

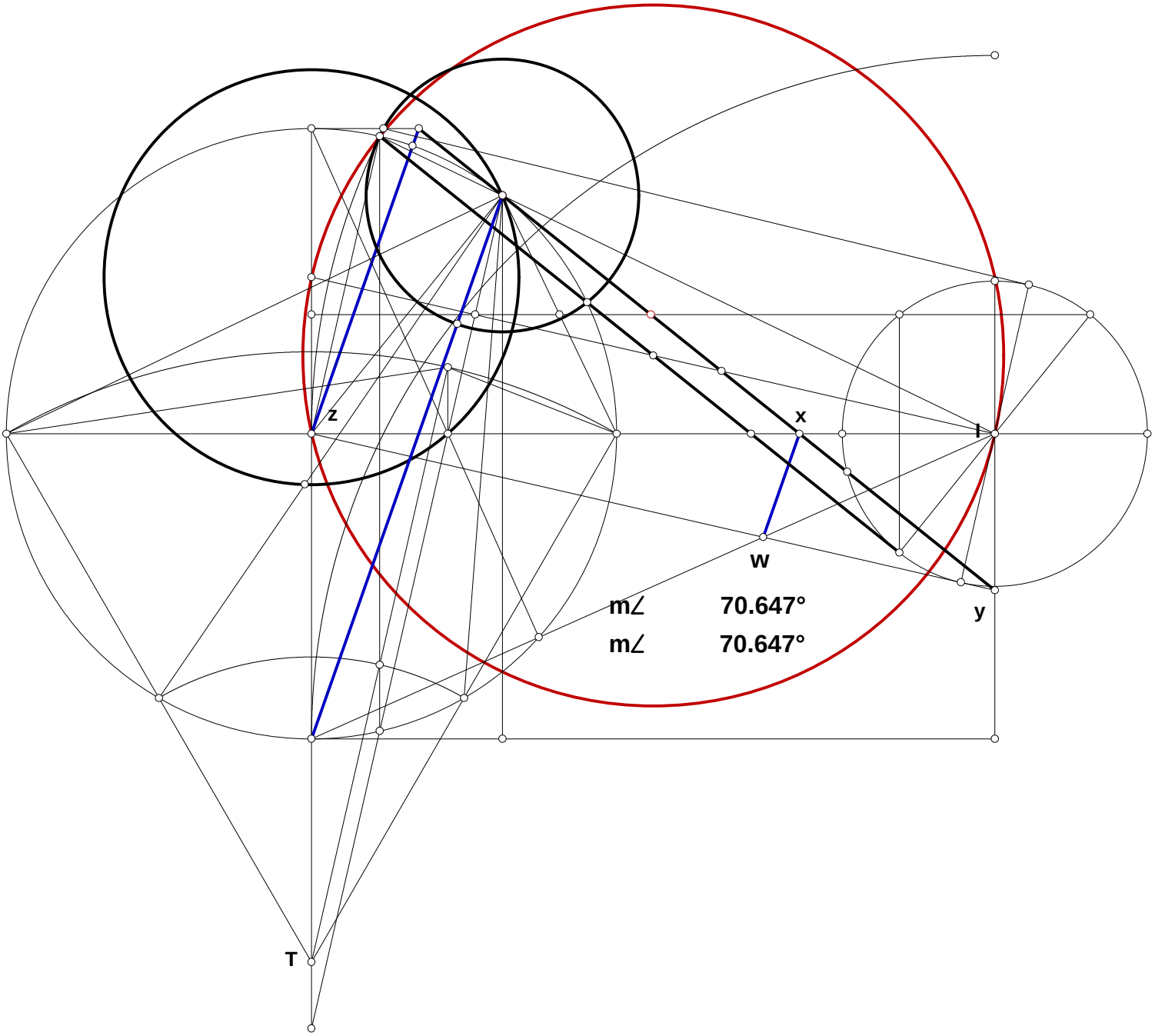
$JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

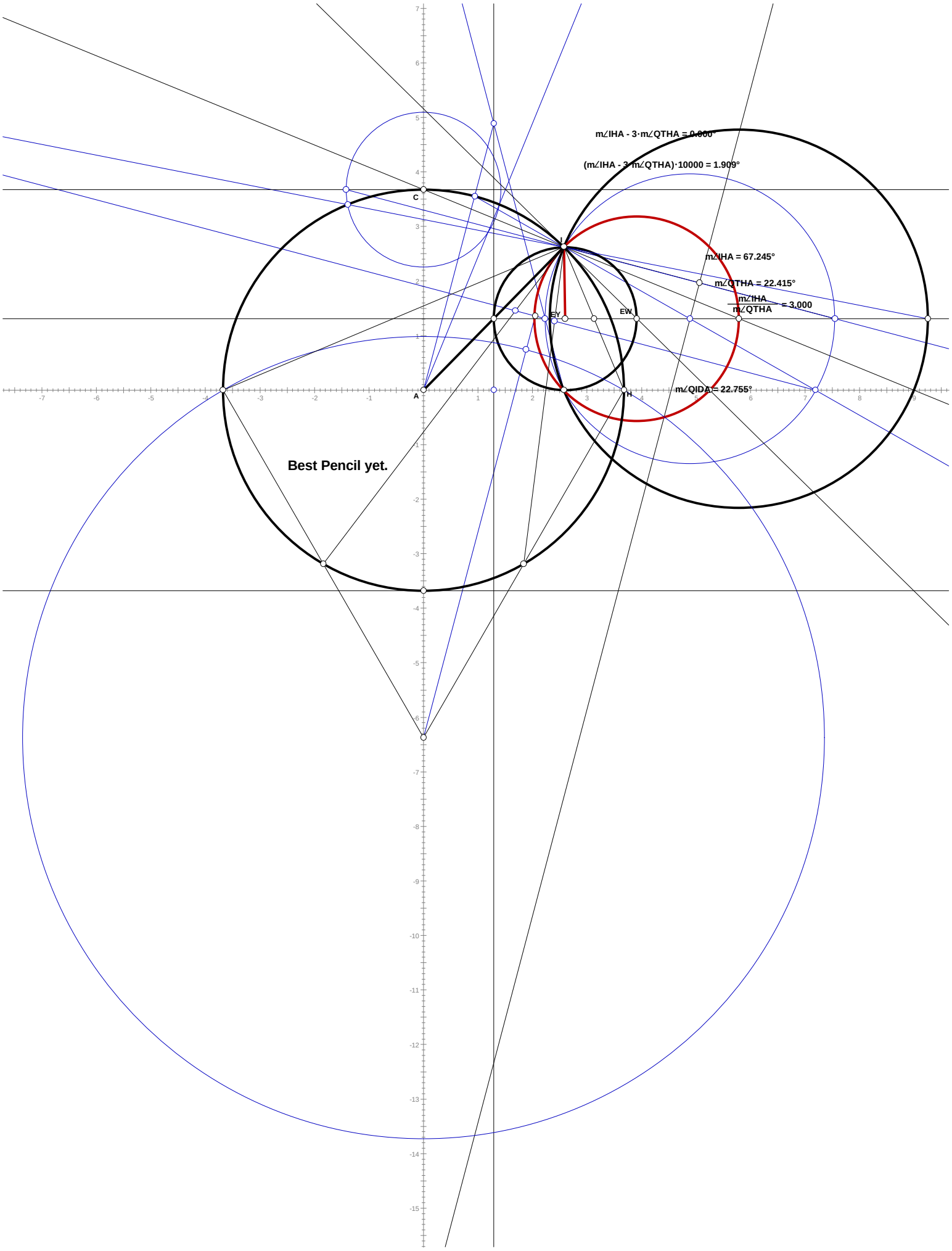
$AJ := \sqrt{AC^2 + CJ^2} \quad GJ := \sqrt{CG^2 + CJ^2}$

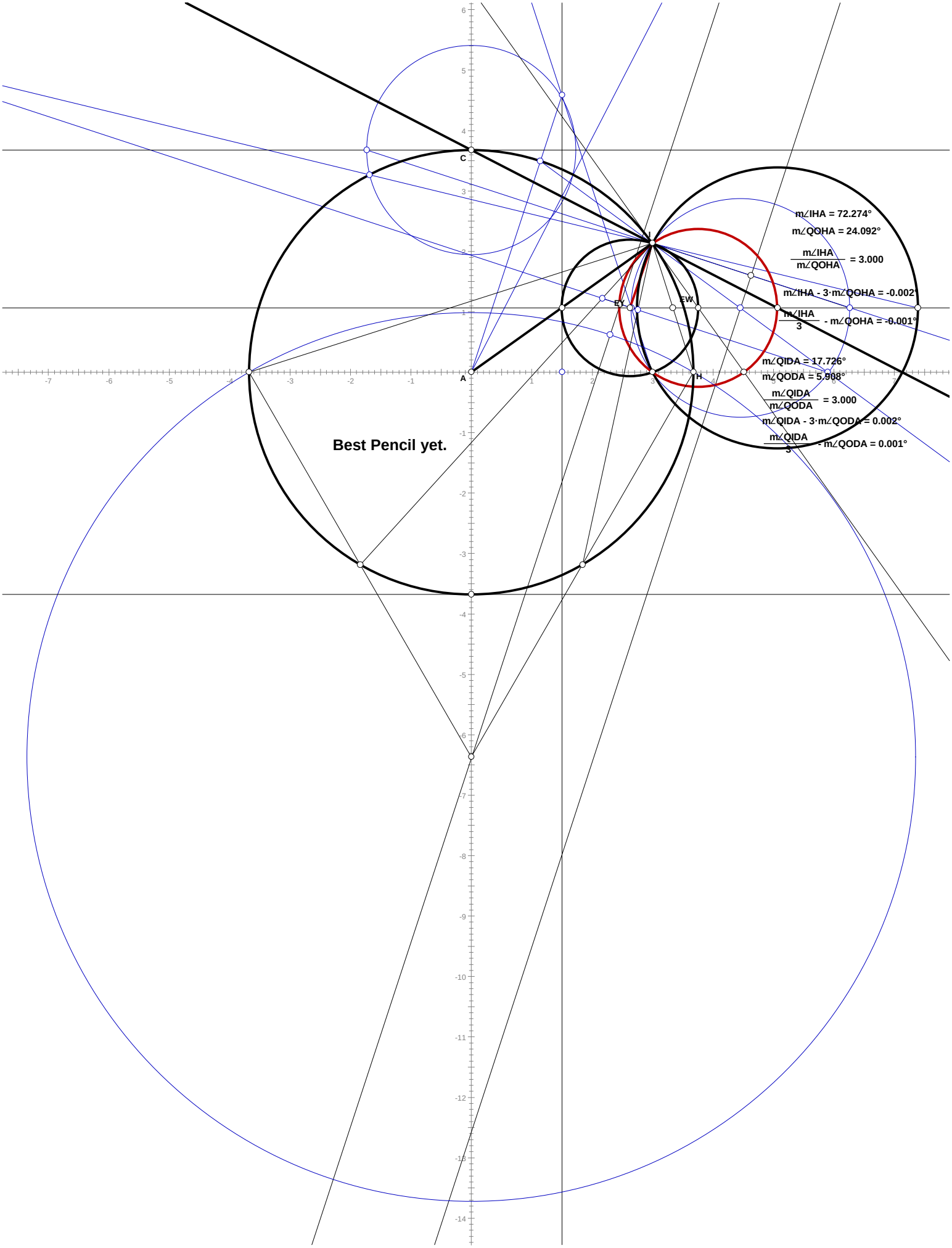
$AL := \sqrt{AE^2 + EL^2}$

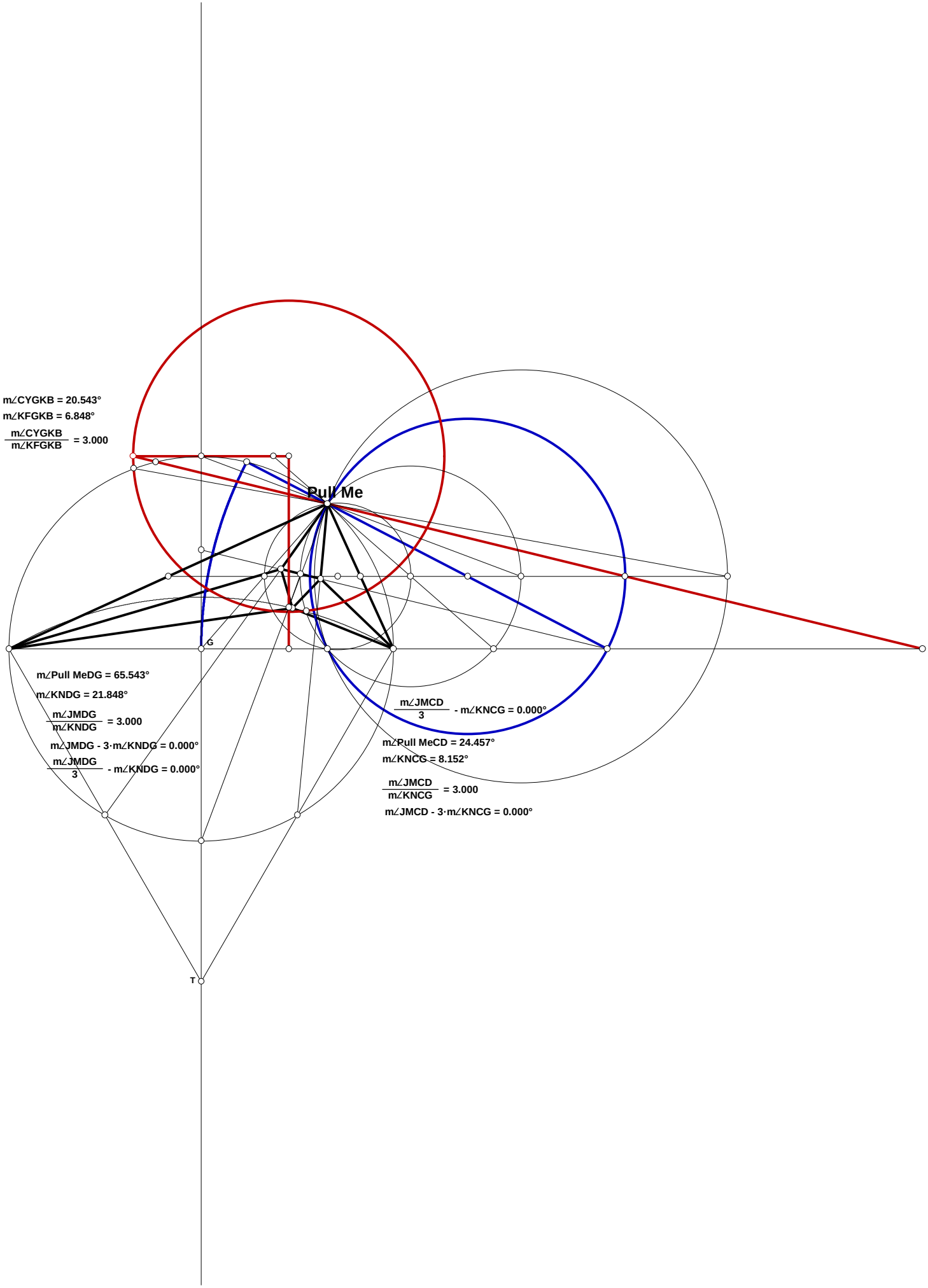
$AE := \frac{1}{2} \quad AC := \frac{1}{4} \quad CG := 1 - \frac{1}{4} \quad CJ := \frac{1}{4} \cdot \sqrt{3} \quad JL := \frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}$

$AJ := \frac{1}{2} \quad GJ := \frac{1}{2} \cdot \sqrt{3}_{*} \quad AL := \frac{1}{2} \cdot \sqrt{2}$


z
x
l
w
y
T
m∠ 70.647°
m∠ 70.647°

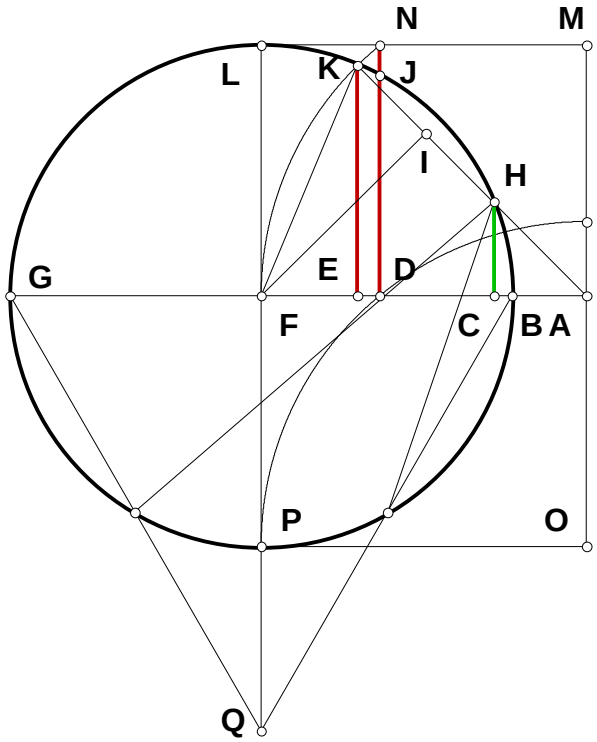

m∠IHA - 3·m∠QTHA = 0.000°
(m∠IHA - 3·m∠QTHA)·10000 = 1.909°
C
I
m∠IHA = 67.245°
m∠QTHA = 22.415°
m∠IHA / m∠QTHA = 3.000
EY
EW
H
A
m∠QIDA = 22.755°
Best Pencil yet.


C
I
EW
H
A
m∠IHA = 72.274°
m∠QOHA = 24.092°
m∠IHA / m∠QOHA = 3.000
m∠IHA - 3·m∠QOHA = -0.002°
m∠IHA / 3 - m∠QOHA = -0.001°
m∠QIDA = 17.726°
m∠QODA = 5.908°
m∠QIDA / m∠QODA = 3.000
m∠QIDA - 3·m∠QODA = 0.002°
m∠QIDA / 3 - m∠QODA = 0.001°
Best Pencil yet.


m∠CYGKB = 20.543°
m∠KFGKB = 6.848°
m∠CYGKB / m∠KFGKB = 3.000
Pull Me
G
T
m∠Pull MeDG = 65.543°
m∠KNDG = 21.848°
m∠JMDG / m∠KNDG = 3.000
m∠JMDG - 3·m∠KNDG = 0.000°
m∠JMDG / 3 - m∠KNDG = 0.000°
m∠JMCD / 3 - m∠KNCG = 0.000°
m∠Pull MeCD = 24.457°
m∠KNCG = 8.152°
m∠JMCD / m∠KNCG = 3.000
m∠JMCD - 3·m∠KNCG = 0.000°

On Trisection 051301

For any given trisection what is the Algebraic names of BC and BE taking BG as unit?



$N := 9 \qquad BG := 1 \qquad AG := BG \cdot N$

$AB := AG - BG \qquad BF := \frac{BG}{2}$

$AF := AB + BF \qquad AN := AF \qquad AK := AN$

$FK := BF \qquad S_1 := AF \qquad S_2 := AK \qquad S_3 := FK$

$AE := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad AI := AE$

$IK := AK - AI \qquad HI := IK \qquad AH := AK - (HI + IK)$

$AC := \frac{AE \cdot AH}{AK} \qquad BC := AC - AB \qquad BE := AE - AB$

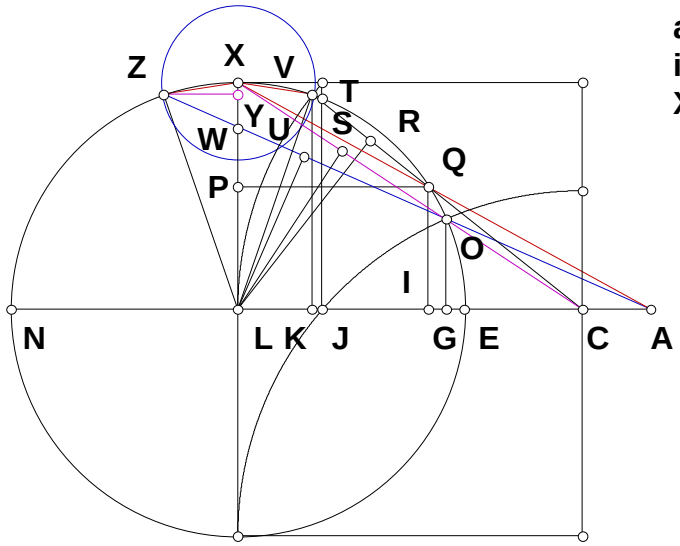
Some Algebraic Names:

$N - 1 - AB = 0 \qquad \frac{1}{2} - BF = 0 \qquad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0 \qquad \frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0$

$\frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0 \qquad 2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \qquad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \qquad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

On Trisection 051401.MCD



For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?

$N := 5 \quad CE := 1 \quad CN := CE \cdot N$

$EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$CL := CE + EL$

$CV := CL \quad S_1 := CL \quad S_2 := CV$

$S_3 := LV \quad CK := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CR := CK \quad RV := CV - CR \quad QR := RV \quad CQ := CV - (QR + RV) \quad CI := \frac{CK \cdot CQ}{CV} \quad EI := CI - CE$

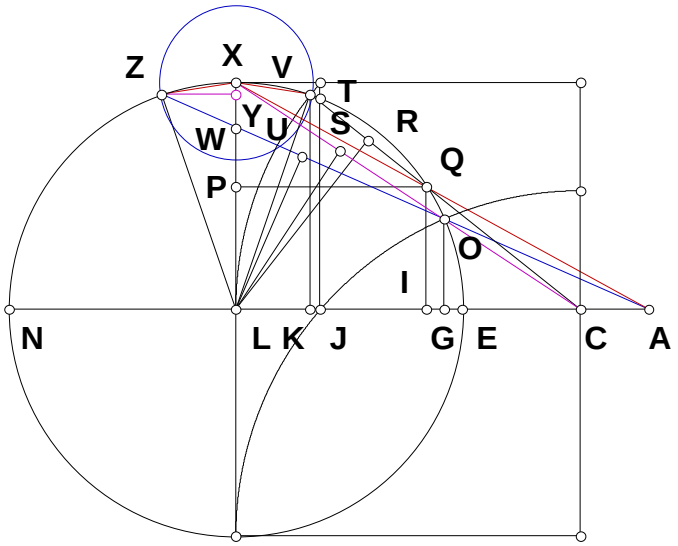
$IN := EN - EI \quad IQ := \sqrt{EI \cdot IN} \quad LP := IQ \quad LX := EL \quad IL := EL - EI \quad PX := LX - LP \quad AL := \frac{IL \cdot LX}{PX}$

$AE := AL - EL \quad AC := AE - CE \quad AI := AC + CI \quad AX := \sqrt{AL^2 + LX^2} \quad AQ := \frac{AX \cdot AI}{AL} \quad QX := AX - AQ$

$CX := \sqrt{CL^2 + LX^2} \quad CS := \frac{CL^2}{CX} \quad SX := CX - CS \quad OS := SX \quad CO := CX - (SX + OS)$

$GO := \frac{LX \cdot CO}{CX} \quad CG := \frac{CL \cdot CO}{CX} \quad EG := CG - CE \quad AG := AE + EG \quad AO := \sqrt{AG^2 + GO^2}$

$AU := \frac{AG \cdot AL}{AO} \quad OU := AU - AO \quad UZ := OU \quad AZ := AO + (OU + UZ)$



$$LW := \frac{GO \cdot AL}{AG} \qquad AW := \frac{AO \cdot AL}{AG}$$

$$WZ := AZ - AW \qquad WY := \frac{GO \cdot WZ}{AO}$$

$$YZ := \frac{AG \cdot WZ}{AO} \qquad YX := LX - (LW + WY)$$

$$XZ := \sqrt{YX^2 + YZ^2}$$

Some Algebraic Names:

$$\frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{(1 + N)} - RV = 0$$

$$2 \cdot CE \cdot \frac{N}{(1 + N)} - CQ = 0 \qquad CE \cdot \frac{N^3 + 6\,N^2 + N}{(N + 1)^3} - CE - EI = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - CI = 0$$

$$CE \cdot N^2 \cdot \frac{\left(-3 + 2 \cdot N + N^2\right)}{(1 + N)^3} - IN = 0 \qquad (N - 1) \cdot N \cdot CE \cdot \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(1 + N)^3} - IQ = 0$$

$$\frac{1}{2} \cdot CE \cdot \frac{\left(2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4 + 1\right)}{(1 + N)^3} - IL = 0$$

$$\left[\frac{1}{2} \cdot CE \cdot (N - 1) \cdot \frac{\left[N^3 + 3 \cdot N^2 - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}{(1 + N)^3}\right] - PX = 0$$

$$\frac{1}{2}\cdot CE\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AL=0$$

$$CE\cdot(N-1)\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N-1\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AE=0$$

$$-CE\cdot N\cdot\frac{\left[-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AC=0$$

$$\frac{CE\cdot(N-1)^2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+4\cdot N+1\right)}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot(1+N)^3}-AI=0$$

$$\frac{1}{2}\cdot CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}-AX=0$$

$$CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}\cdot N\cdot\frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{(1+N)^3}-AQ=0$$

$$\sqrt{\frac{\left[2\cdot(1+N)^3\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}}\cdot CE\cdot(N-1)\cdot\frac{\left[\left(N^3+3\cdot N^2\right)-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}{2\cdot(1+N)^3}\ \ldots=0$$

$$\left.+3\cdot N+1\right]$$

$$\left.+3\cdot N+1\right]$$

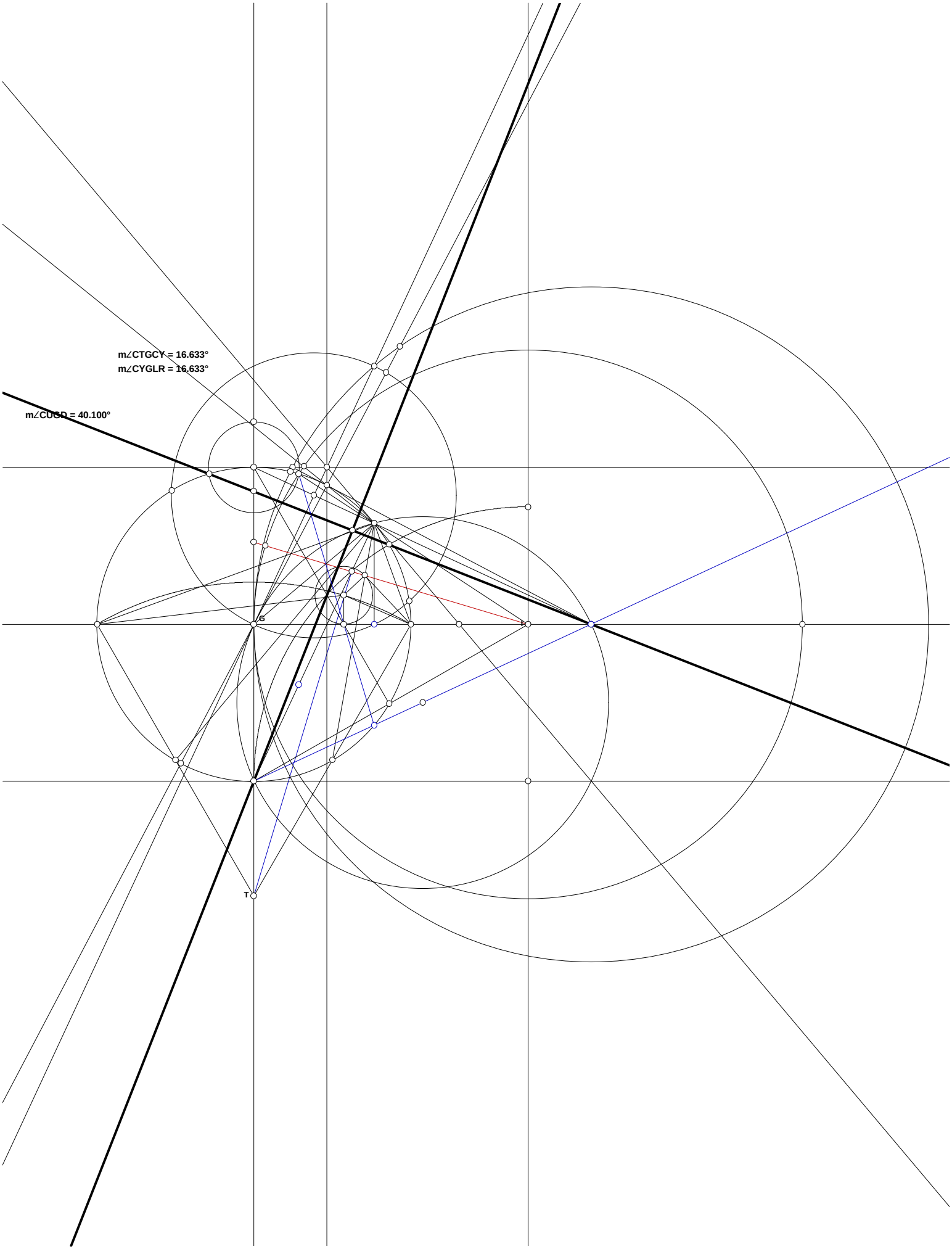
$$+-QX$$

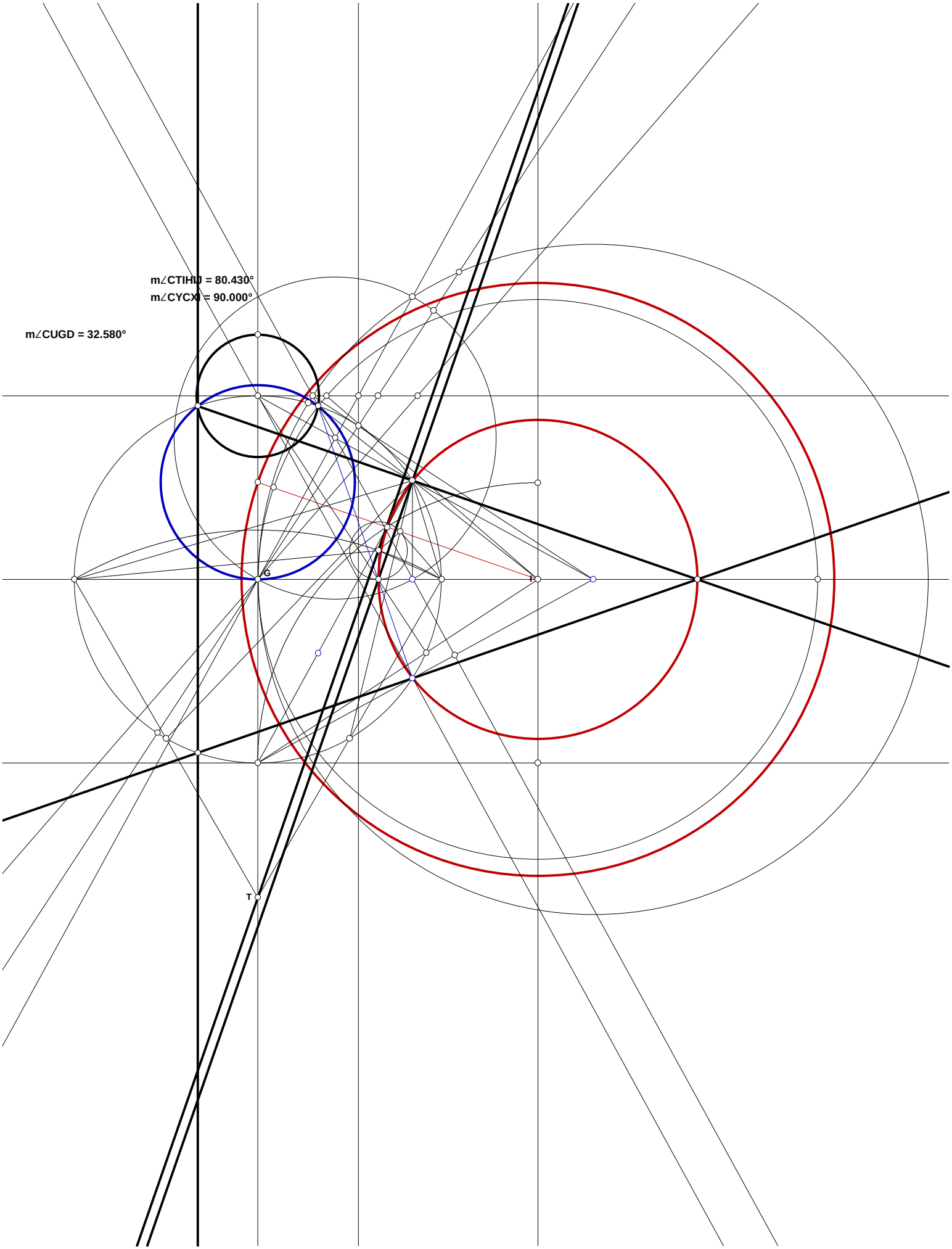
$$\frac{CE}{2}\cdot\sqrt{2}\cdot\sqrt{\left(1+N^2\right)}-CX=0 \qquad \frac{1}{4}\cdot CE\cdot(1+N)^2\cdot\frac{\sqrt{2}}{\sqrt{1+N^2}}-CS=0 \qquad \frac{1}{4}\cdot\sqrt{2}\cdot CE\cdot\frac{(N-1)^2}{\sqrt{1+N^2}}-SX=0$$

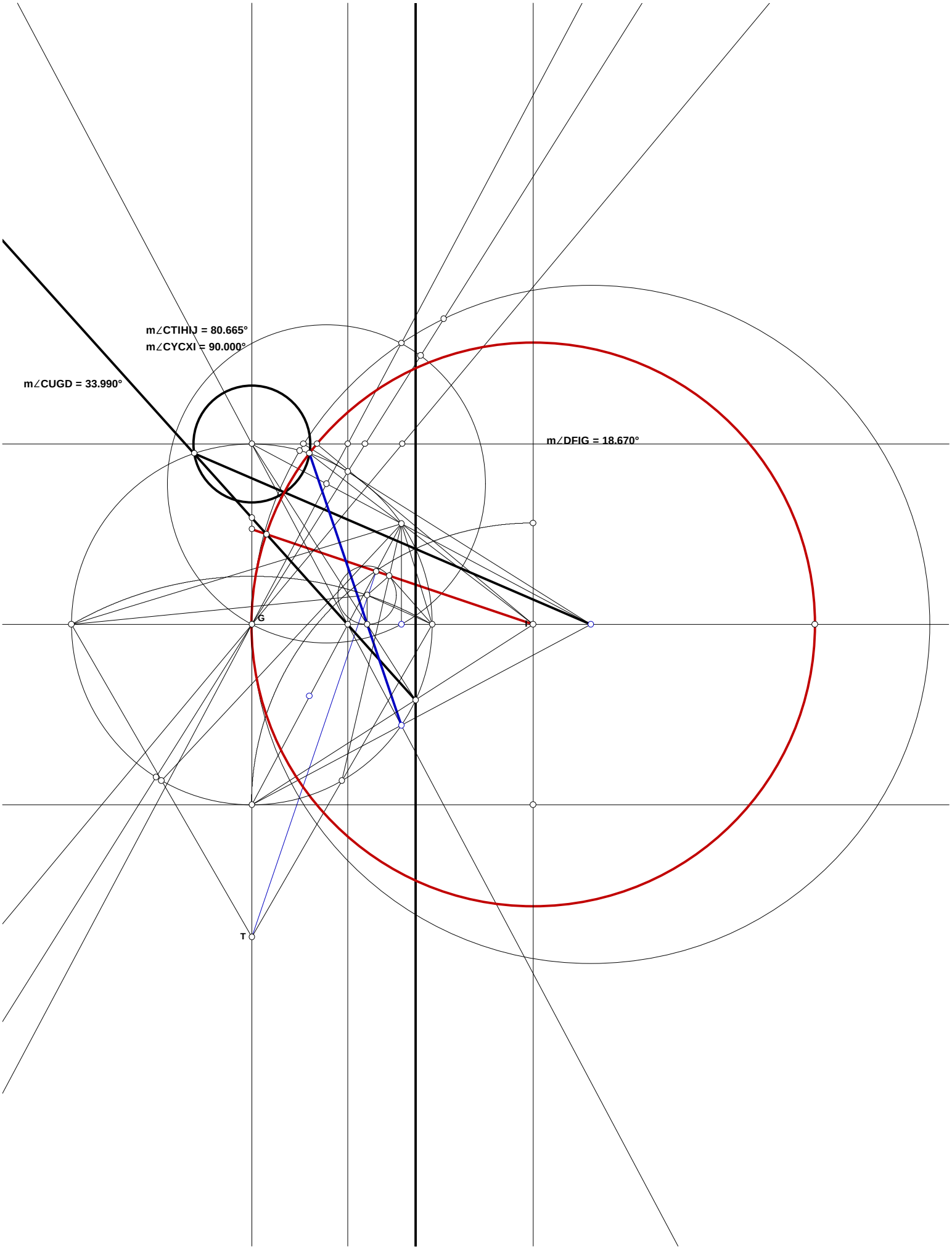
$$\frac{\sqrt{2}}{\sqrt{1+N^2}}\cdot CE\cdot N-CO=0 \qquad CE\cdot N\cdot\frac{(N-1)}{\left(1+N^2\right)}-GO=0 \qquad CE\cdot\frac{N^2+N}{\left(1+N^2\right)}-CG=0 \qquad CE\cdot\frac{(N-1)}{\left(1+N^2\right)}-EG=0$$

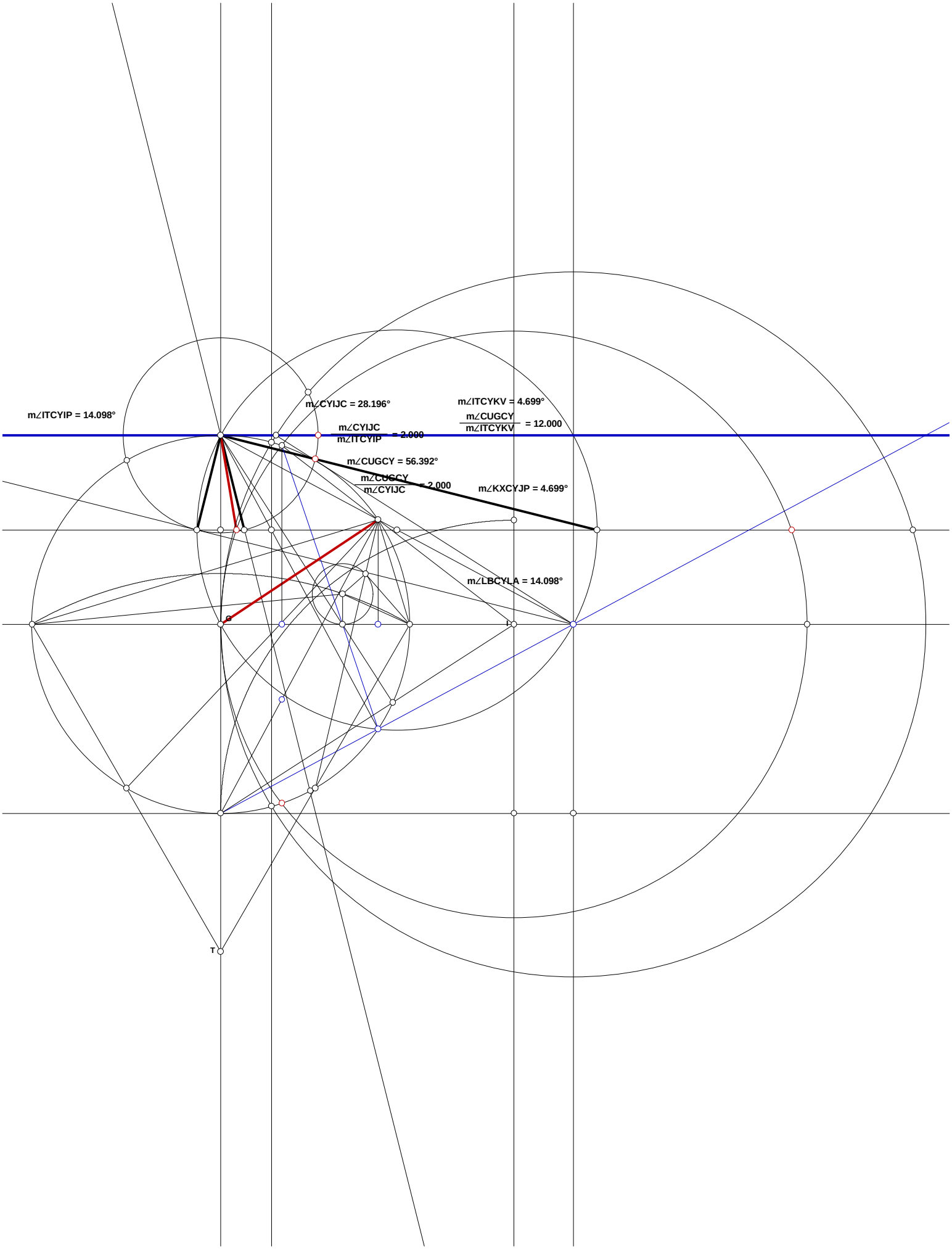
$$CE\cdot(N-1)^2\cdot N\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-2\cdot N+\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot\left(1+N^2\right)\right]}-AG=0$$

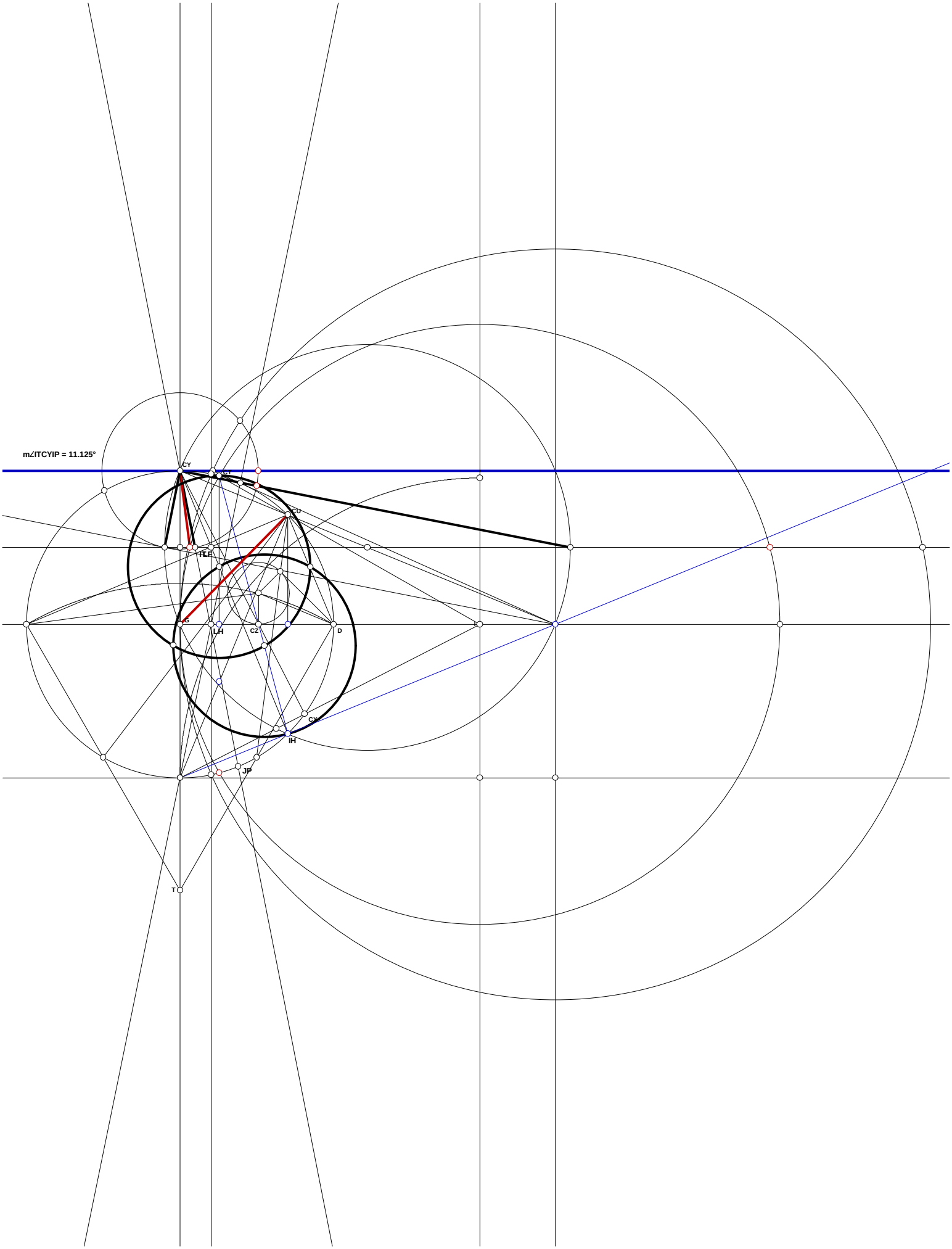
$$\frac{CE\cdot(N-1)}{2}\cdot\sqrt{\frac{\left[\begin{array}{l}\left[2\cdot N^4+14\cdot N^3-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+32\cdot N^2\right]-7\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\ +\,-7\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+14\cdot N+2-\sqrt{(N+3)\cdot(3\cdot N+1)}\end{array}\right]}{\left[(1+N)\cdot\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\right]}}-XZ=0$$

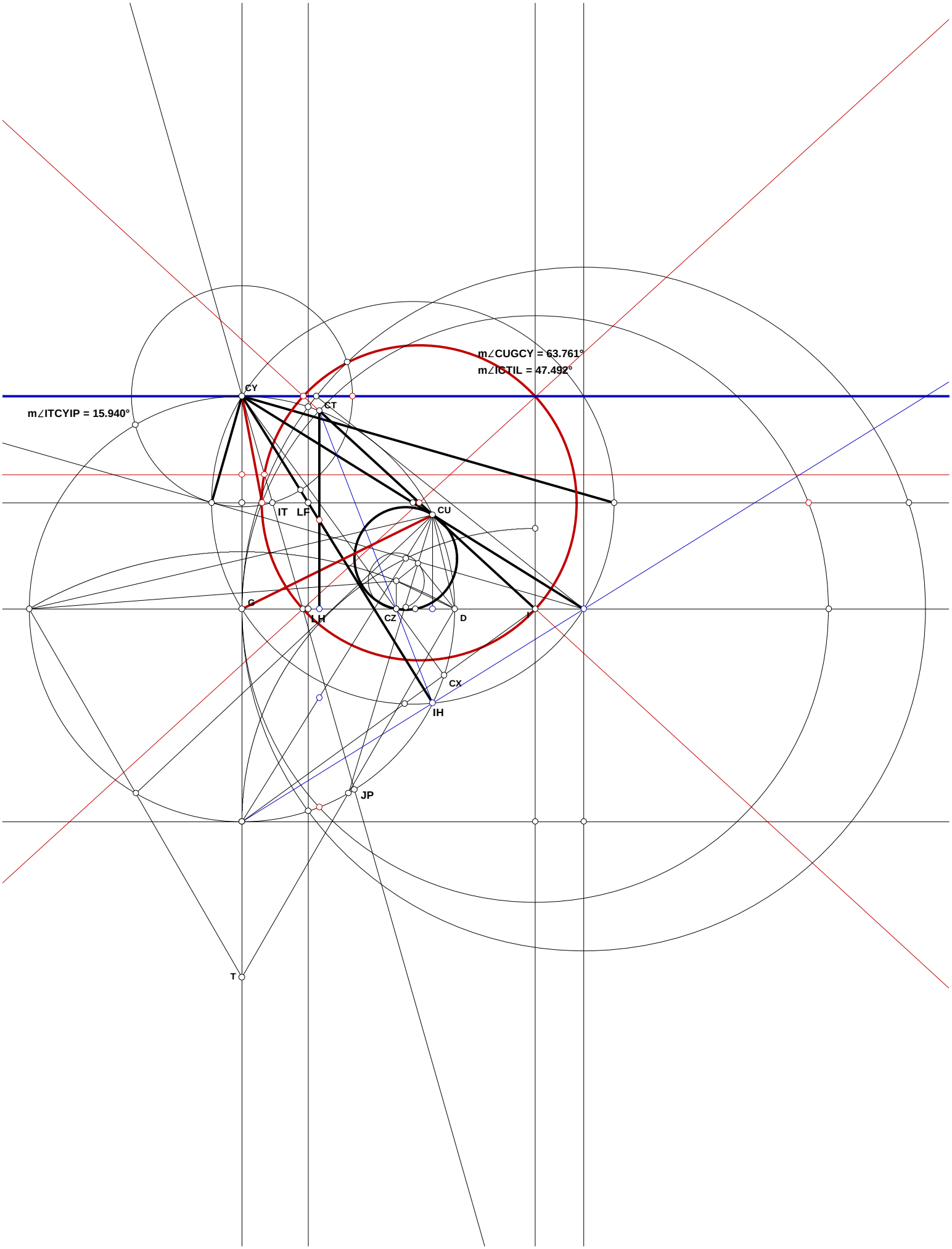

m∠CTGCY = 16.633°
m∠CYGLR = 16.633°
m∠CUGD = 40.100°
G
T

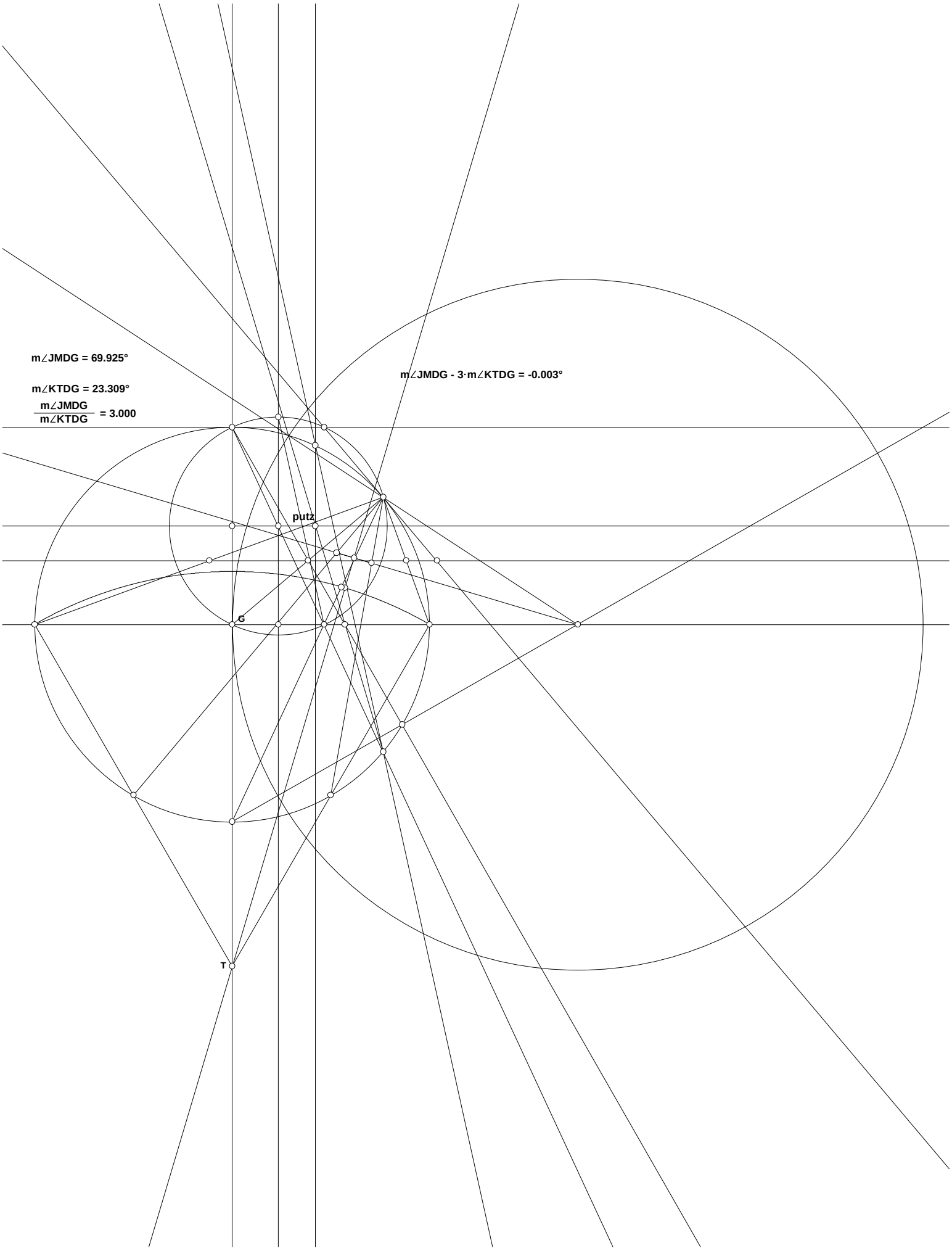

m∠CTIHIJ = 80.430°
m∠CYCX = 90.000°
m∠CUGD = 32.580°
G
I
T

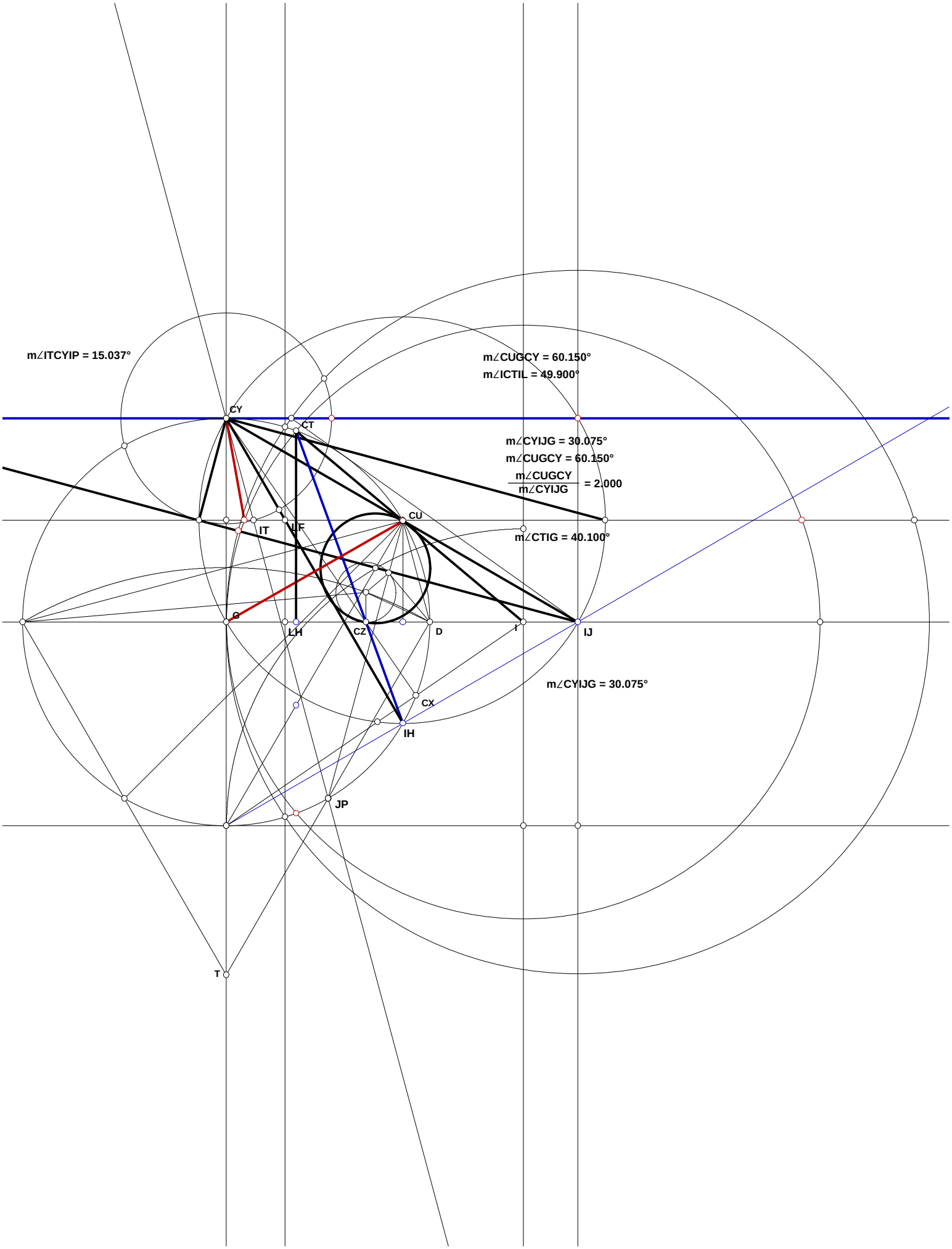

m∠CTIHIJ = 80.665°
m∠CYCXI = 90.000°
m∠CUGD = 33.990°
m∠DFIG = 18.670°
G
T

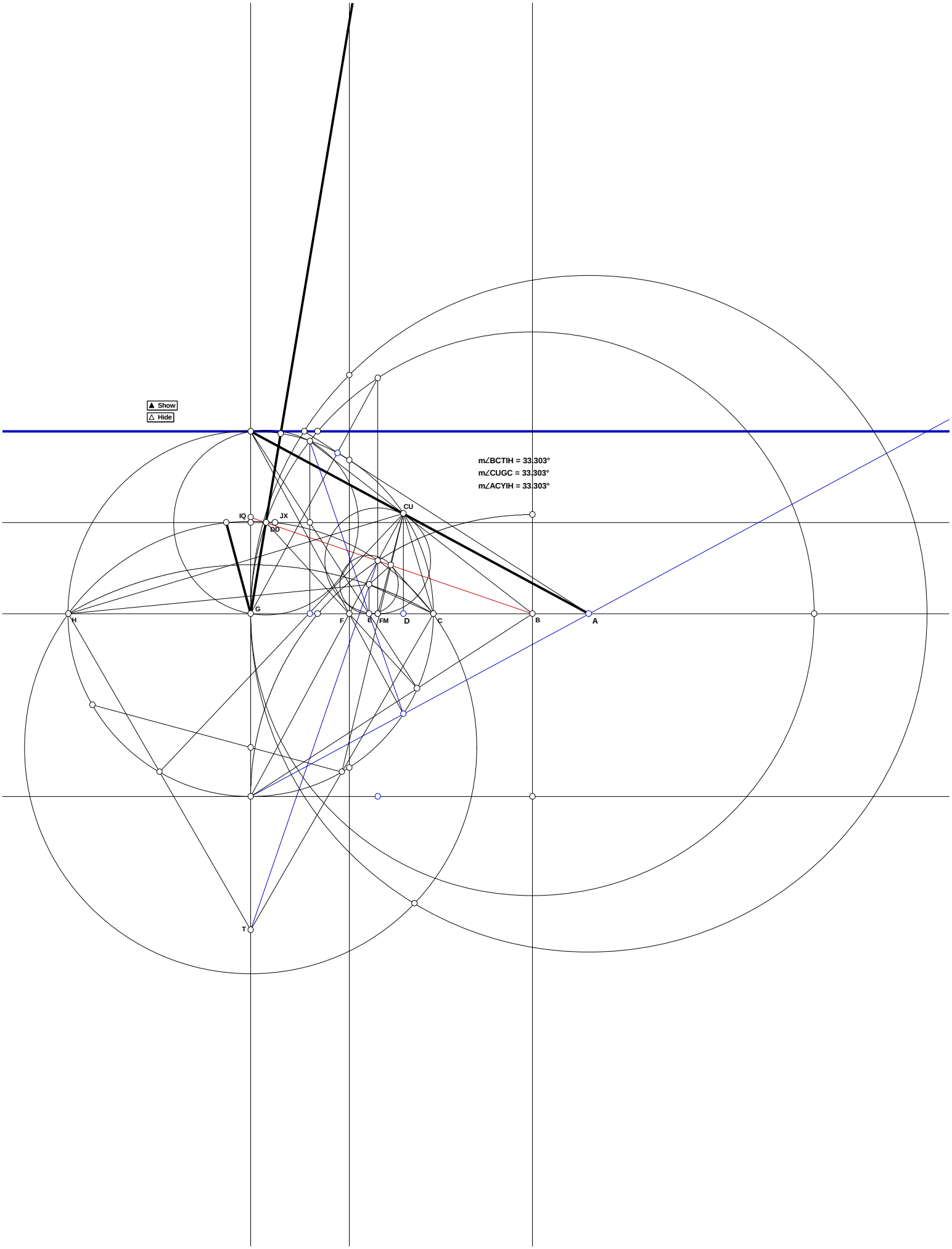

m∠ITCYIP = 14.098°
m∠CYIJC = 28.196°
m∠ITCYKV = 4.699°
m∠CUGCY / m∠ITCYKV = 12.000
m∠CYIJC / m∠ITCYIP = 2.000
m∠CUGCY = 56.392°
m∠CUGCY / m∠CYIJC = 2.000
m∠KXCYJP = 4.699°
m∠LBCYLA = 14.098°
G
I
T

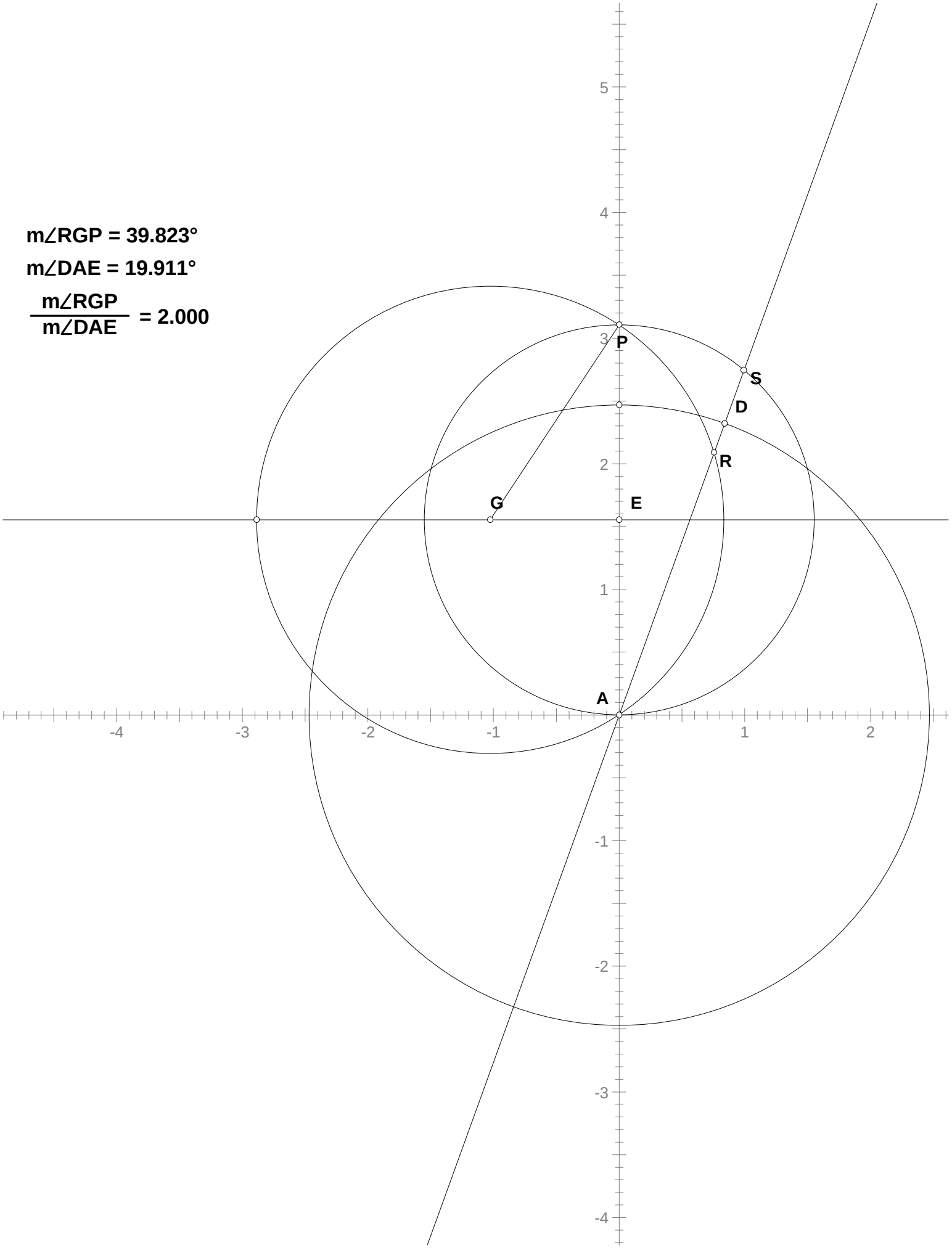

m∠ITCYIP = 11.125°
CY
CT
CO
ITLE
C
LH
CZ
D
CX
IH
JP
T

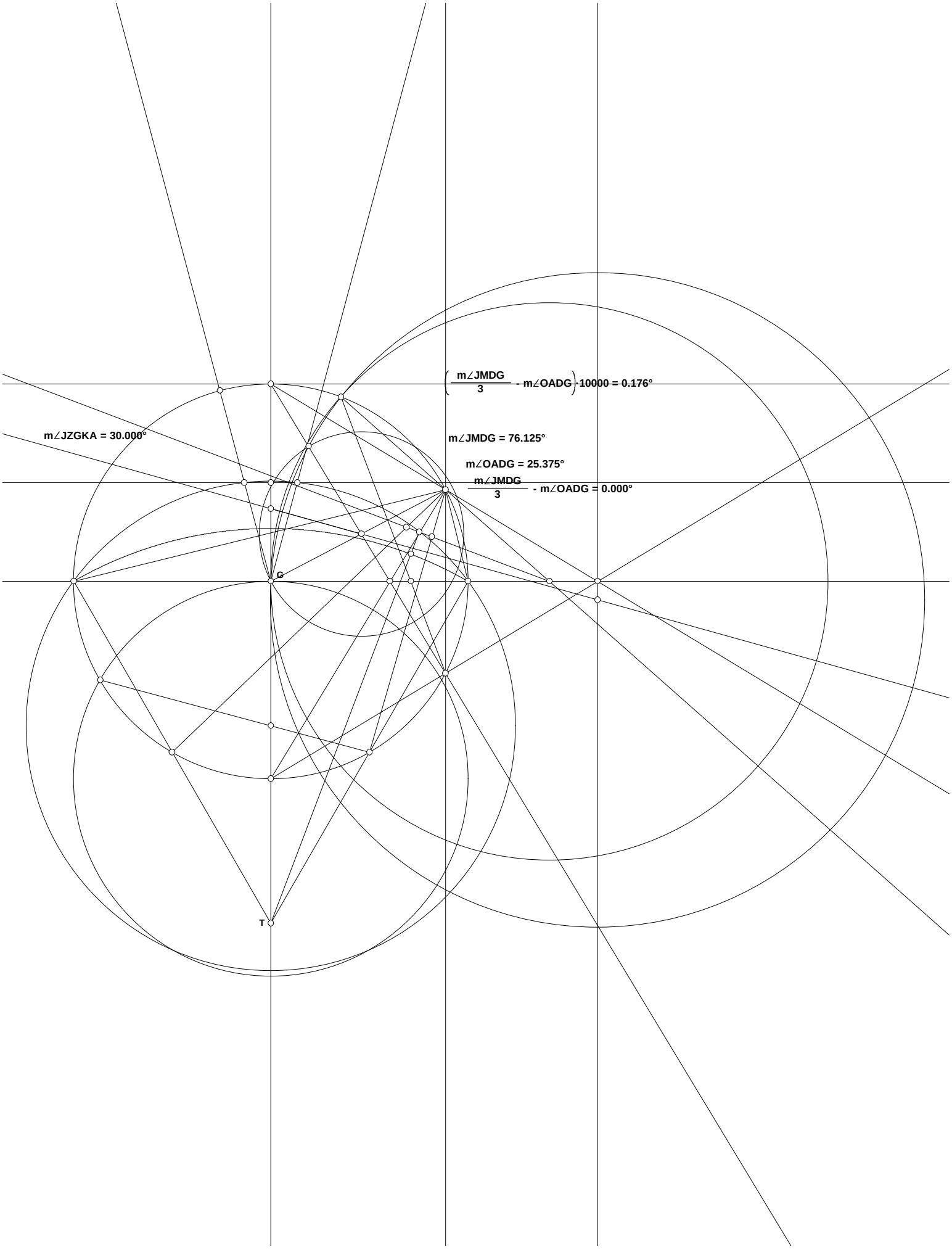

m∠CUGCY = 63.761°
m∠ICTIL = 47.492°
m∠ITCYIP = 15.940°
CY
CT
IT
LF
CU
C
LH
CZ
D
I
CX
IH
JP
T

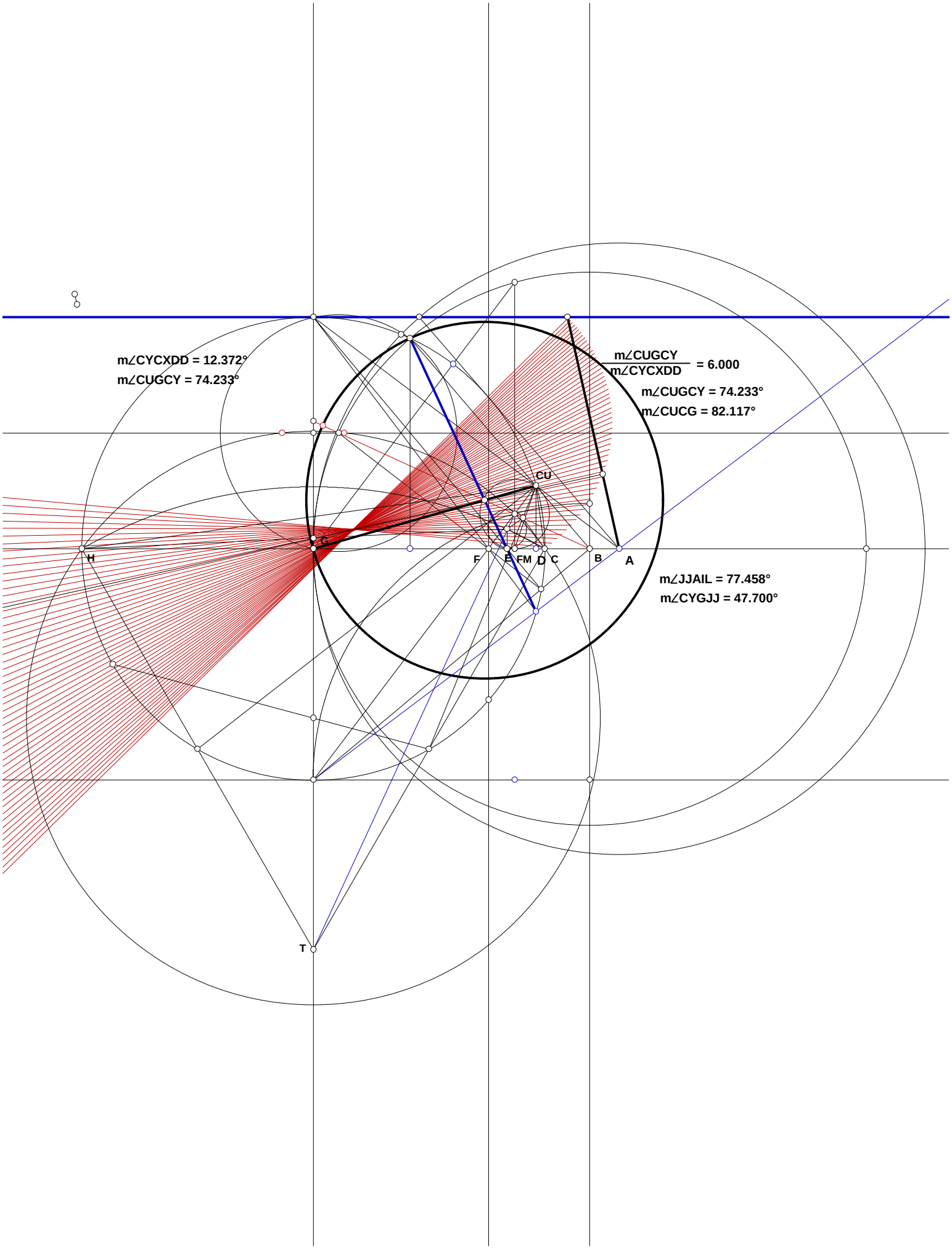

m∠JMDG = 69.925°
m∠KTDG = 23.309°
m∠JMDG / m∠KTDG = 3.000
m∠JMDG - 3·m∠KTDG = -0.003°
putz
G
T

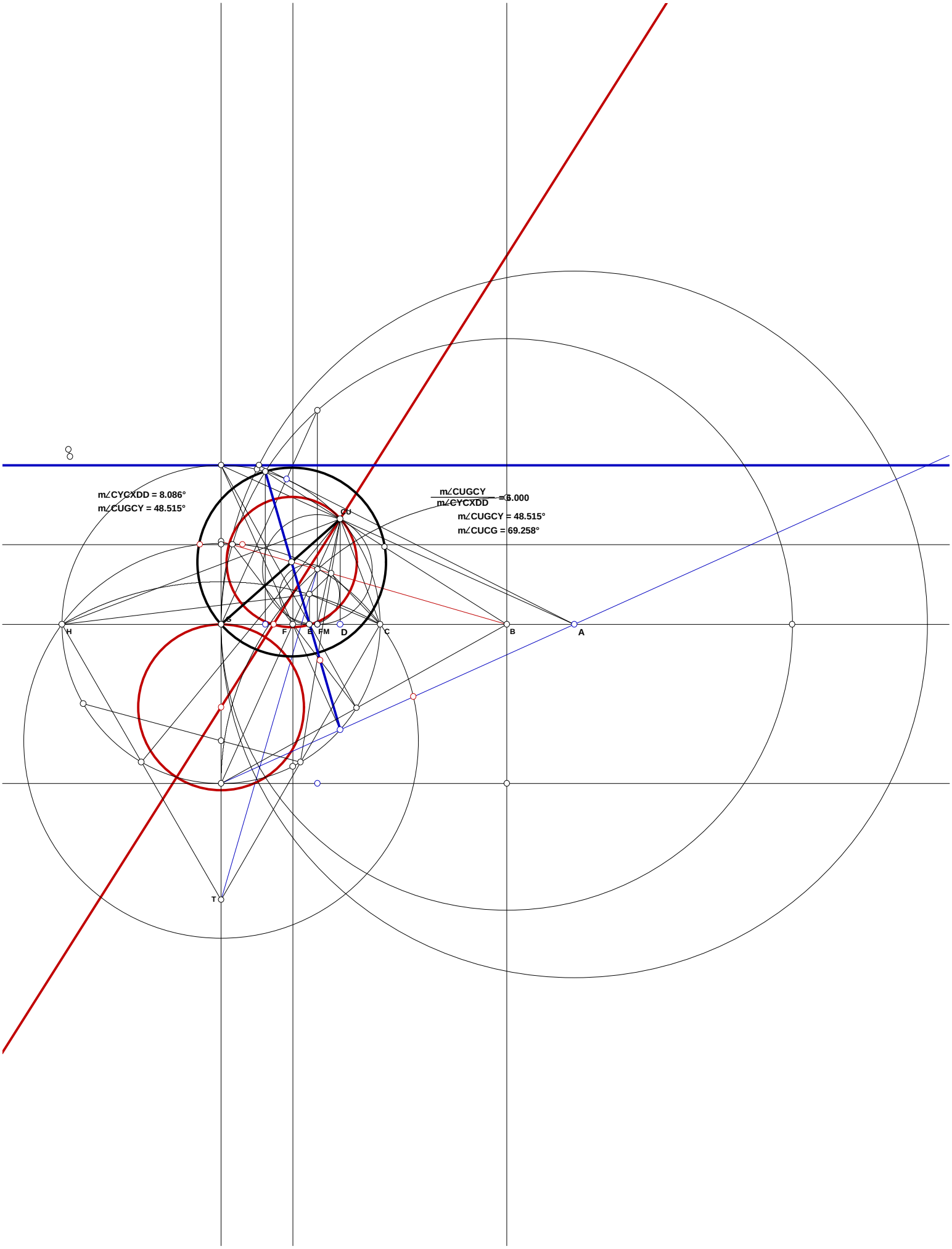

m∠ITCYIP = 15.037°
m∠CUGCY = 60.150°
m∠ICTIL = 49.900°
CY
CT
m∠CYIJG = 30.075°
m∠CUGCY = 60.150°
m∠CUGCY / m∠CYIJG = 2.000
CU
IT
LF
m∠CTIG = 40.100°
G
LH
CZ
D
I
IJ
m∠CYIJG = 30.075°
CX
IH
JP
T

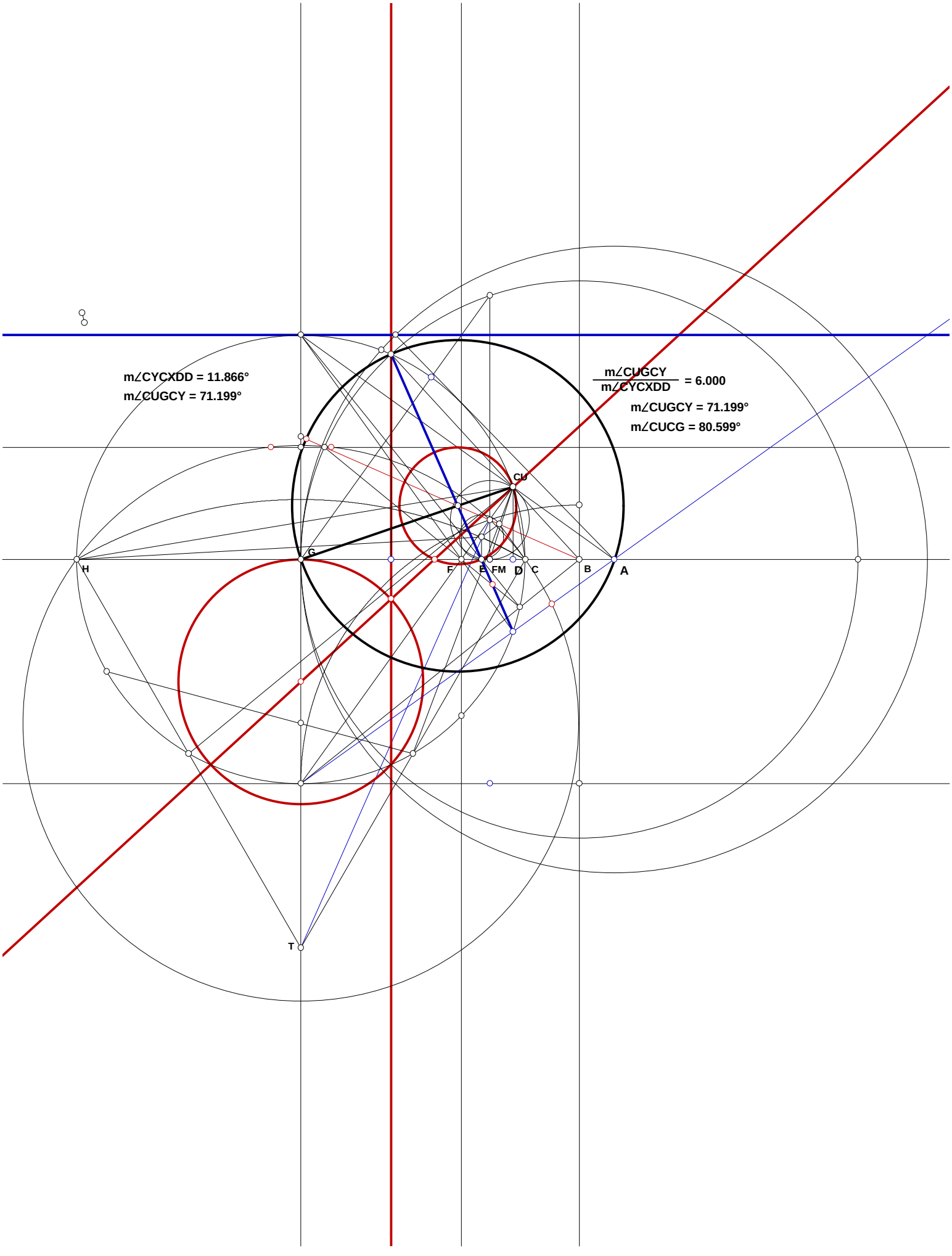

Show
Hide
m∠BCTIH = 33.303°
m∠CUGC = 33.303°
m∠ACYIH = 33.303°
IQ
JX
DD
CU
G
H
F
E
FM
D
C
B
A
T

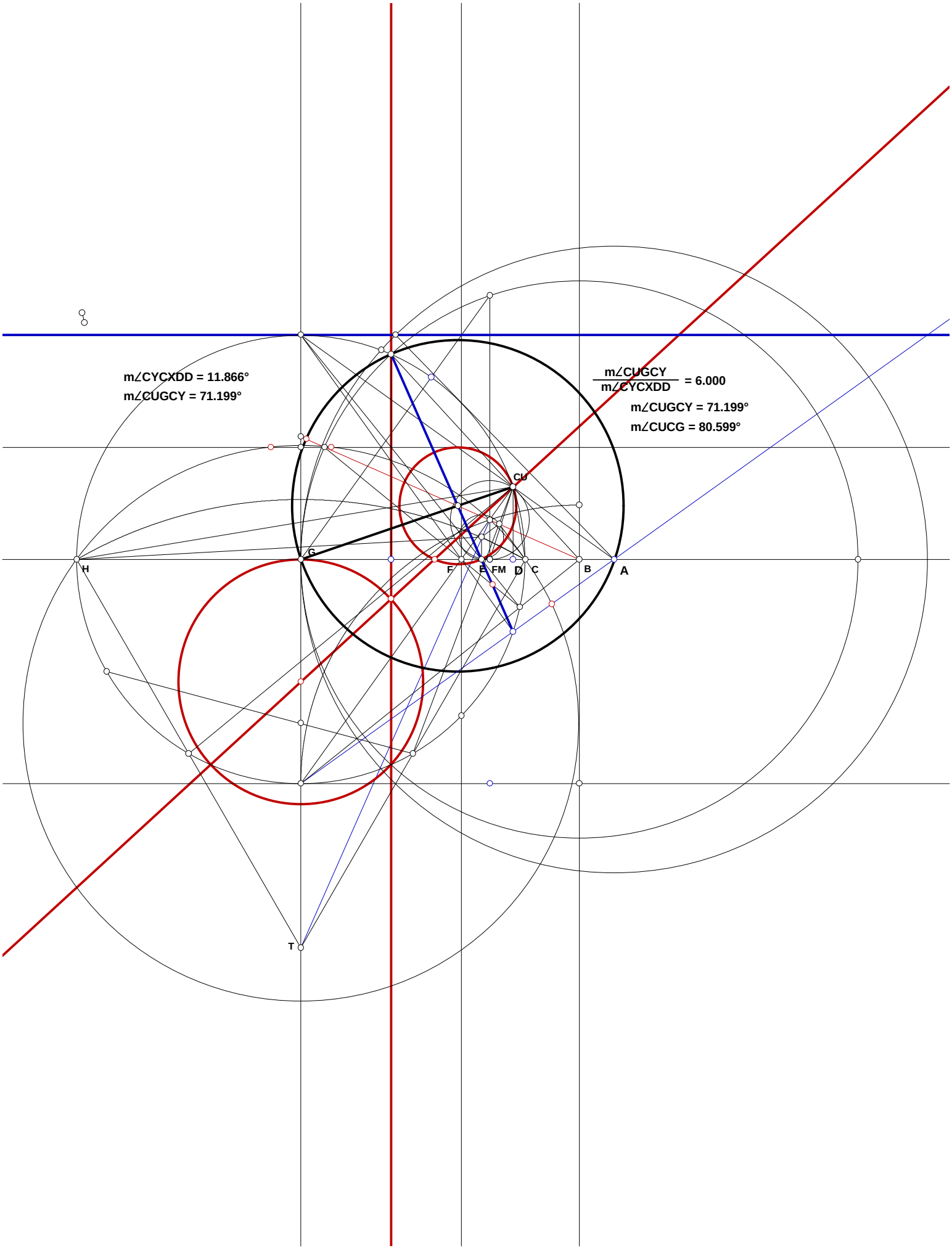

m∠RGP = 39.823°
m∠DAE = 19.911°
m∠RGP / m∠DAE = 2.000
P
S
D
R
G
E
A
5
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2

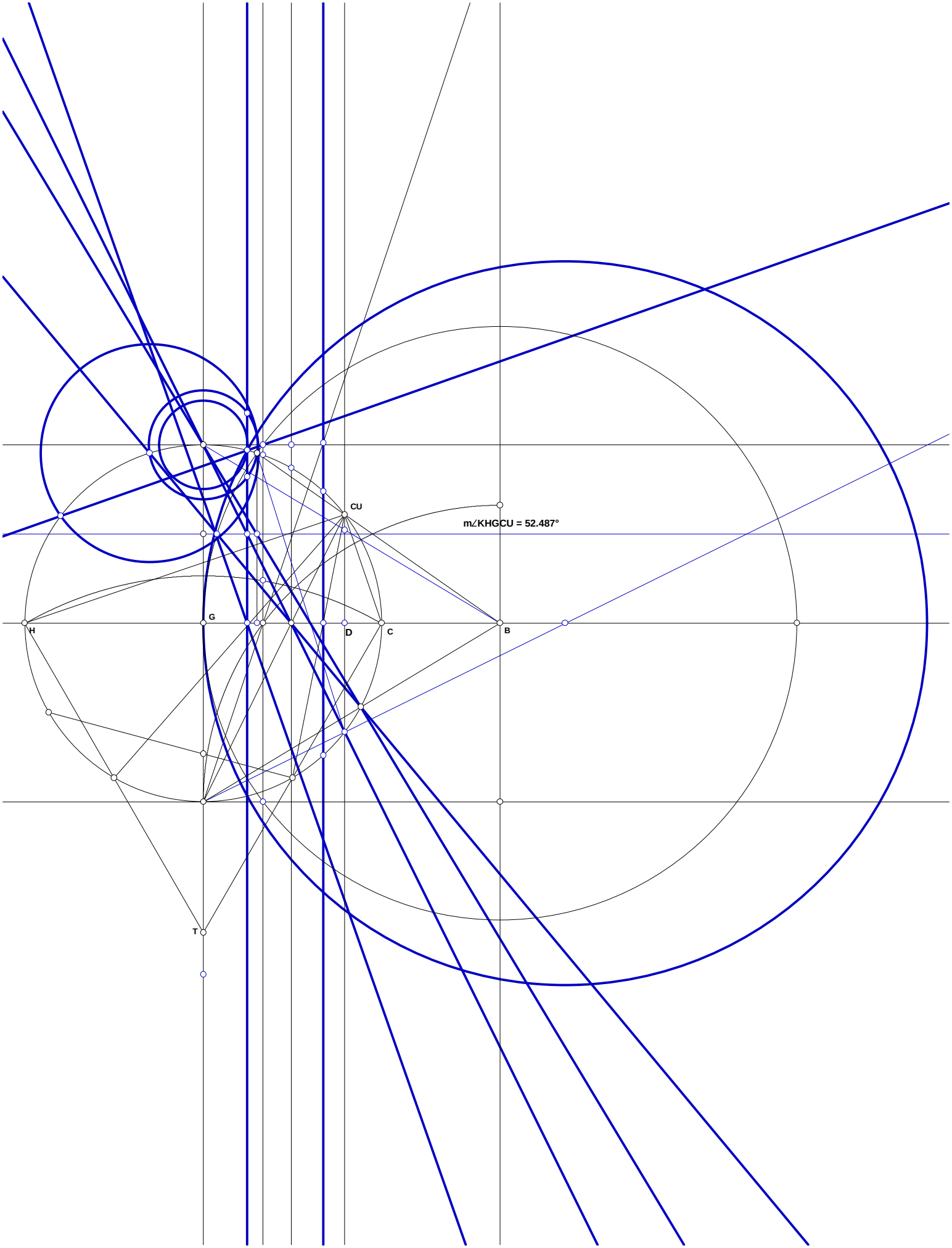

$\left(\frac{m\angle JMDG}{3} - m\angle OADG\right) 10000 = 0.176°$
$m\angle JZGKA = 30.000°$
$m\angle JMDG = 76.125°$
$m\angle OADG = 25.375°$
$\frac{m\angle JMDG}{3} - m\angle OADG = 0.000°$
G
T


m∠CYCXDD = 12.372°
m∠CUGCY = 74.233°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 74.233°
m∠CUCG = 82.117°
m∠JJAIL = 77.458°
m∠CYGJJ = 47.700°
CU
G
H
F
E
FM
D
C
B
A
T

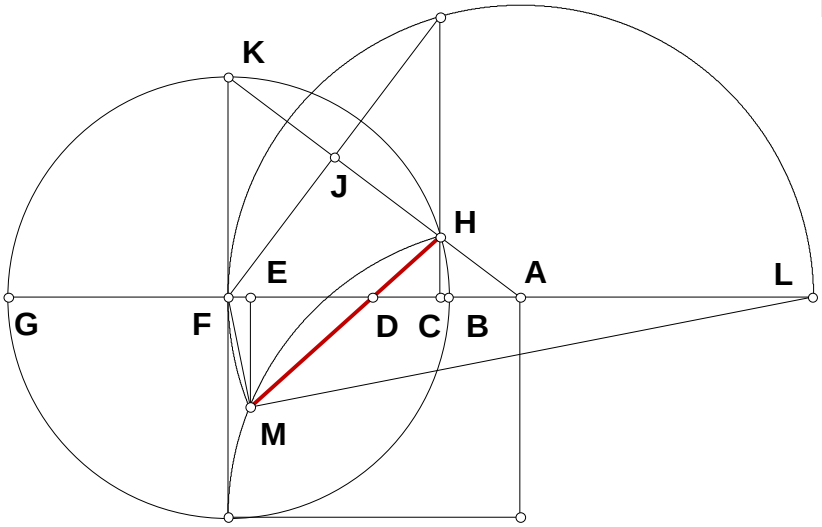

m∠CYCXDD = 8.086°
m∠CUGCY = 48.515°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 48.515°
m∠CUCG = 69.258°
U
H
G
F
E
FM
D
C
B
A
T


m∠CYCXDD = 11.866°
m∠CUGCY = 71.199°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 71.199°
m∠CUCG = 80.599°
CU
G
H
F
E
FM
D
C
B
A
T


m∠CYCXDD = 11.866°
m∠CUGCY = 71.199°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 71.199°
m∠CUCG = 80.599°
CU
G
H
F
E
FM
D
C
B
A
T


CU
m∠KHGCU = 52.487°
G
H
D
C
B
T

Segment DF And HM 052201



Given AB and AG, what is HM and DF?

$N := 7.111$ $AB := .375$

$AG := AB \cdot N$ $BG := AG - AB$

$BF := \frac{BG}{2}$ $FK := BF$

$AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2}$

$AJ := \frac{AF^2}{AK}$ $JK := AK - AJ$

$HJ := JK$ $AH := AK - (JK + HJ)$ $AC := \frac{AF \cdot AH}{AK}$ $EM := \frac{BF}{2}$ $FL := 2 \cdot AF$ $EF := \frac{FL}{2} - \frac{\sqrt{-4\ EM^2 + FL^2}}{2}$

$AE := AF - EF$ $CE := AE - AC$ $CH := \frac{FK \cdot AH}{AK}$ $HM := \sqrt{(EM + CH)^2 + CE^2}$ $DE := \frac{CE \cdot EM}{EM + CH}$

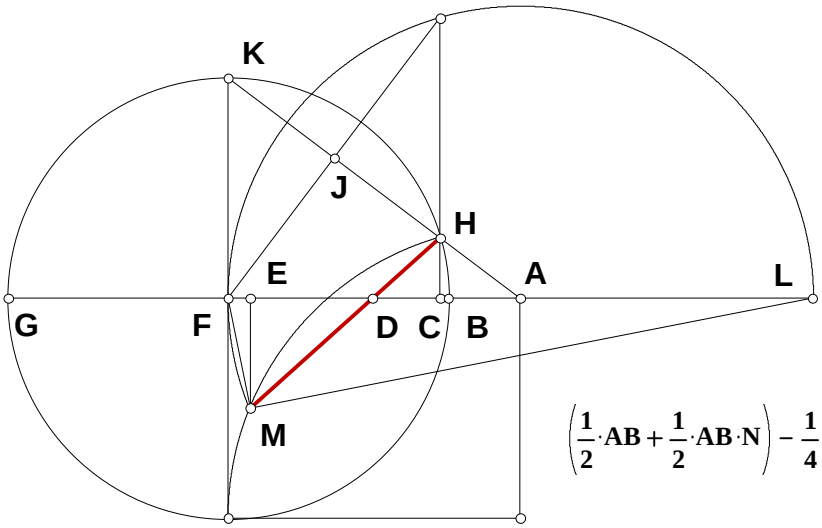
$DF := DE + EF$

Some Algebraic Names:

$AB \cdot N - AG = 0$ $AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - FK = 0$

$\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$ $\frac{1}{2} \cdot AB \cdot \sqrt{2} \cdot \sqrt{1 + N^2} - AK = 0$ $\frac{1}{4} \cdot AB \cdot (1 + N)^2 \cdot \frac{\sqrt{2}}{\sqrt{1 + N^2}} - AJ = 0$

$\frac{1}{4} \cdot \sqrt{2} \cdot AB \cdot \frac{(1 + N^2 - 2 \cdot N)}{\sqrt{1 + N^2}} - JK = 0$ $AB \cdot \sqrt{2} \cdot \frac{N}{\sqrt{1 + N^2}} - AH = 0$ $AB \cdot (1 + N) \cdot \frac{N}{1 + N^2} - AC = 0$



$$\frac{1}{4}\cdot AB\cdot N-\frac{1}{4}\cdot AB-EM=0$$

$$AB+AB\cdot N-FL=0$$

$$\left(\frac{1}{2}\cdot AB+\frac{1}{2}\cdot AB\cdot N\right)-\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-EF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-AE=0$$

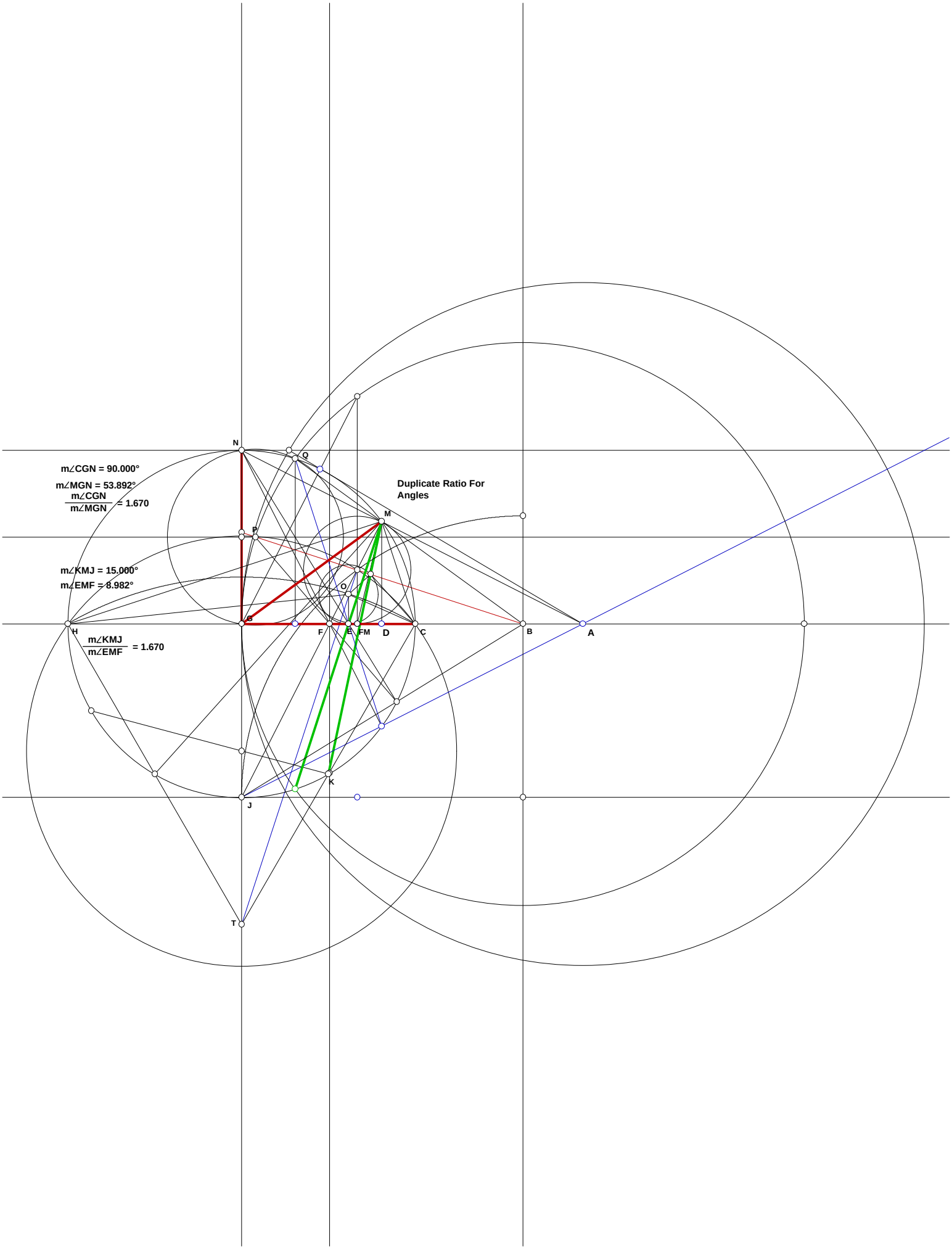
$$\frac{1}{4}\cdot AB\cdot\frac{\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)}{\left(1+N^2\right)}-CE=0$$

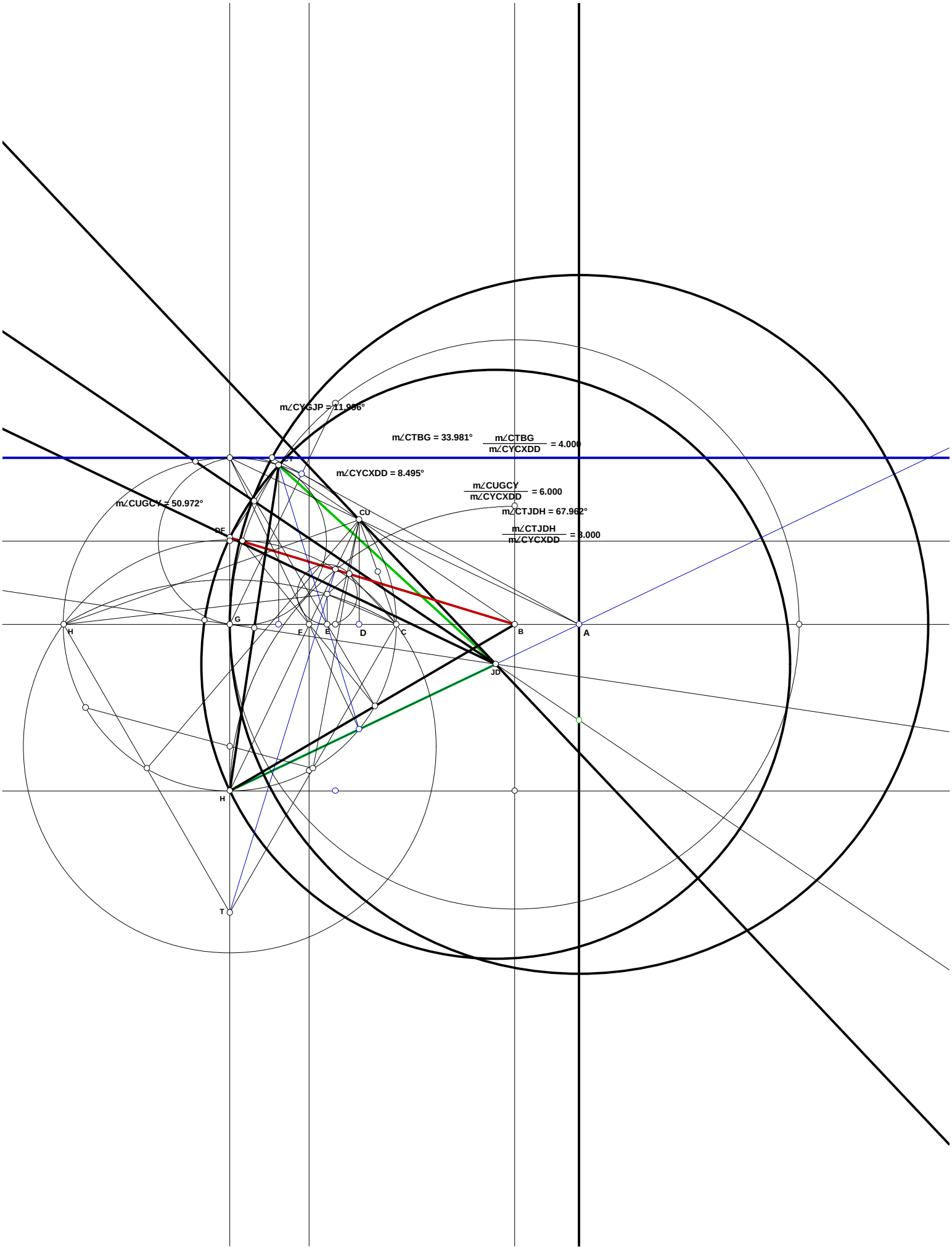
$$(N-1)\cdot AB\cdot\frac{N}{\left(1+N^2\right)}-CH=0$$

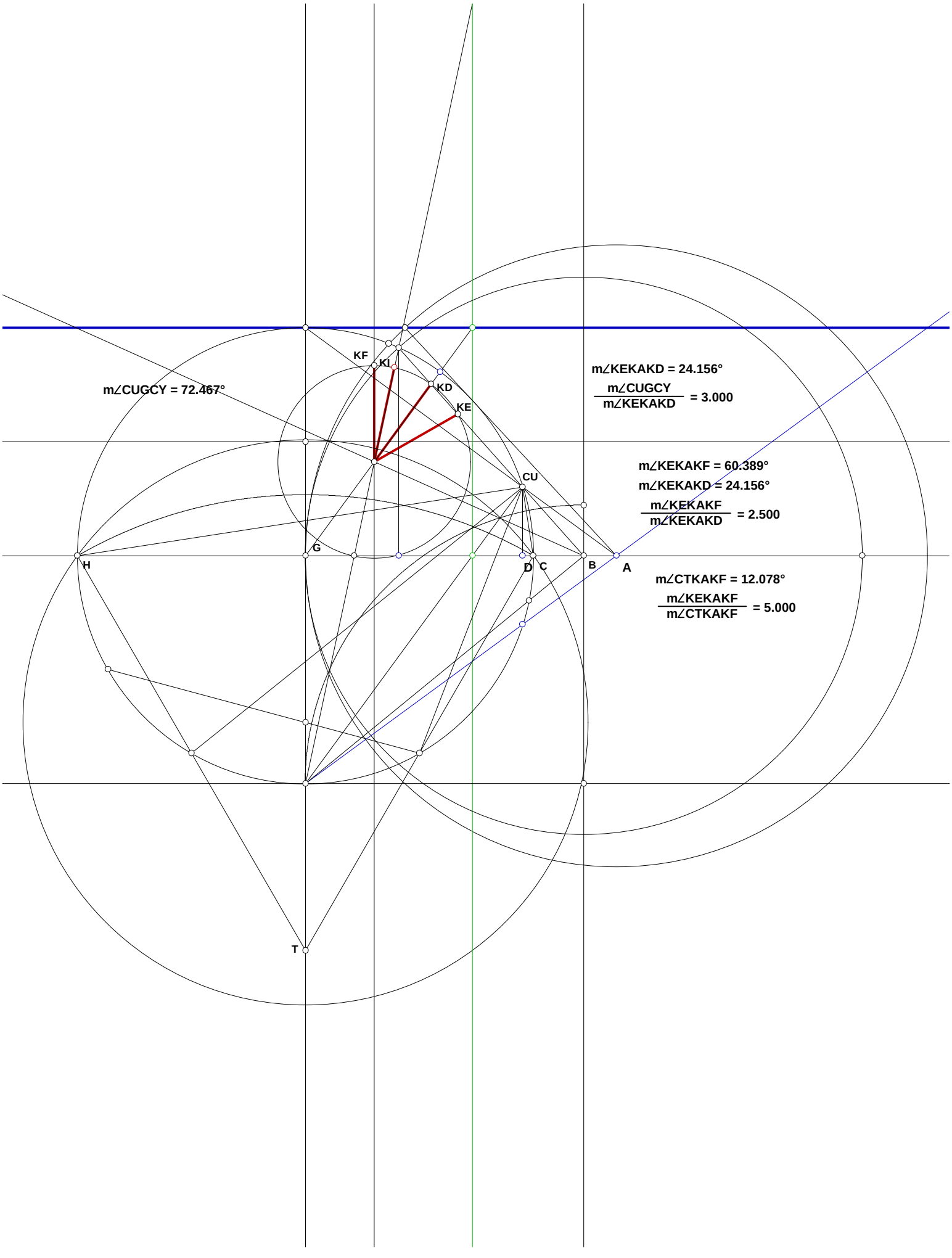
$$\frac{1}{2}\cdot AB\cdot\sqrt{(1+N)\cdot\frac{\left[N^3+3\cdot N^2-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+3\cdot N+1\right]}{\left(1+N^2\right)}}-HM=0$$

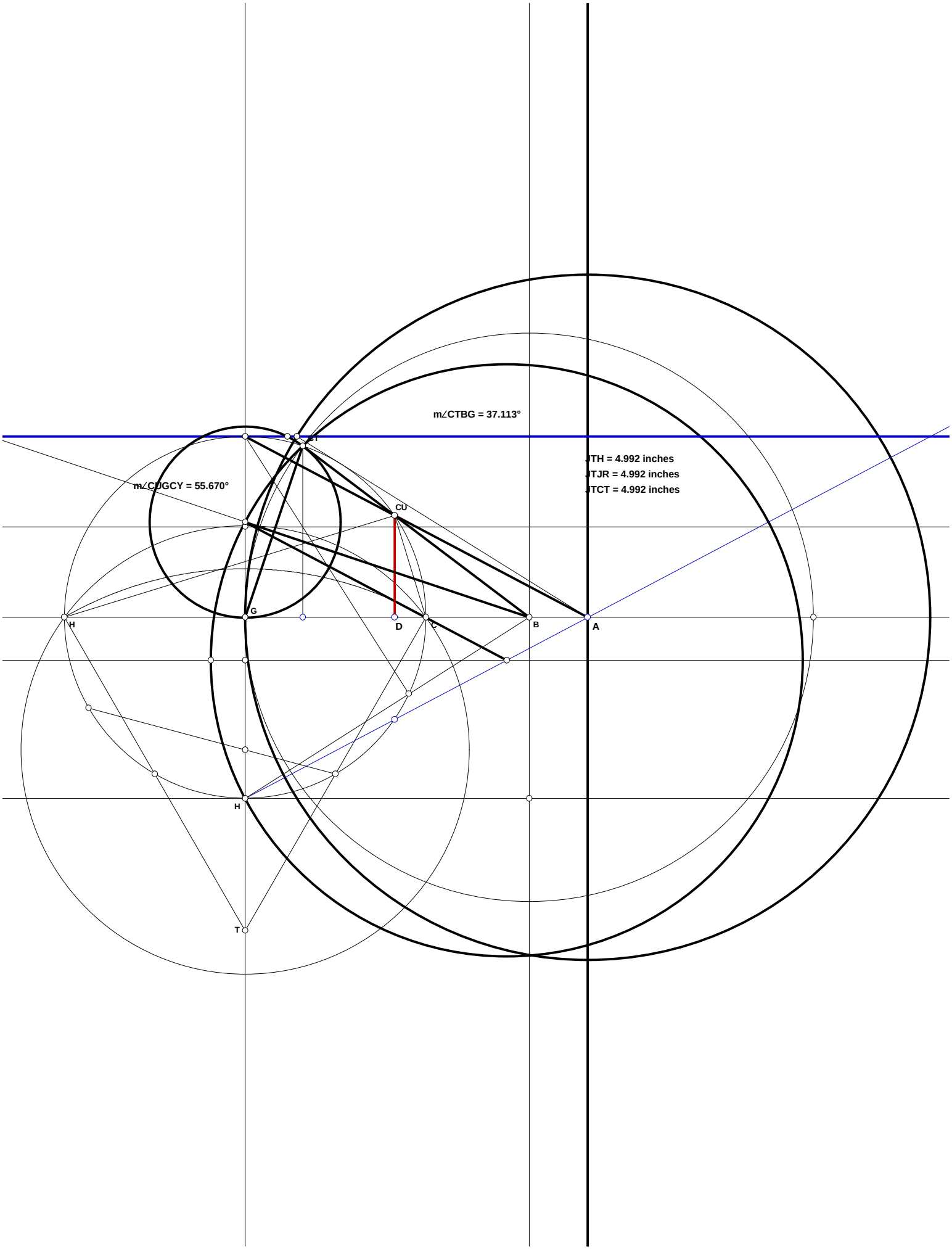
$$\frac{1}{4}\cdot\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)\cdot\frac{AB}{\left(N^2+4\cdot N+1\right)}-DE=0$$

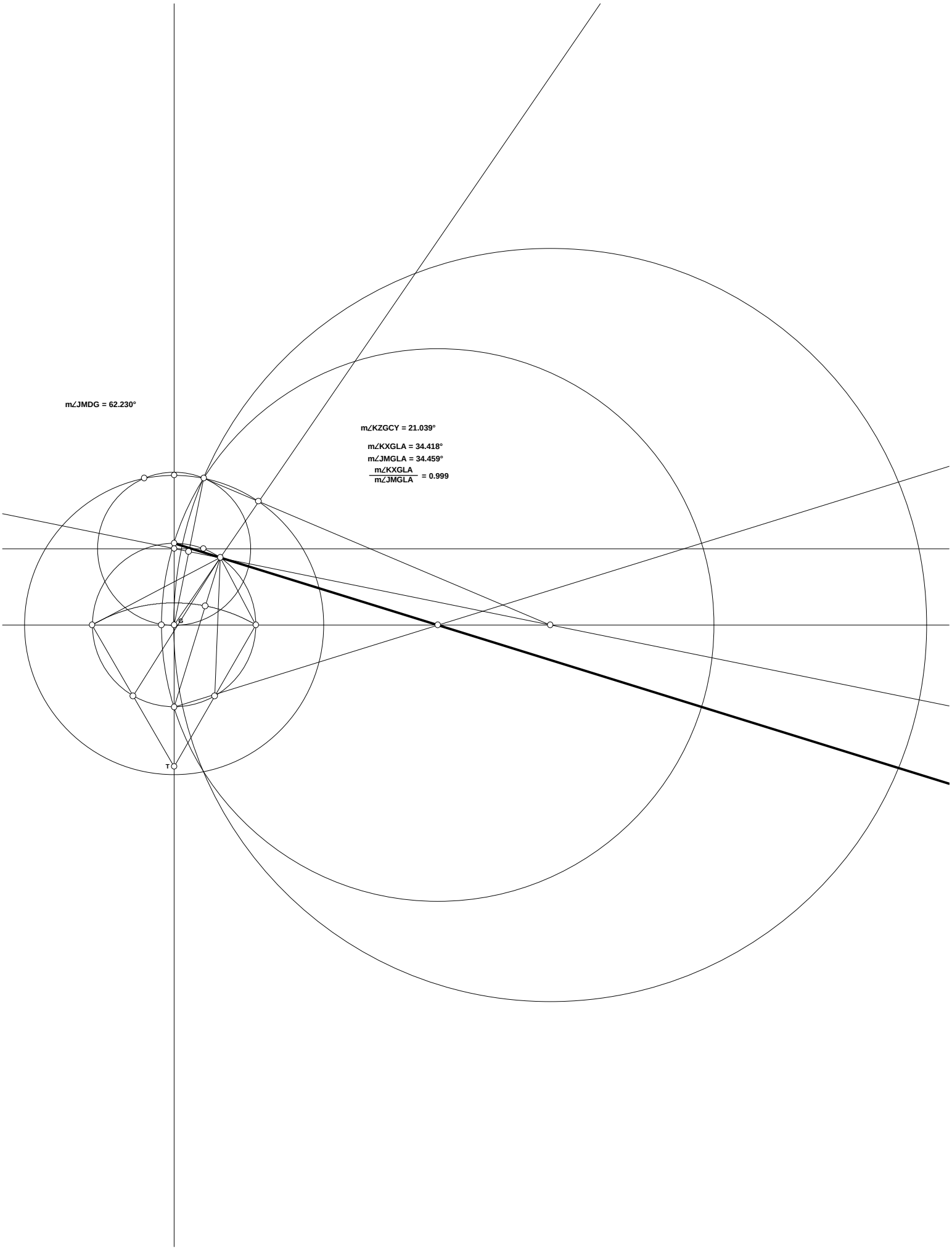
$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N+3\cdot N^2+1+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\right)}{\left(N^2+4\cdot N+1\right)}-DF=0$$

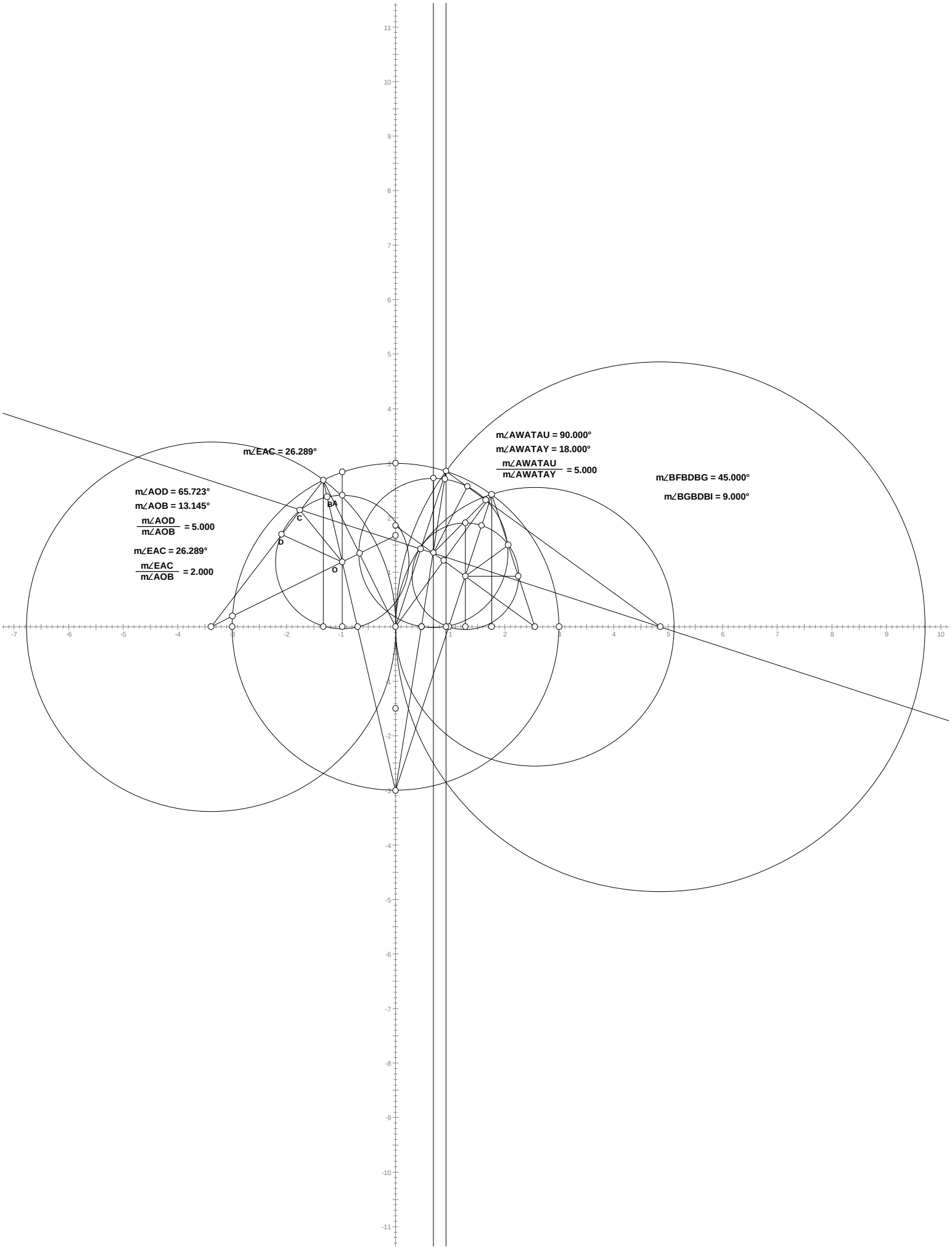

m∠CGN = 90.000°
m∠MGN = 53.892°
m∠CGN / m∠MGN = 1.670
Duplicate Ratio For Angles
m∠KMJ = 15.000°
m∠EMF = 8.982°
m∠KMJ / m∠EMF = 1.670
N
Q
M
P
O
H
G
F
E
FM
D
C
B
A
K
J
T

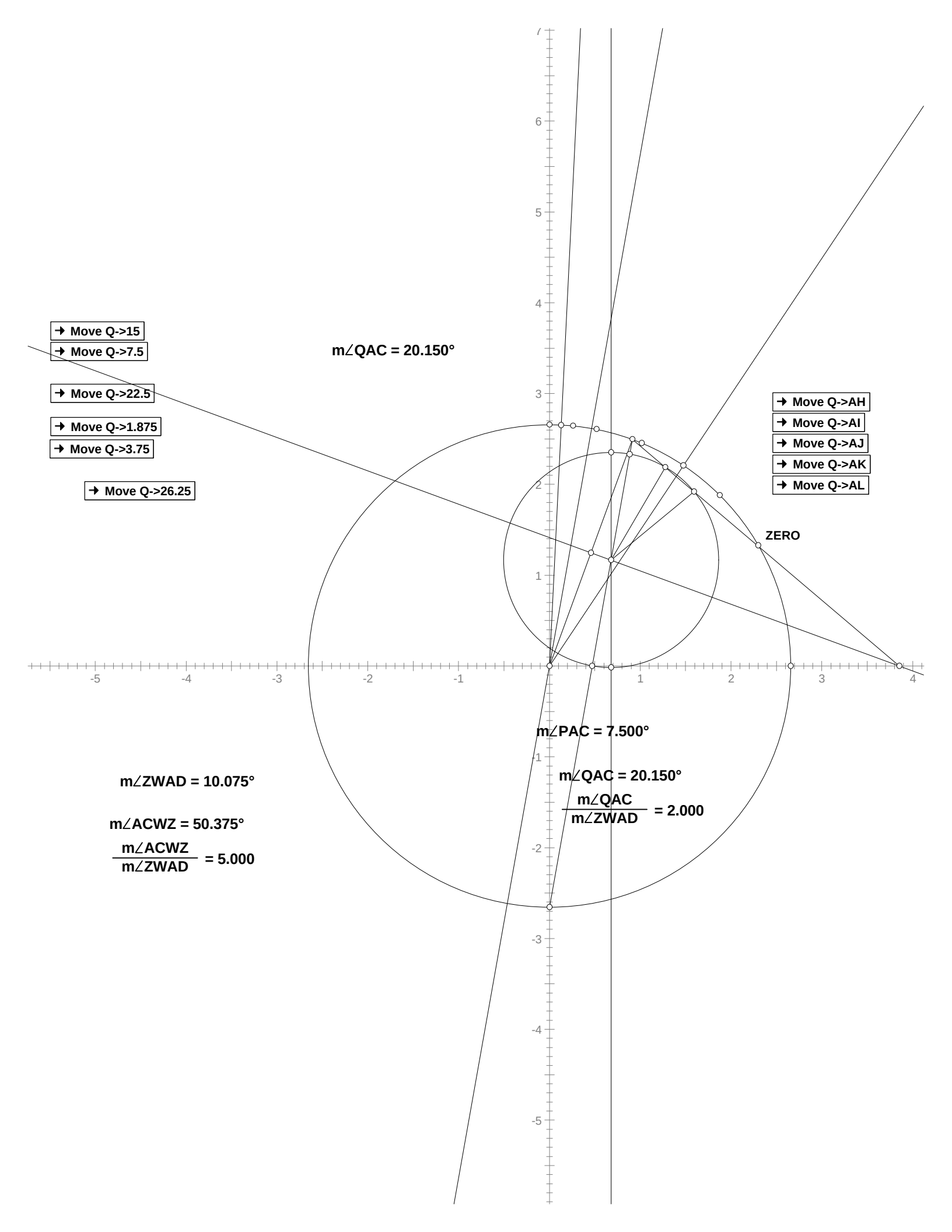

m∠CYGJP = 11.996°
m∠CTBG = 33.981°
m∠CTBG / m∠CYCXDD = 4.000
m∠CYCXDD = 8.495°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 50.972°
m∠CTJDH = 67.962°
m∠CTJDH / m∠CYCXDD = 8.000
CT
CU
DE
G
H
F
E
D
C
B
A
JD
H
T

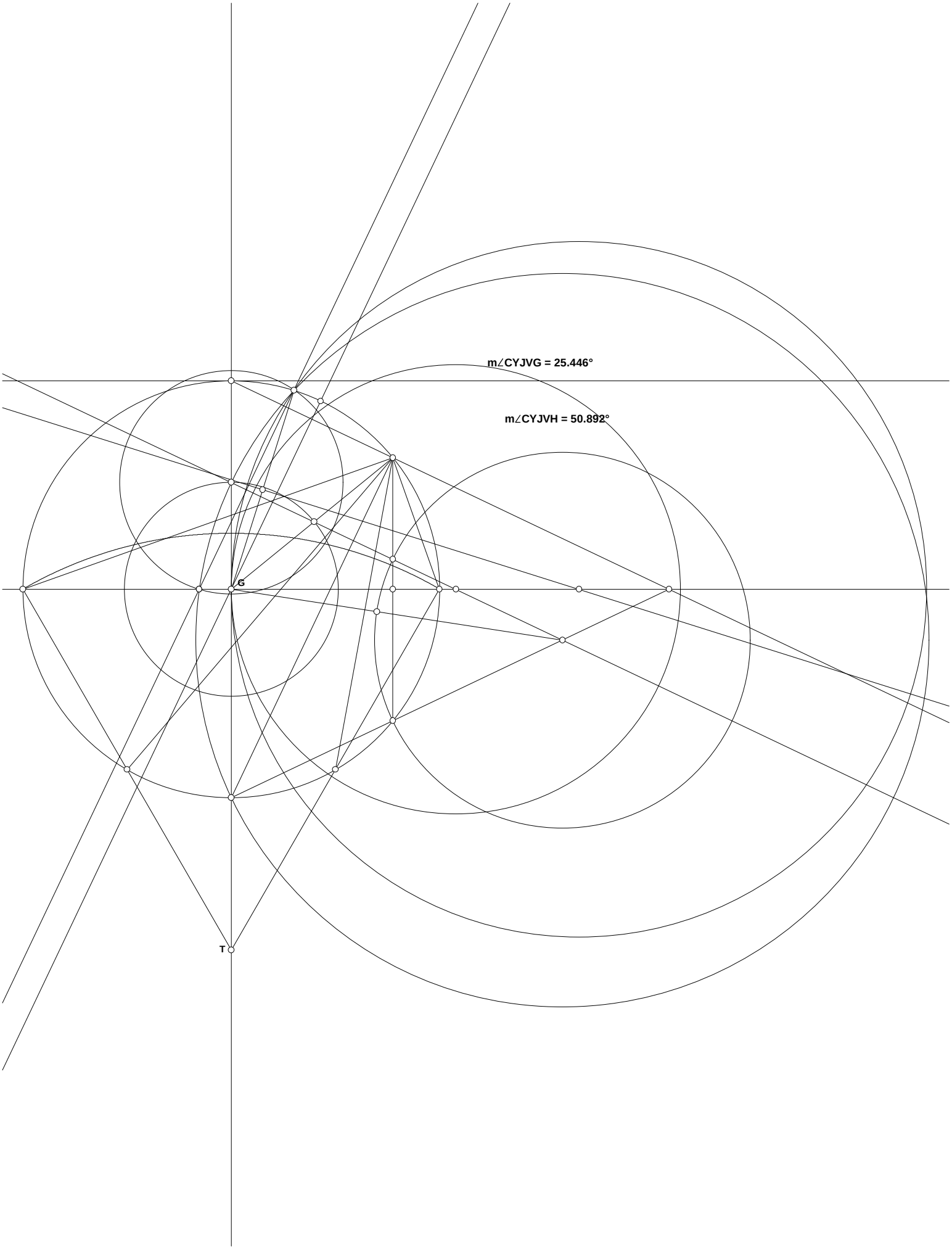

KF
KI
KD
KE
CU
G
H
D
C
B
A
T
m∠CUGCY = 72.467°
m∠KEKAKD = 24.156°
m∠CUGCY / m∠KEKAKD = 3.000
m∠KEKAKF = 60.389°
m∠KEKAKD = 24.156°
m∠KEKAKF / m∠KEKAKD = 2.500
m∠CTKAKF = 12.078°
m∠KEKAKF / m∠CTKAKF = 5.000

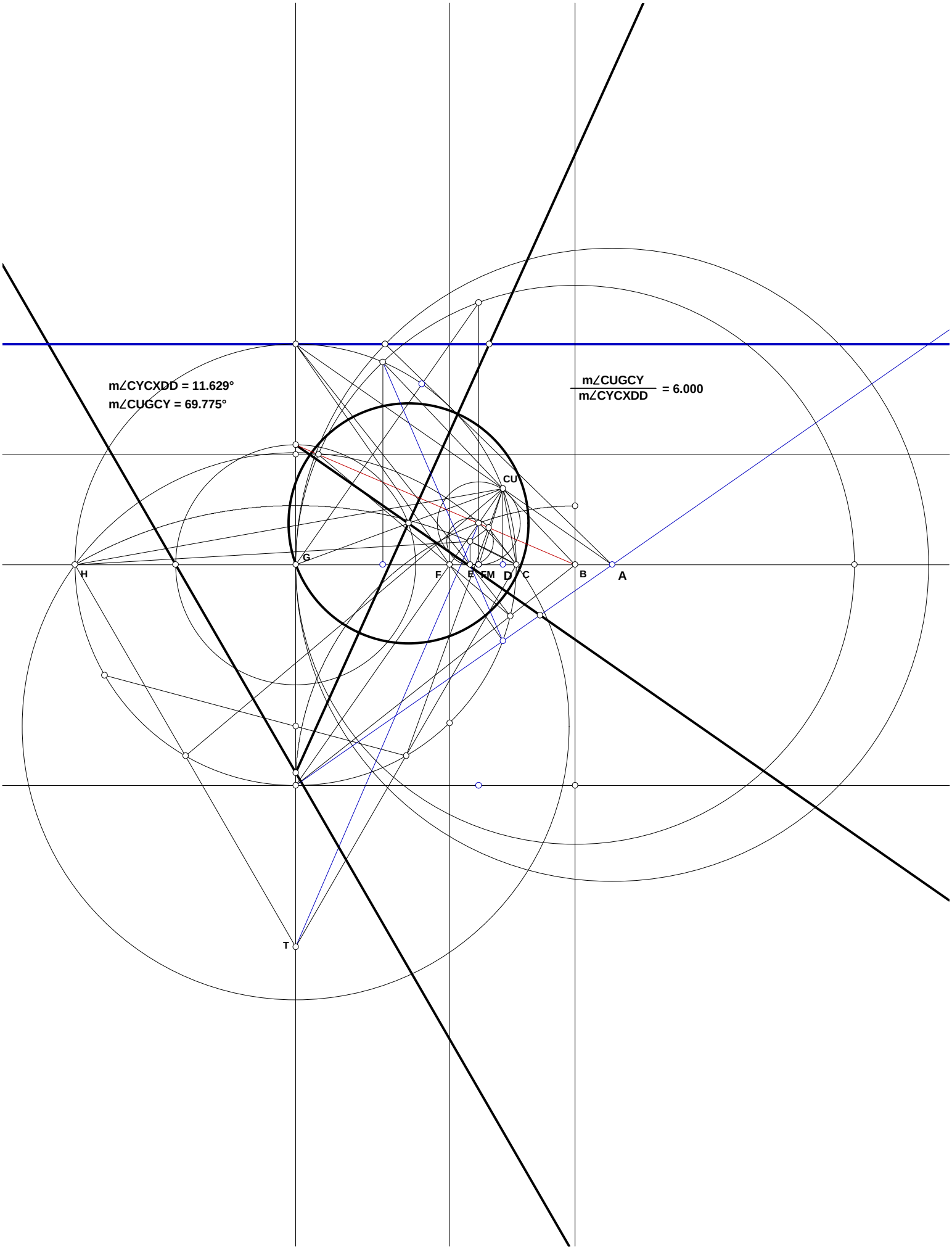

m∠CTBG = 37.113°
m∠CUGCY = 55.670°
JTH = 4.992 inches
JTJR = 4.992 inches
JTCT = 4.992 inches
CU
G
H
D
C
B
A
H
T

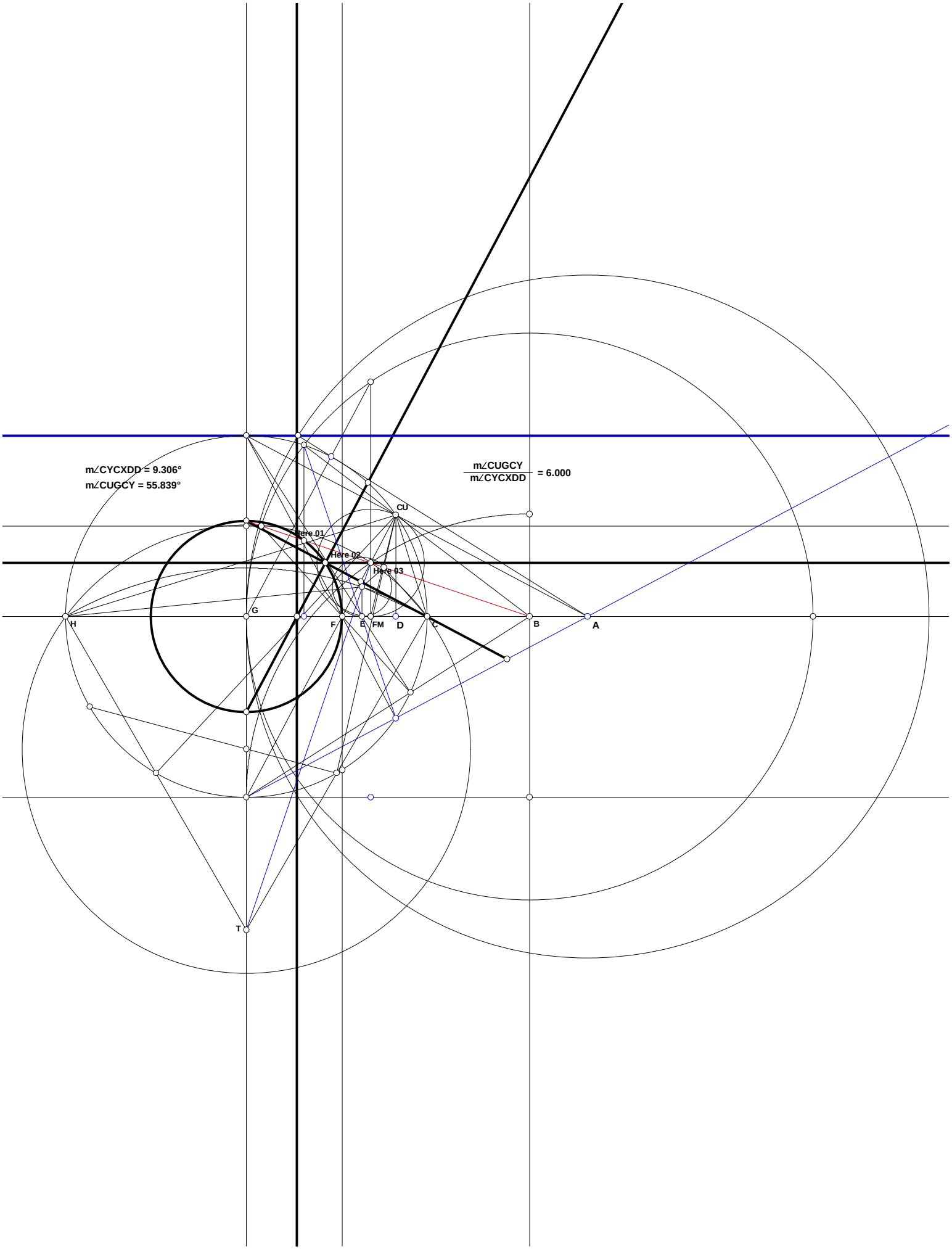

m∠JMDG = 62.230°
m∠KZGCY = 21.039°
m∠KXGLA = 34.418°
m∠JMGLA = 34.459°
m∠KXGLA / m∠JMGLA = 0.999
T


m∠EAC = 26.289°
m∠AOD = 65.723°
m∠AOB = 13.145°
m∠AOD / m∠AOB = 5.000
m∠EAC = 26.289°
m∠EAC / m∠AOB = 2.000
m∠AWATAU = 90.000°
m∠AWATAY = 18.000°
m∠AWATAU / m∠AWATAY = 5.000
m∠BFBDBG = 45.000°
m∠BGBDBI = 9.000°
A
B
C
D
O

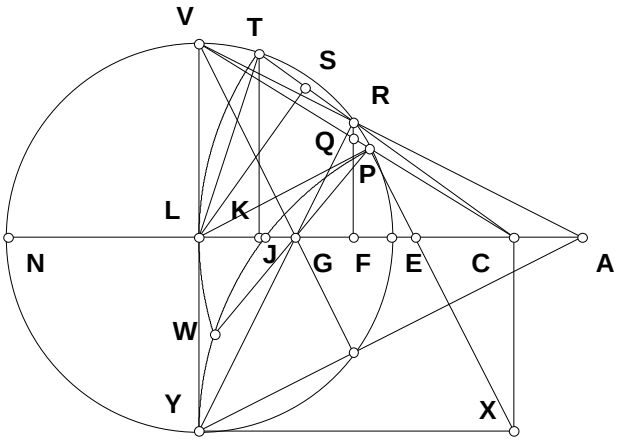

Move Q->15
Move Q->7.5
Move Q->22.5
Move Q->1.875
Move Q->3.75
Move Q->26.25
m∠QAC = 20.150°
Move Q->AH
Move Q->AI
Move Q->AJ
Move Q->AK
Move Q->AL
ZERO
m∠PAC = 7.500°
m∠QAC = 20.150°
m∠QAC / m∠ZWAD = 2.000
m∠ZWAD = 10.075°
m∠ACWZ = 50.375°
m∠ACWZ / m∠ZWAD = 5.000


m∠CYJVG = 25.446°
m∠CYJVH = 50.892°
G
T


m∠CYCXDD = 11.629°
m∠CUGCY = 69.775°
m∠CUGCY / m∠CYCXDD = 6.000
CU
G
H
F
E
FM
D
C
B
A
T


m∠CYCXDD = 9.306°
m∠CUGCY = 55.839°
m∠CUGCY / m∠CYCXDD = 6.000
CU
Here 01
Here 02
Here 03
G
H
F
E
FM
D
C
B
A
T

Point of Intersection 052701



Do RY and PW intersect at G?

$N := 5 \qquad CE := 1 \qquad CN := CE \cdot N$

$EN := CN - CE \qquad EL := \frac{EN}{2} \qquad LV := EL$

$LT := EL \qquad LY := EL \qquad CL := CE + EL \qquad CT := CL$

$S_1 := CL \qquad S_2 := CT \qquad S_3 := LT$

$KL := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \qquad CK := CL - KL \qquad CS := CK \qquad ST := CT - CS \qquad RS := ST$

$CR := CT - (ST + RS) \qquad CF := \frac{CK \cdot CR}{CT} \qquad FR := \sqrt{CR^2 - CF^2} \qquad FQ := \frac{LV \cdot CF}{CL} \qquad FL := CL - CF$

$GL := \frac{FL \cdot LY}{(LY + FR)}$

Some Algebraic Names:

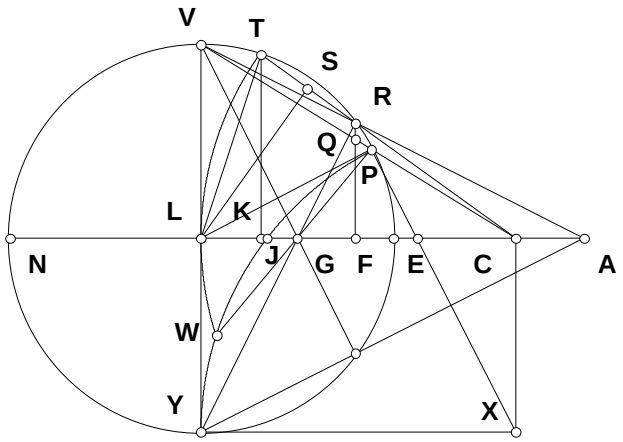
$CE \cdot N - CN = 0 \qquad CE \cdot N - CE - EN = 0 \qquad \frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{2} \cdot CE + \frac{1}{2} \cdot CE \cdot N - CL = 0$

$\frac{1}{4} \cdot CE \cdot \frac{(N-1)^2}{(1+N)} - KL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1+N)} - ST = 0$

$2 \cdot CE \cdot \frac{N}{(1+N)} - CR = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)^3} \cdot N - CF = 0$

$CE \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \frac{(N-1)}{(1+N)^3} - FR = 0 \qquad CE \cdot (N-1) \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1+N)^4} - FQ = 0$

$\frac{1}{2} \cdot CE \cdot \frac{\left(1 + 2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4\right)}{(1+N)^3} - FL = 0$



$$GL := \frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}$$

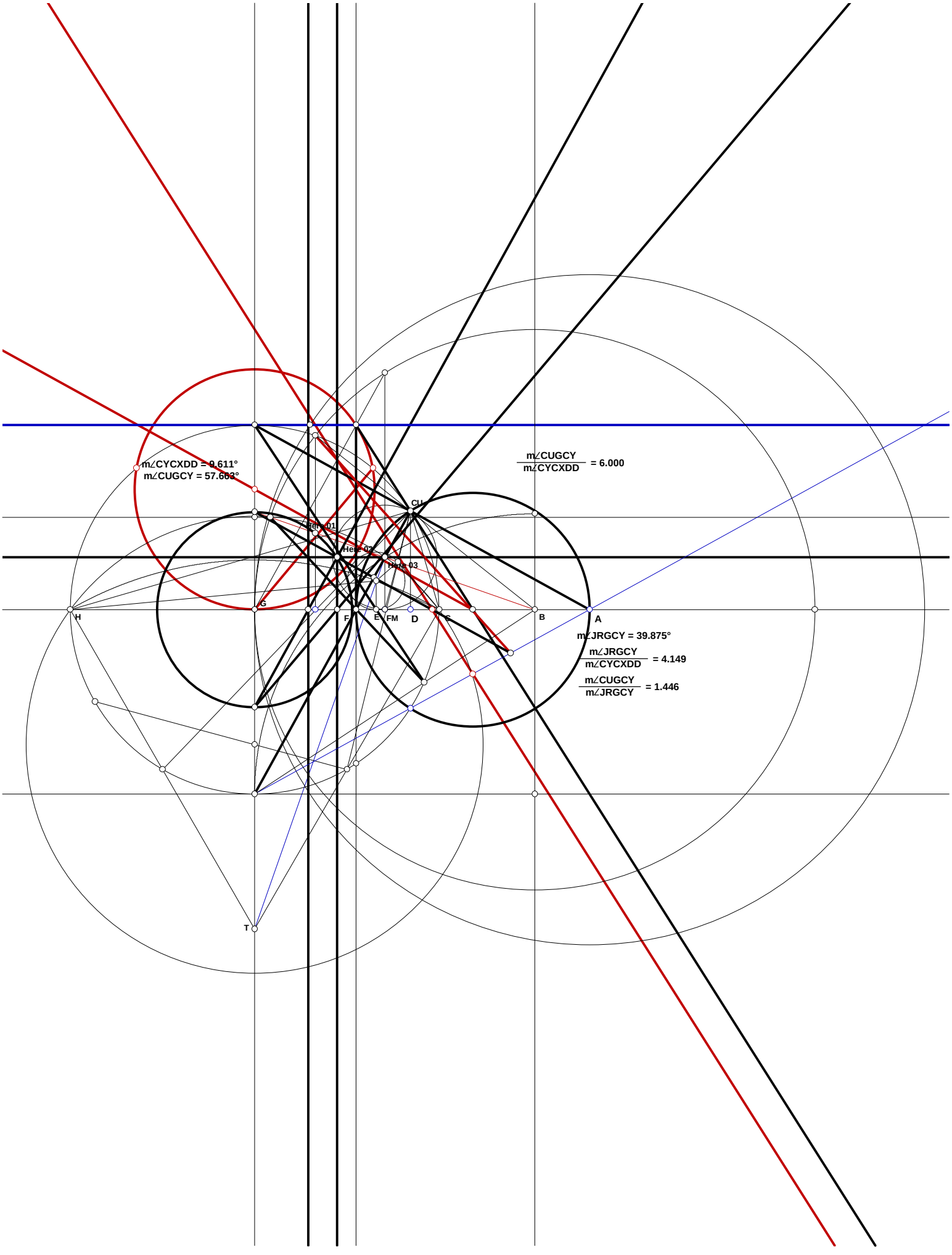
From Segment DF And HM 052201:

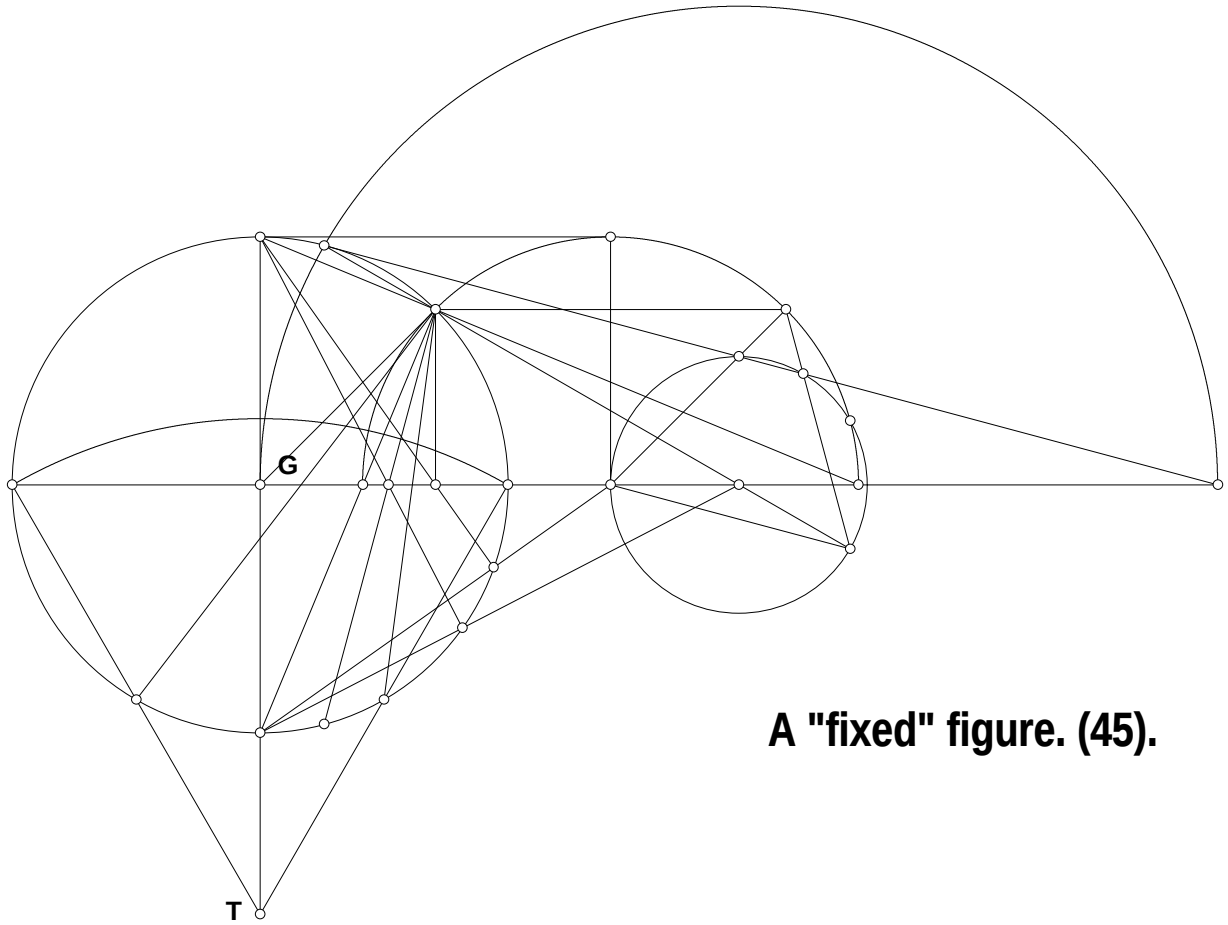
$$\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)} - GL = 0$$

$$\frac{\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)}}{\frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}} = 1$$

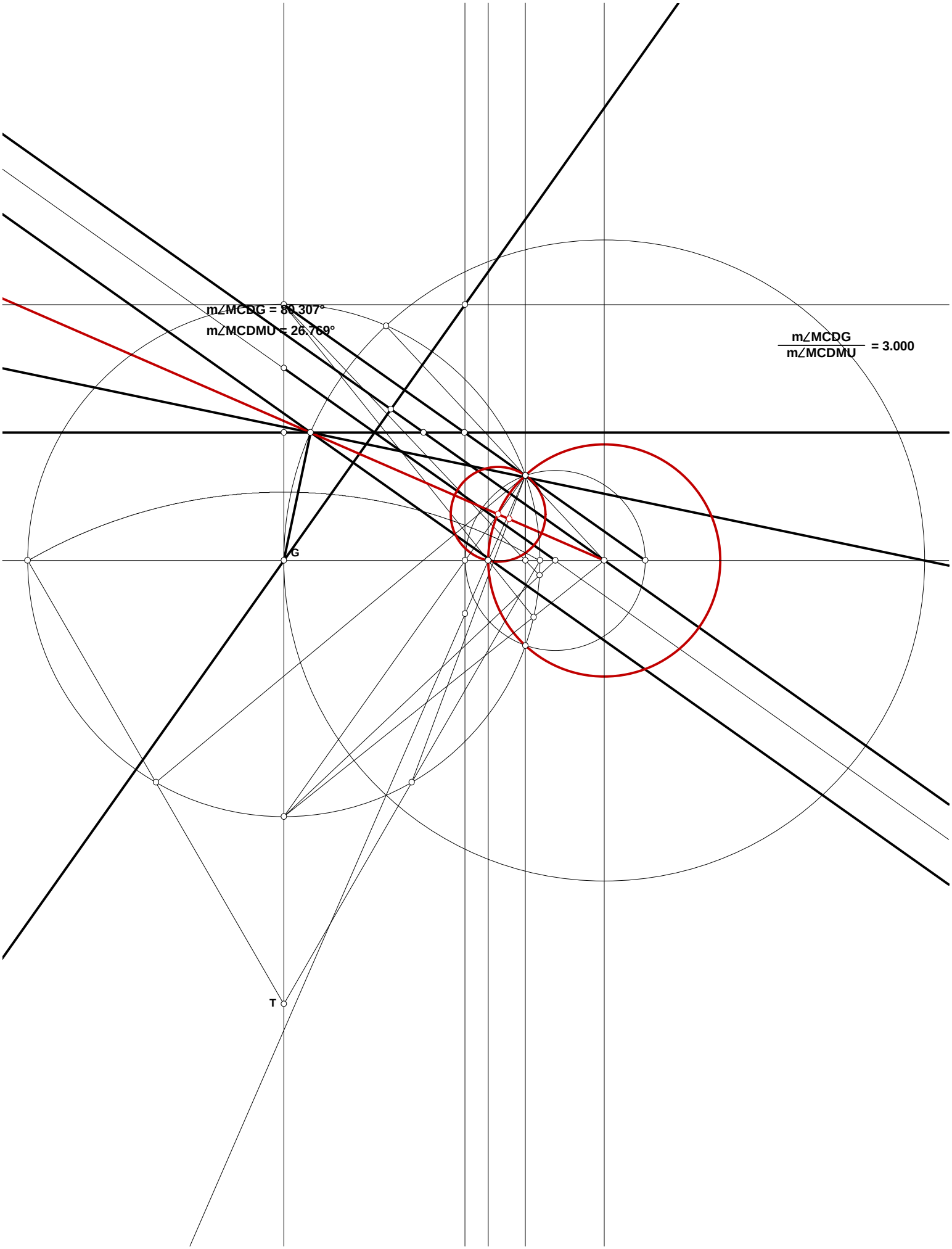
Which does reduce to,

$$1 = 1$$


m∠CYCXDD = 9.611°
m∠CUGCY = 57.668°
m∠CUGCY / m∠CYCXDD = 6.000
CU
Hero 01
Hero 02
Hero 03
G
H
F
E
FM
D
C
B
A
T
m∠JRGCY = 39.875°
m∠JRGCY / m∠CYCXDD = 4.149
m∠CUGCY / m∠JRGCY = 1.446

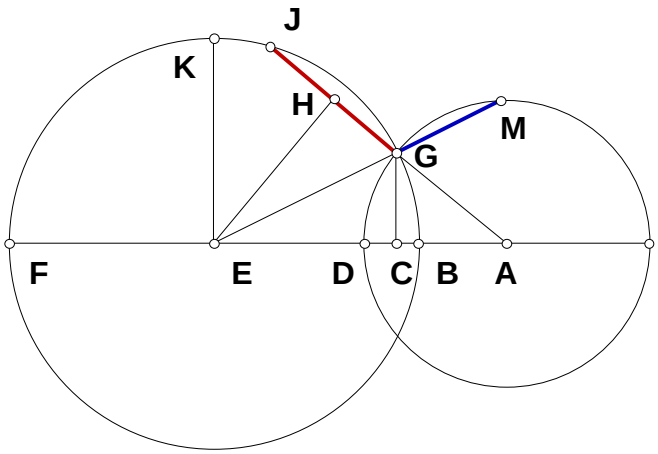


A "fixed" figure. (45).


m∠MCDG = 80.307°
m∠MCDMU = 26.769°
m∠MCDG / m∠MCDMU = 3.000
G
T

A Small Extrapolation 060101

Given AE, AG, and EG, what is the Algebraic name of the segment GJ?



$S_1 := 3 \quad S_2 := 2 \quad S_3 := 1$

$AE := S_1 \quad AG := S_2 \quad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2\,AE}$$

$$AH := \frac{AC \cdot AE}{AG} \quad GH := AH - AG$$

$HJ := GH \quad GJ := GH + HJ$

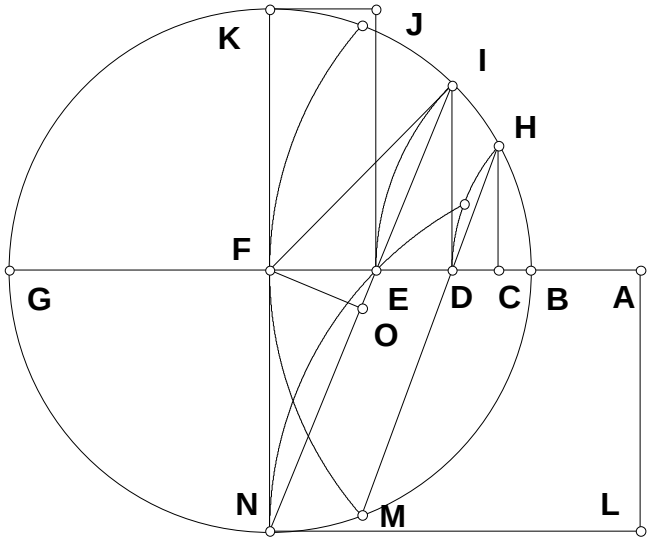
Some Algebraic Names:

$$\frac{S_2^2 + S_1^2 - S_3^2}{2\,S_1} - AC = 0 \qquad \frac{S_1^2 + S_2^2 - S_3^2}{2\,S_2} - AH = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{2\,S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{S_3} - GM = \;?$$

Units From Both Sides 060201

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.



$N := 5.727 \qquad AB := .573 \qquad AG := AB \cdot N$

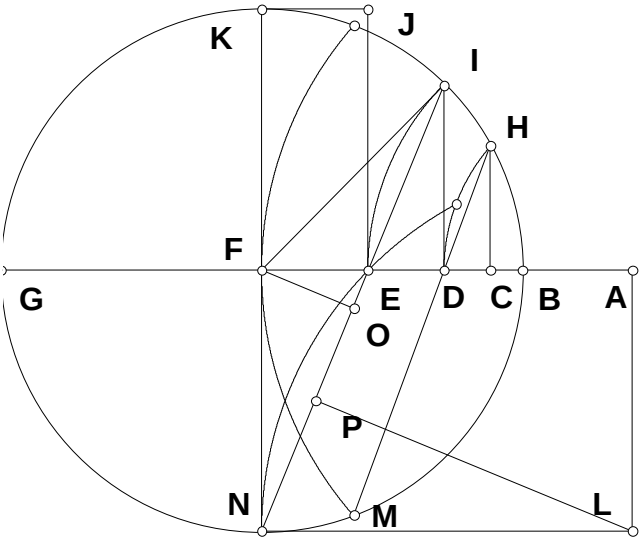
$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

$AB \cdot (N - 1) - BG = 0$

$\frac{1}{2} \cdot AB \cdot (N - 1) - BF = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AF = 0 \qquad AB\sqrt{N} - AE = 0 \qquad AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$



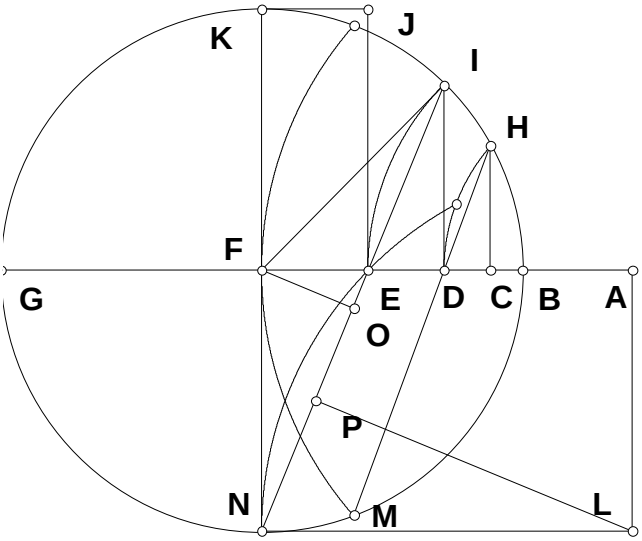
$N_2 := 2 \qquad BG_2 := 1 \qquad BF_2 := \frac{BG_2}{2}$

$BE_2 := \frac{BF_2}{N_2} \qquad EF_2 := BF_2 - BE_2$

$FN_2 := BF_2 \qquad EN_2 := \sqrt{EF_2^2 + FN_2^2}$

$NP_2 := \frac{EN_2}{2} \qquad LN_2 := \frac{EN_2 \cdot NP_2}{EF_2}$

$AF_2 := LN_2 \qquad AB_2 := AF_2 - BF_2$



$$\frac{BG_2}{2} - BF_2 = 0 \qquad \frac{BG_2}{(2 \cdot N_2)} - BE_2 = 0$$

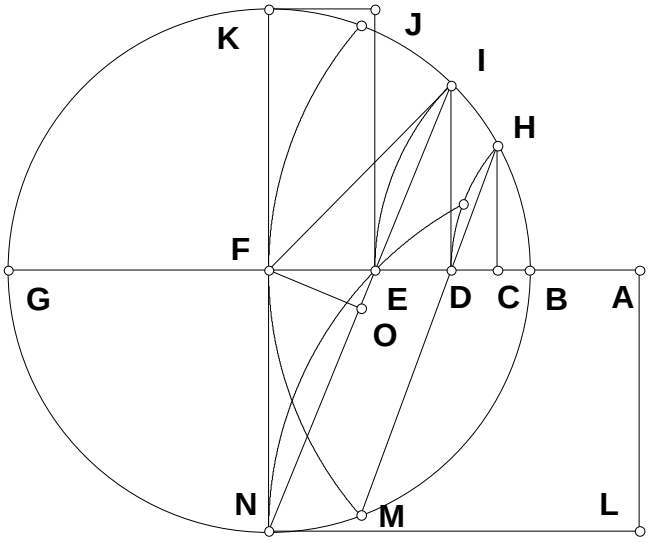
$$\frac{BG_2}{2} \cdot \frac{(N_2 - 1)}{N_2} - EF_2 = 0$$

$$\frac{BG_2}{2} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - EN_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - NP_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{(2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{[N_2 \cdot (N_2 - 1)]} - LN_2 = 0$$

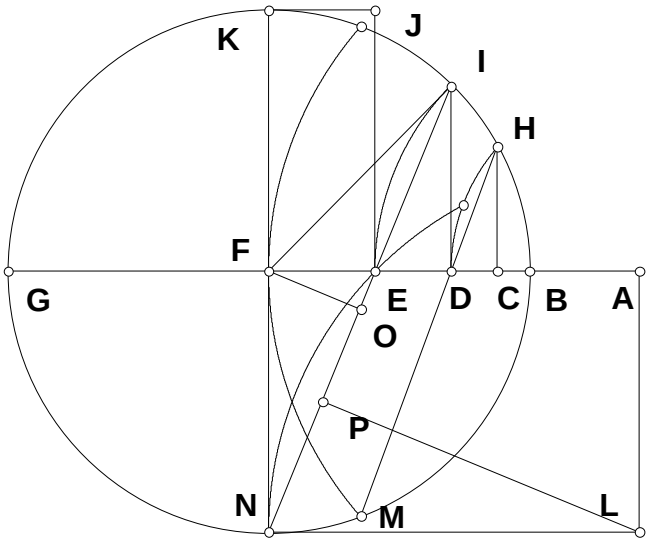
$$\frac{BG_2}{4 \cdot N_2 \cdot (N_2 - 1)} - AB_2 = 0$$



$$AI := AE \quad FI := BF \quad AD := \frac{AI^2 + AF^2 - FI^2}{2\,AF}$$

$$BD := AD - AB$$

$$AD := 2 \cdot AB \cdot \frac{N}{(N+1)} \qquad BD := AB \cdot \frac{(N-1)}{(1+N)}$$



$N_3 := 3.364 \quad BG_3 := 2.708 \quad BF_3 := \frac{BG_3}{2}$

$BD_3 := \frac{BF_3}{N_3} \quad DG_3 := BG_3 - BD_3$

$DI_3 := \sqrt{BD_3 \cdot DG_3} \quad DF_3 := BF_3 - BD_3$

$FN_3 := BF_3 \quad EF_3 := \frac{DF_3 \cdot FN_3}{FN_3 + DI_3}$

$EN_3 := \sqrt{EF_3^2 + FN_3^2} \quad NP_3 := \frac{EN_3}{2}$

$LN_3 := \frac{EN_3 \cdot NP_3}{EF_3} \quad AF_3 := LN_3$

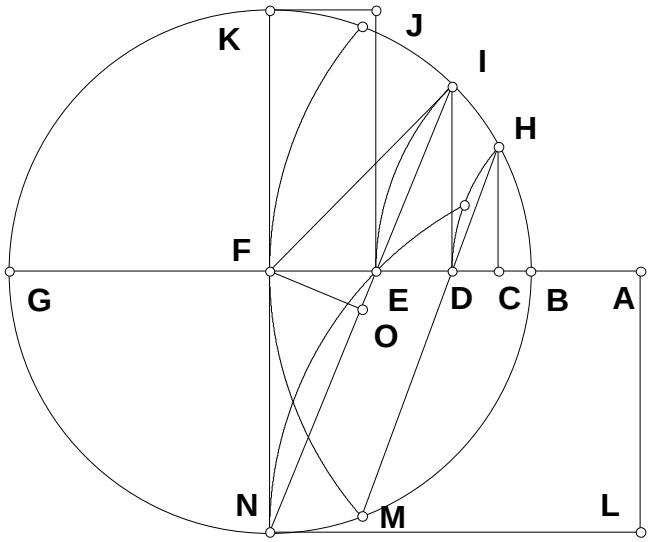
$AB_3 := AF_3 - BF_3 \quad AB_3 = 0.573$

$BD_3 := \frac{1}{2} \cdot \frac{BG_3}{N_3} \quad DG_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(2 \cdot N_3 - 1)}{N_3} \quad DI_3 := \frac{1}{(2 \cdot N_3)} \cdot BG_3 \cdot \sqrt{2 \cdot N_3 - 1}$

$DF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{N_3} \quad EF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{(N_3 + \sqrt{2 \cdot N_3 - 1})}$

$EN_3 := \frac{1}{2} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}} \quad NP_3 := \frac{1}{4} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}}$

$LN_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{N_3}{(N_3 - 1)} \quad AB_3 := \frac{1}{2} \cdot \frac{BG_3}{(N_3 - 1)}$



$AH := AD$

$FH := BF \quad AC := \frac{AH^2 + AF^2 - FH^2}{2 \cdot AF}$

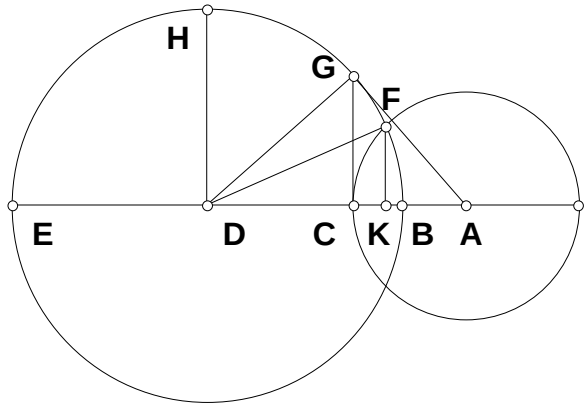
$BC := AC - AB$

$AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$

$AC := AB \cdot N \cdot \frac{\left(6 \cdot N + 1 + N^2\right)}{(N+1)^3} \qquad BC := AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3}$

Isolating A Problem 060301

If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?



$N := 4 \qquad BE := 1$

$BD := \frac{BE}{2} \qquad BC := \frac{BE}{N} \qquad CE := BE - BC$

$CG := \sqrt{BC \cdot CE} \qquad CD := BD - BC \qquad AC := \frac{CG^2}{CD}$

$AF := AC \qquad AD := AC + CD \qquad DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2\ AD} \qquad BK := BD - DK$

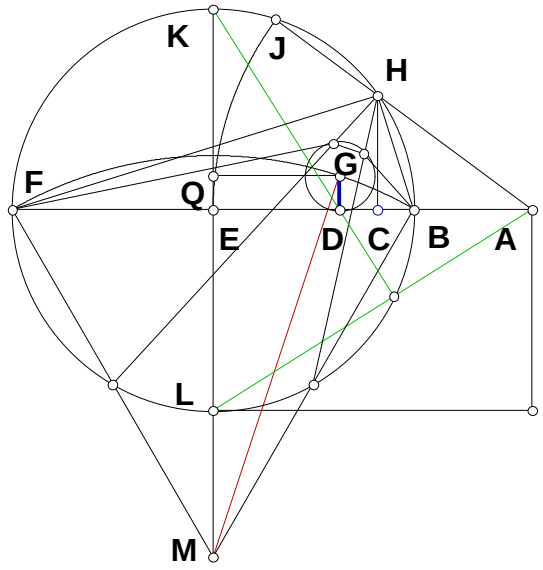
$CK := BC - BK$

$BE \cdot \frac{(N-1)}{N} - CE = 0 \qquad BE \cdot \frac{\sqrt{(N-1)}}{N} - CG = 0 \qquad BE \cdot \frac{(N-2)}{2 \cdot N} - CD = 0$

$BE \cdot \frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0 \qquad BE \cdot \frac{N}{2 \cdot (N-2)} - AD = 0 \qquad BE \cdot \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$BE \cdot \frac{(3 \cdot N - 2)}{N^3} - BK = 0 \qquad BE \cdot \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0 \qquad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

For Any AB 061001



For any AB, AF what is DG?

$N := 4.39 \quad AB := .615 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad EK := BE$

$AE := AB + BE \quad DE := \frac{EK^2}{AE} \quad EF := BE$

$FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM$

$DG := EQ$

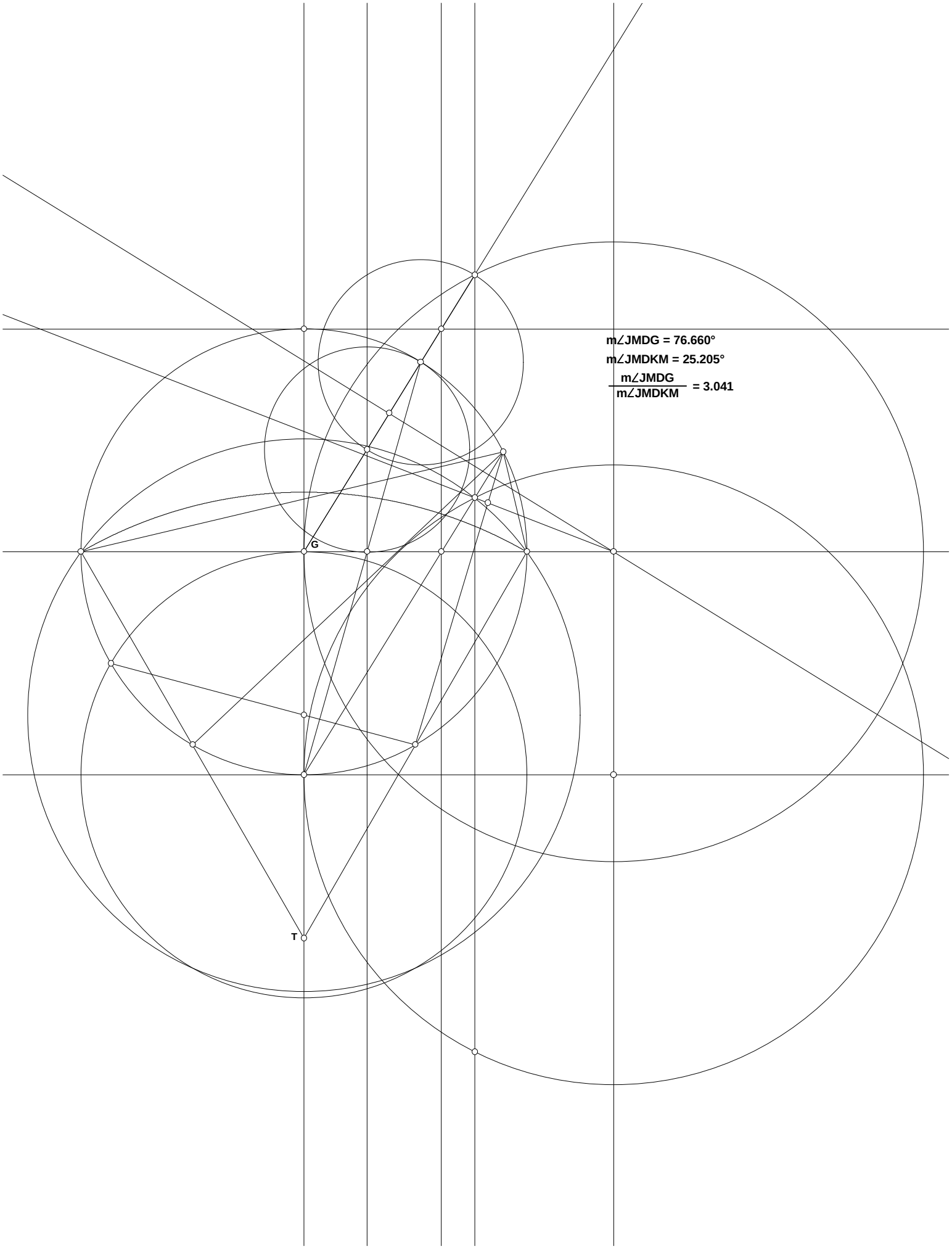
Some Algebraic Names:

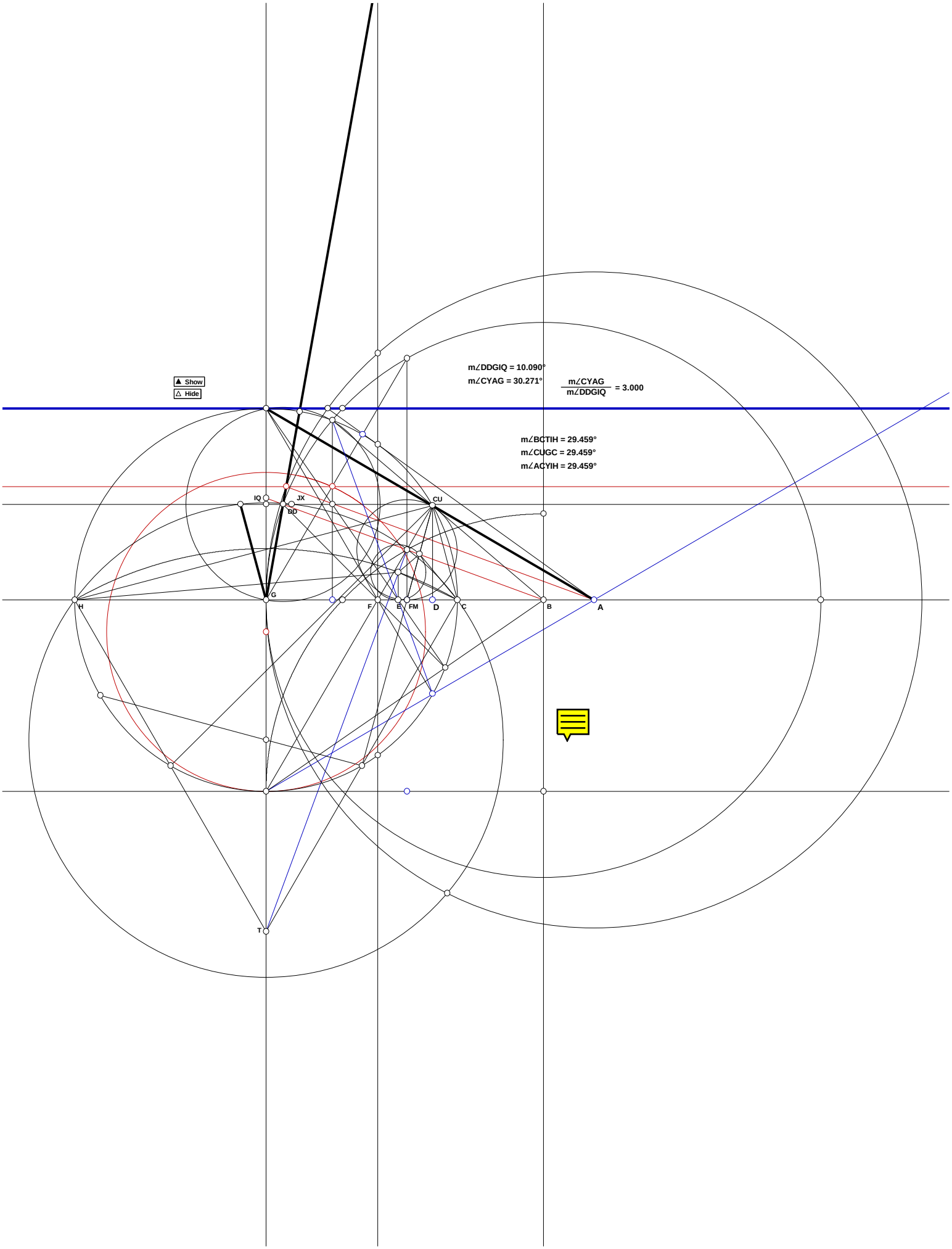
$AB \cdot (N - 1) - BF = 0 \qquad \frac{AB \cdot (N - 1)}{2} - BE = 0 \qquad \frac{1}{2} \cdot AB \cdot (N + 1) - AE = 0$

$\frac{1}{2} \cdot AB \cdot \frac{(N - 1)^2}{(N + 1)} - DE = 0 \qquad \frac{1}{2} \cdot \sqrt{3} \cdot AB \cdot (N - 1) - EM = 0$

$$\frac{AB \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)\right]}{2 \cdot (N + 1)} - MQ = 0$$

$$\frac{AB \cdot (N - 1) \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \sqrt{3} - \sqrt{3} \cdot N\right]}{2 \cdot (N + 1)} - DG = 0$$

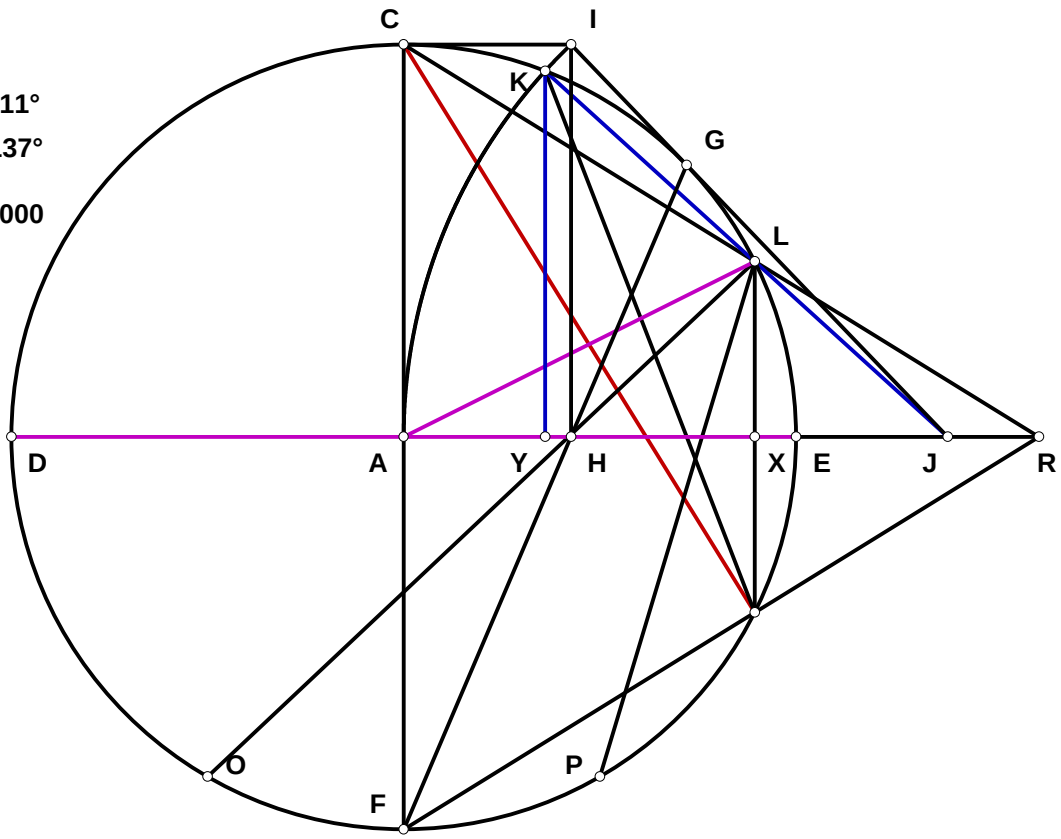

m∠JMDG = 76.660°
m∠JMDKM = 25.205°
m∠JMDG / m∠JMDKM = 3.041
G
T


Show
Hide
m∠DDGIQ = 10.090°
m∠CYAG = 30.271°
m∠CYAG / m∠DDGIQ = 3.000
m∠BCTIH = 29.459°
m∠CUGC = 29.459°
m∠ACYIH = 29.459°
IQ
JX
DD
CU
G
H
F
E
FM
D
C
B
A
T

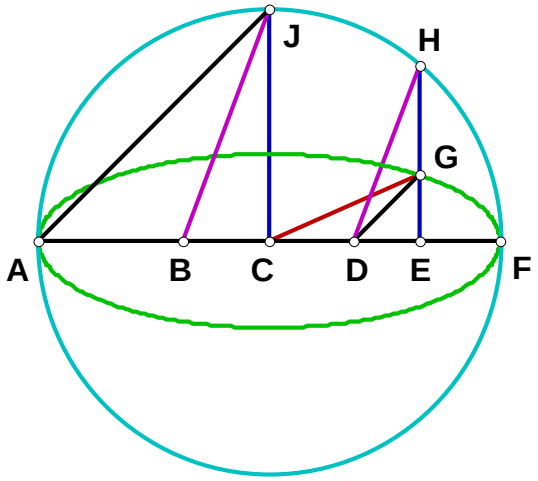
$m\angle LAC = 63.411°$

$m\angle KAC = 21.137°$

$\frac{m\angle LAC}{m\angle KAC} = 3.000$



Elipse By Parallels 082601



$N_1 := 4 \quad N_2 := 1$

$AF := 1 \quad AC := \frac{AF}{2} \quad CJ := AC$

$BC := \frac{AC}{N_1} \qquad AE := \frac{AF}{N_2}$

$EF := AF - AE \quad EH := \sqrt{AE \cdot EF} \quad EG := \frac{BC \cdot EH}{CJ}$

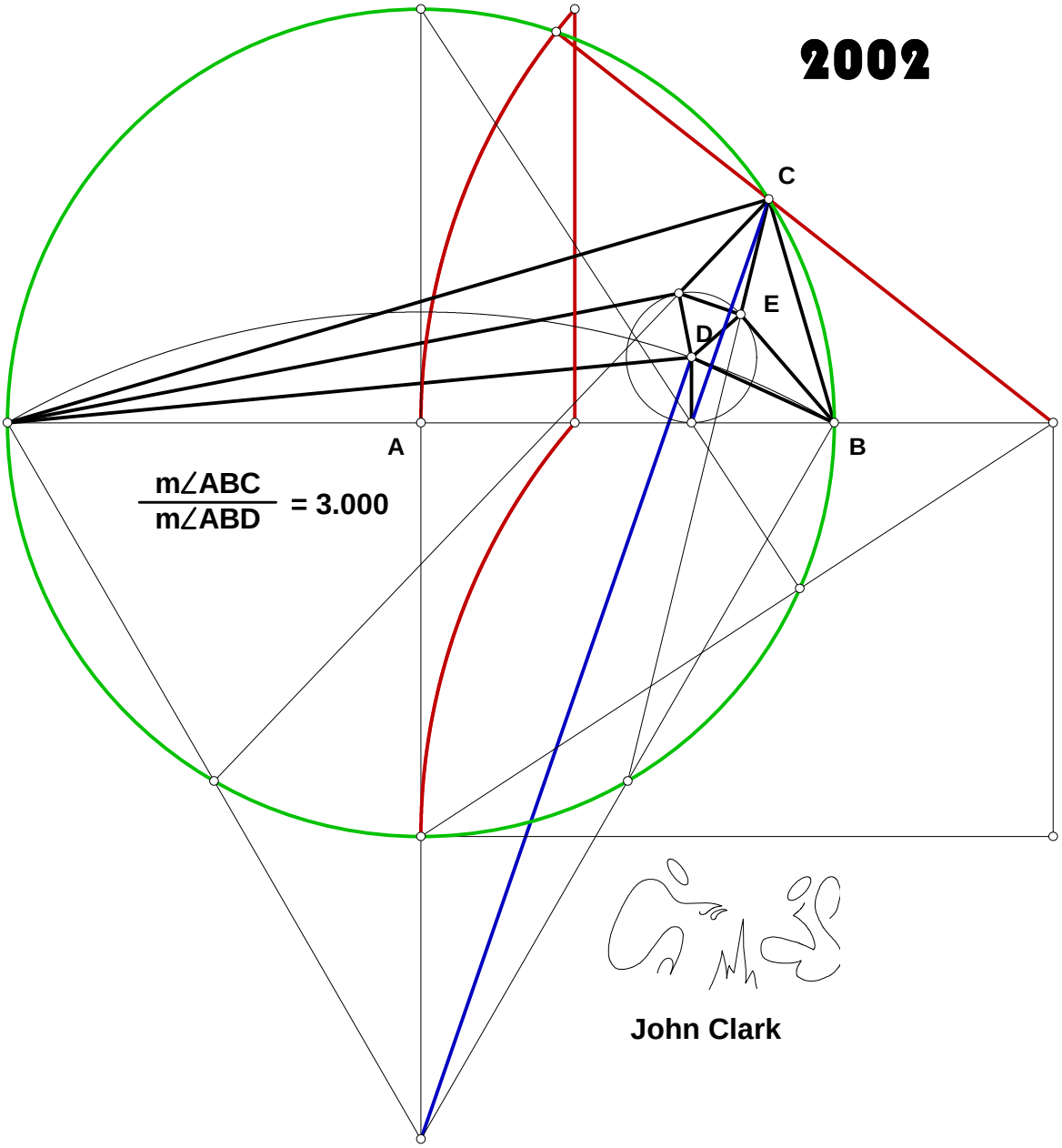
$CE := AE - AC \quad CG := \sqrt{CE^2 + EG^2}$

Some Algebraic Names:

$BC := \frac{1}{(2 \cdot N_1)} \quad AE := \frac{1}{N_2} \quad EF := 1 - \frac{1}{N_2} \quad EH := \frac{\sqrt{N_2 - 1}}{N_2} \quad EG := \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2}$

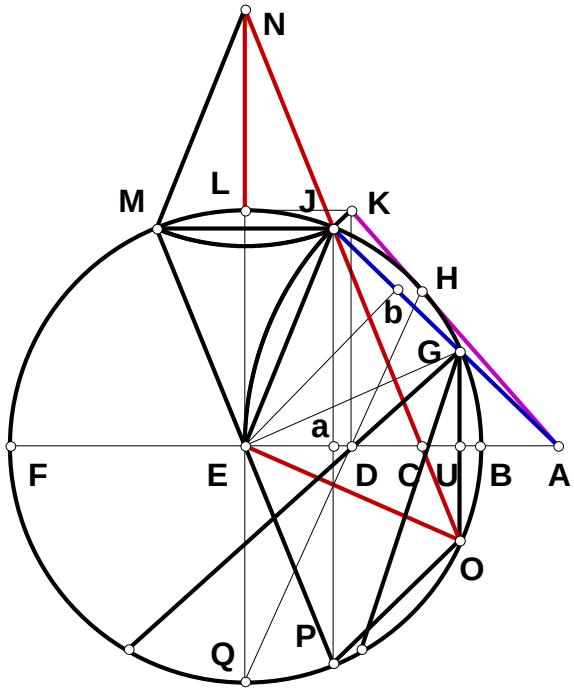
$CE := \frac{1}{N_2} - \frac{1}{2} \quad CG := \frac{1}{2} \cdot \frac{\sqrt{4 \cdot N_1^2 - 4 \cdot N_1^2 \cdot N_2 + N_1^2 \cdot N_2^2 + 4 \cdot N_2 - 4}}{(N_1 \cdot N_2)}$

Three Pieces Of Paper



John Clark

Just Another Proof Of Paper 010202



$N := 7 \quad AB := 1 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad AE := BE + AB$

$AJ := AE \quad EJ := BE \quad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \quad GJ := 2 \cdot Gb \quad AG := AJ - GJ$

$Aa := AE - Ea \quad AU := \frac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \quad GU := \frac{Ja \cdot AG}{AJ}$

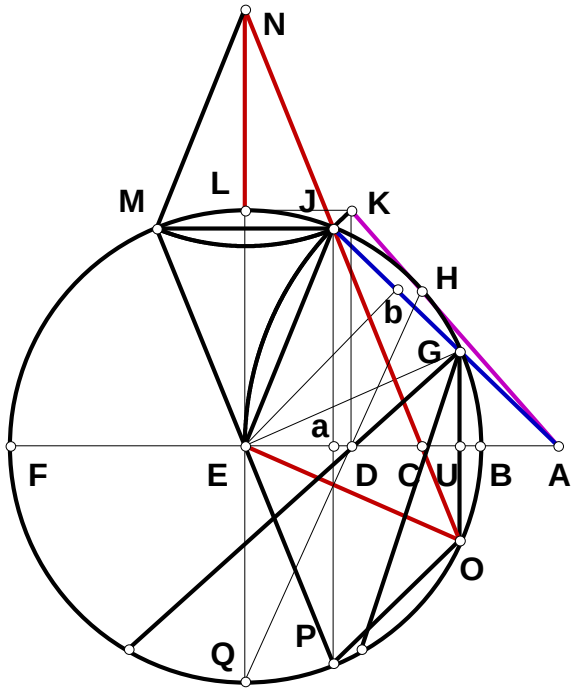
$Ua := Aa - AU \quad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \quad JN - BE = 0$

From 4/29/94 $OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \quad NO := JO + JN \quad EU := Ua + Ea \quad EN := \sqrt{NO^2 - EU^2} \quad EL := BE \quad LN := EN - EL$

Some Algebraic Names;



$$N - 1 - BF = 0 \qquad \frac{N-1}{2} - BE = 0$$

$$\frac{N+1}{2} - AE = 0 \qquad \frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0$$

$$\frac{2\cdot N}{N+1} - AG = 0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0$$

$$\frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0$$

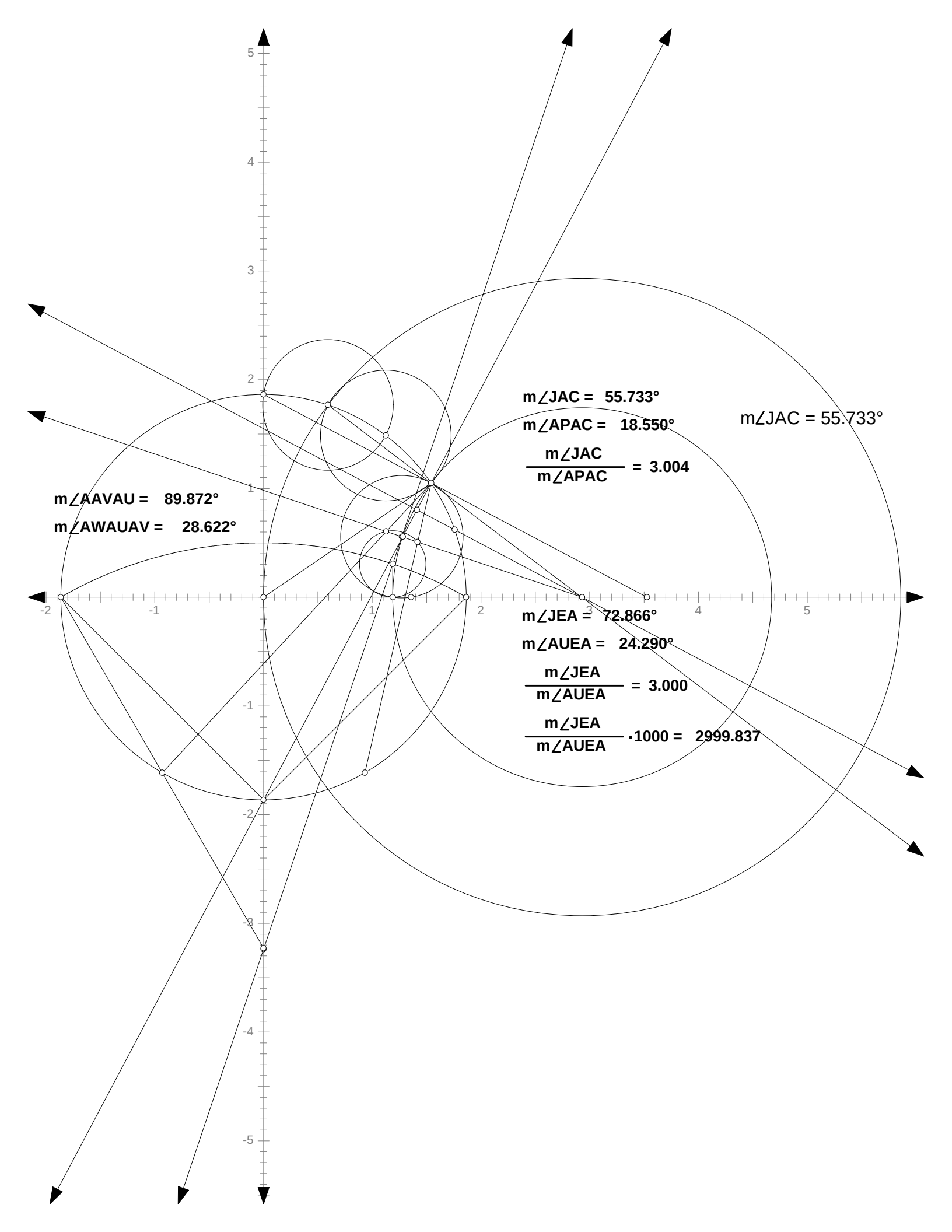
$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0$$

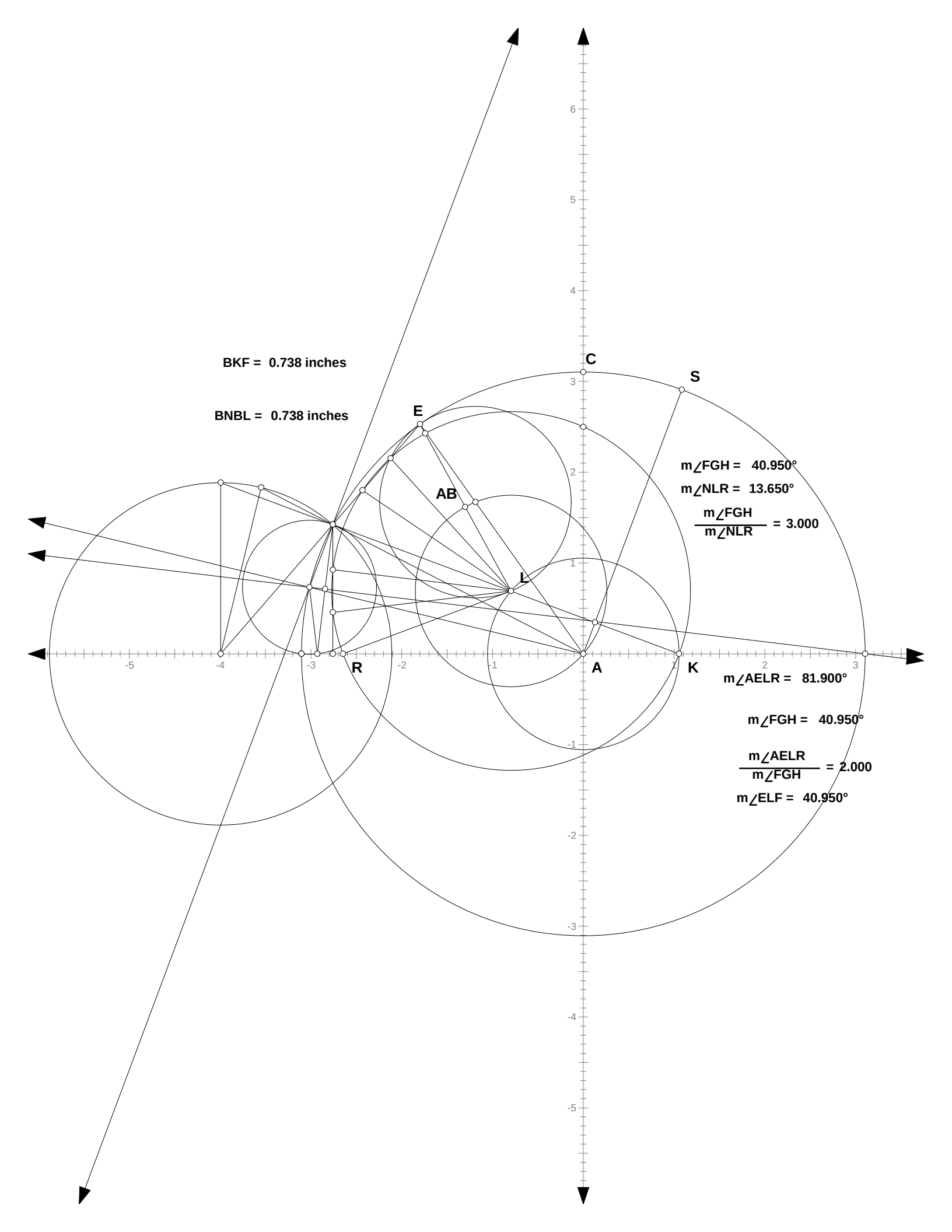
$$\frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0 \qquad \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \qquad \frac{N-1}{2} - JN = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - OP = 0 \qquad \frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

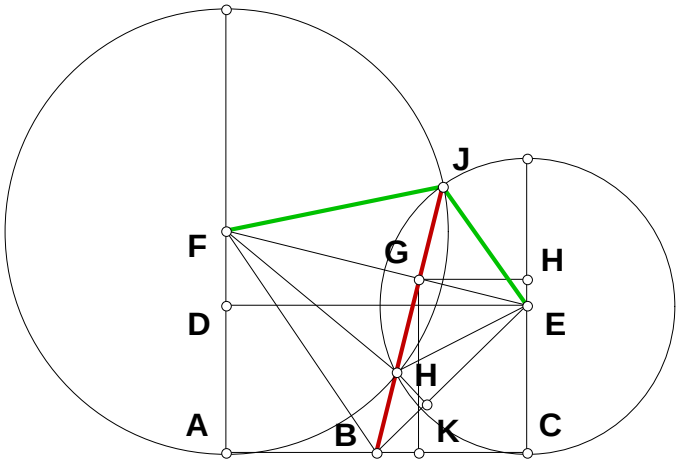
$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$


m∠JAC = 55.733°
m∠JAC = 55.733°
m∠APAC = 18.550°
m∠JAC / m∠APAC = 3.004
m∠AAVAU = 89.872°
m∠AWAUAV = 28.622°
m∠JEA = 72.866°
m∠AUEA = 24.290°
m∠JEA / m∠AUEA = 3.000
m∠JEA / m∠AUEA ·1000 = 2999.837


BKF = 0.738 inches
BNBL = 0.738 inches
C
S
E
AB
L
R
A
K
m∠FGH = 40.950°
m∠NLR = 13.650°
m∠FGH / m∠NLR = 3.000
m∠AELR = 81.900°
m∠FGH = 40.950°
m∠AELR / m∠FGH = 2.000
m∠ELF = 40.950°

Easy Power-Line 06_20_02

For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.



$AC := 5 \qquad AF := 4$

$CE := 3 \qquad FH := AF \quad EH := CE$

$AD := CE \quad DF := AF - AD$

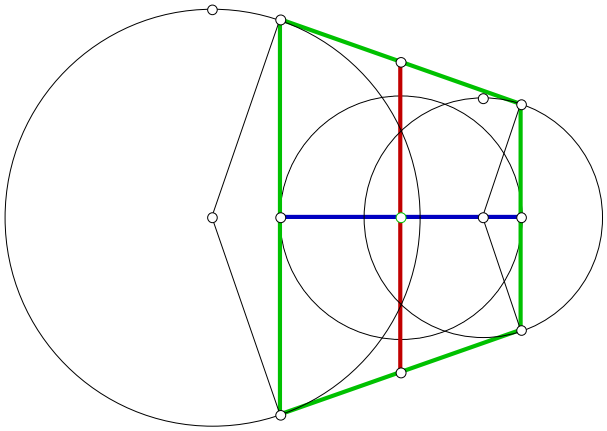
$EJ := CE \quad FJ := AF$

$DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

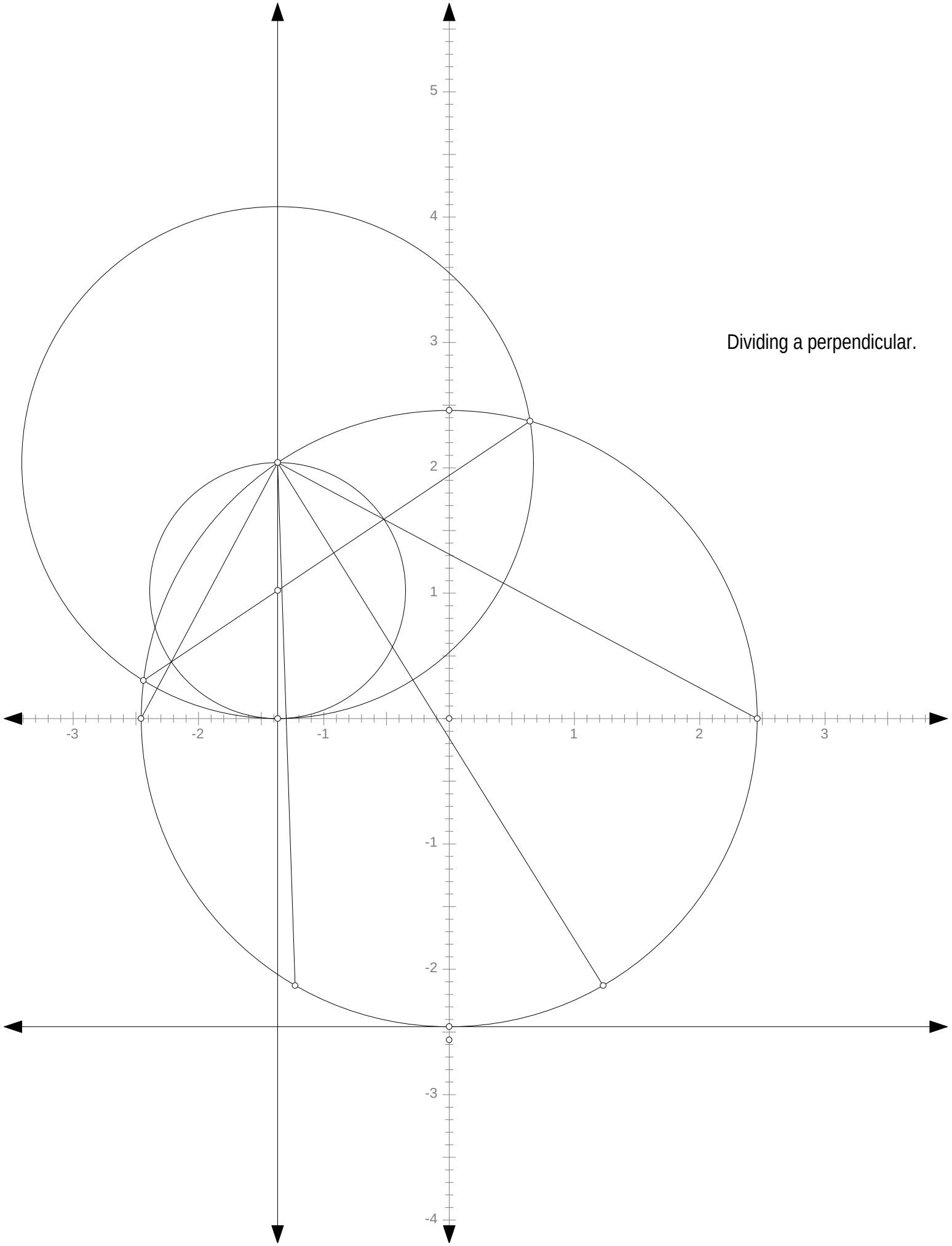
$EG := \dfrac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF}$

$EH := \dfrac{DF \cdot EG}{EF} \quad CH := CE + EH \quad GH := \dfrac{DE \cdot EG}{EF} \quad CK := GH \quad KG := CH \quad BK := \dfrac{DF \cdot KG}{DE} \quad BC := CK + BK$

$BC - \dfrac{AC}{2} = 0$



The power line is the line upon which lays a segment between the two midpoints on the two tangents of two given circles or between a point and a circle.
The intersection of two circles lays halfway between their two tangents.
A power plane lays between two spheres or a sphere and a point.

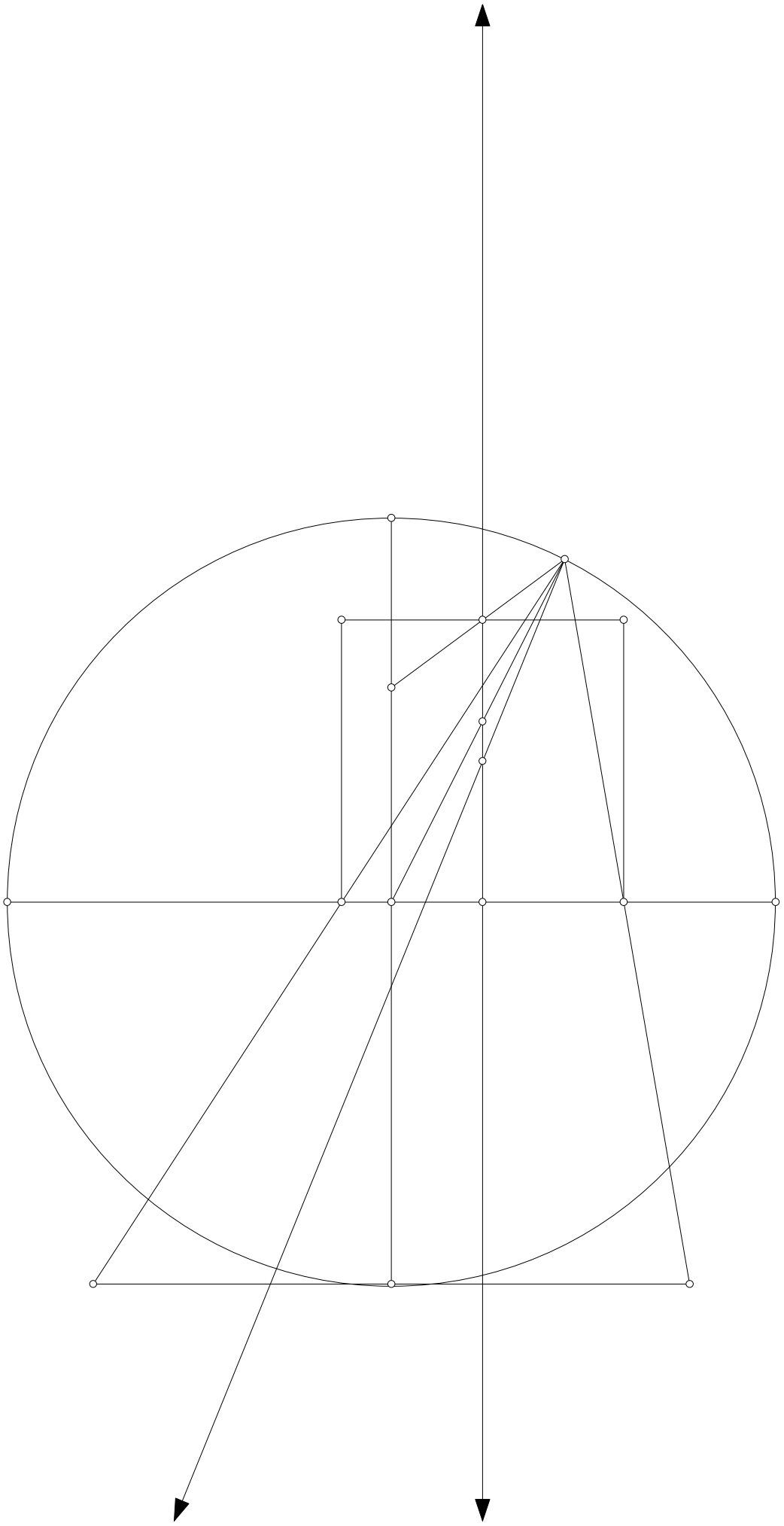


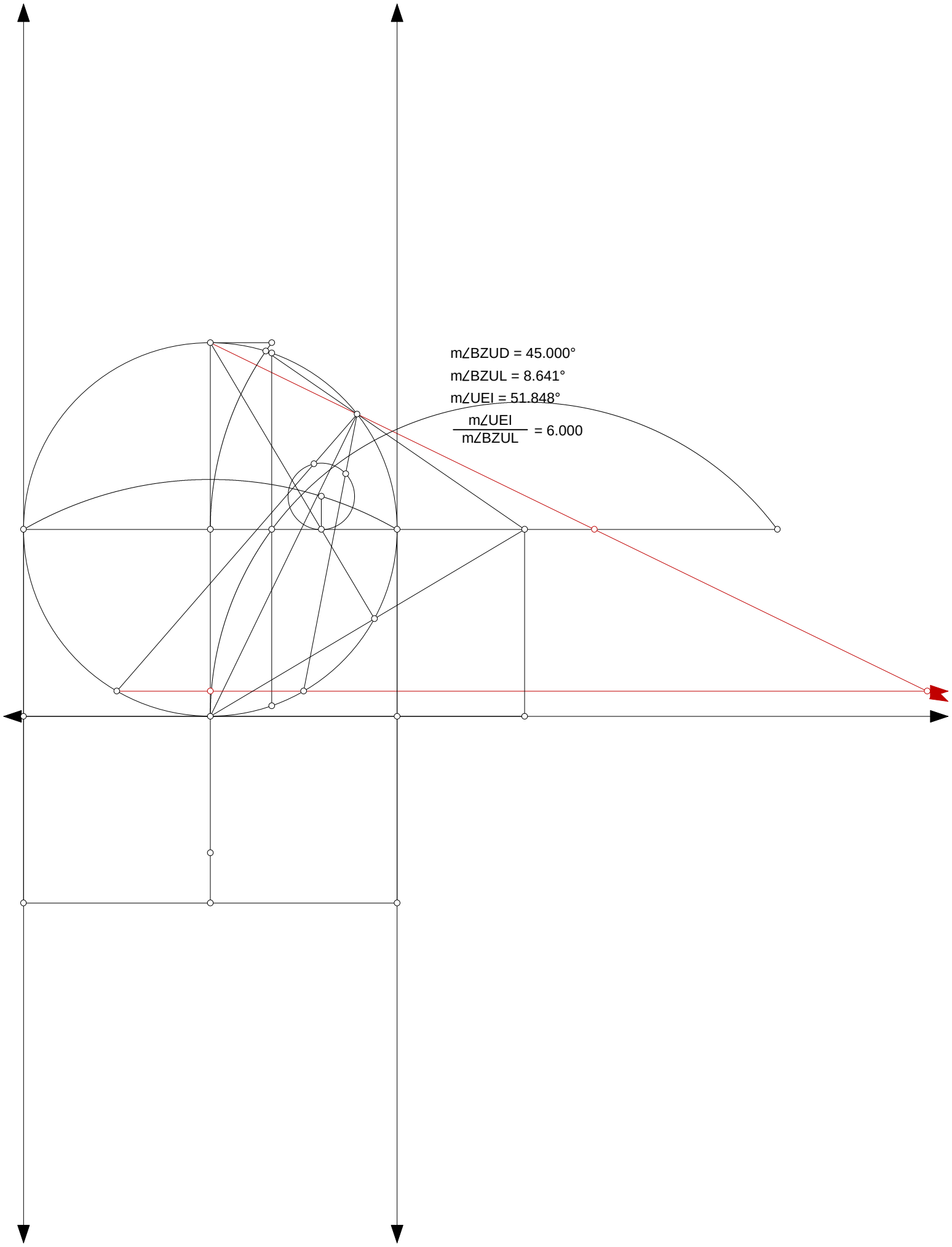
Dividing a perpendicular.

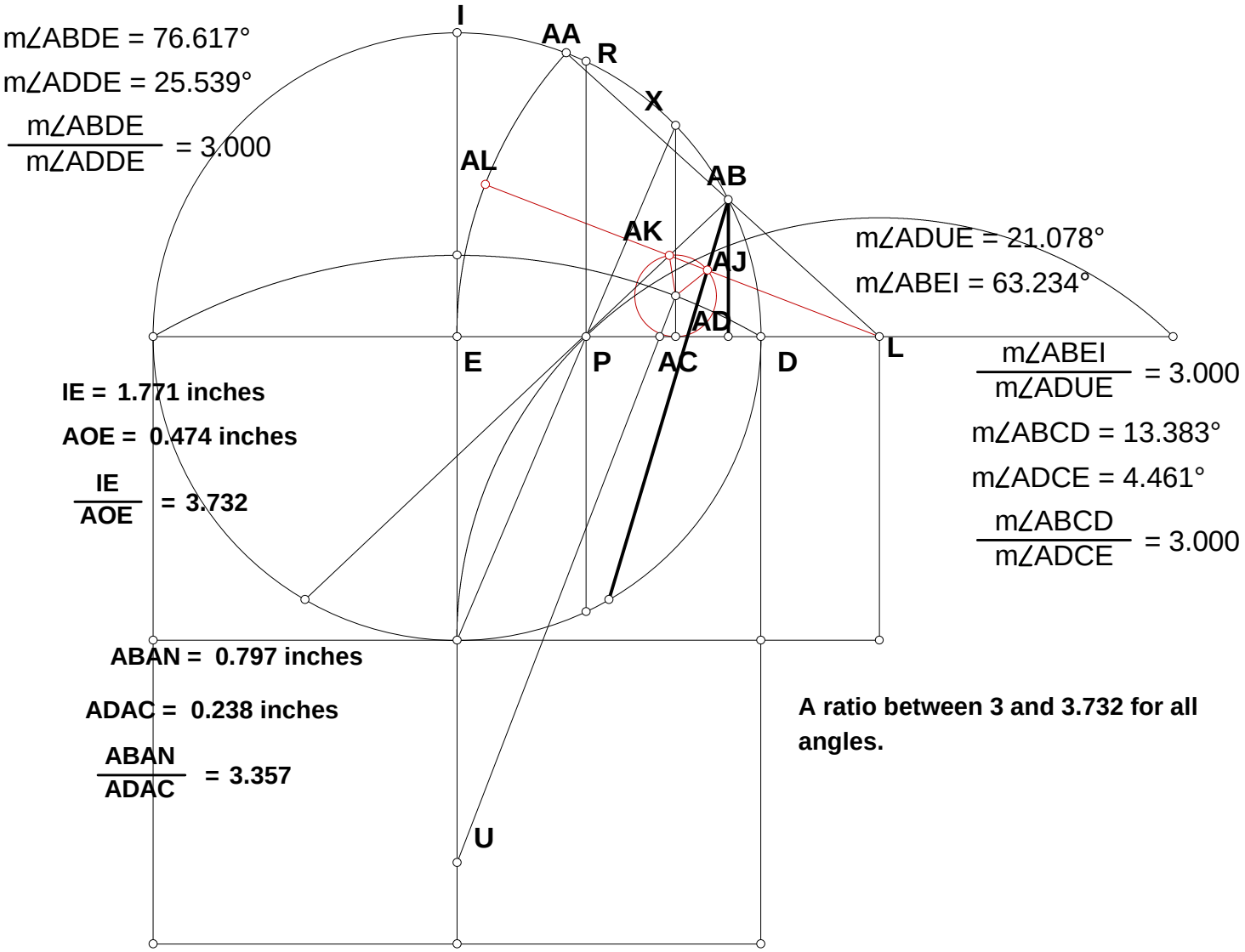
ZX = 1.951 inches

ABX = 0.702 inches

$$\frac{ZX}{ABX} = 2.780$$




m∠BZUD = 45.000°
m∠BZUL = 8.641°
m∠UEI = 51.848°
m∠UEI / m∠BZUL = 6.000



m∠ABDE = 76.617°

m∠ADDE = 25.539°

$\frac{m\angle ABDE}{m\angle ADDE} = 3.000$

m∠ADUE = 21.078°

m∠ABEI = 63.234°

$\frac{m\angle ABEI}{m\angle ADUE} = 3.000$

IE = 1.771 inches

AOE = 0.474 inches

$\frac{IE}{AOE} = 3.732$

m∠ABCD = 13.383°

m∠ADCE = 4.461°

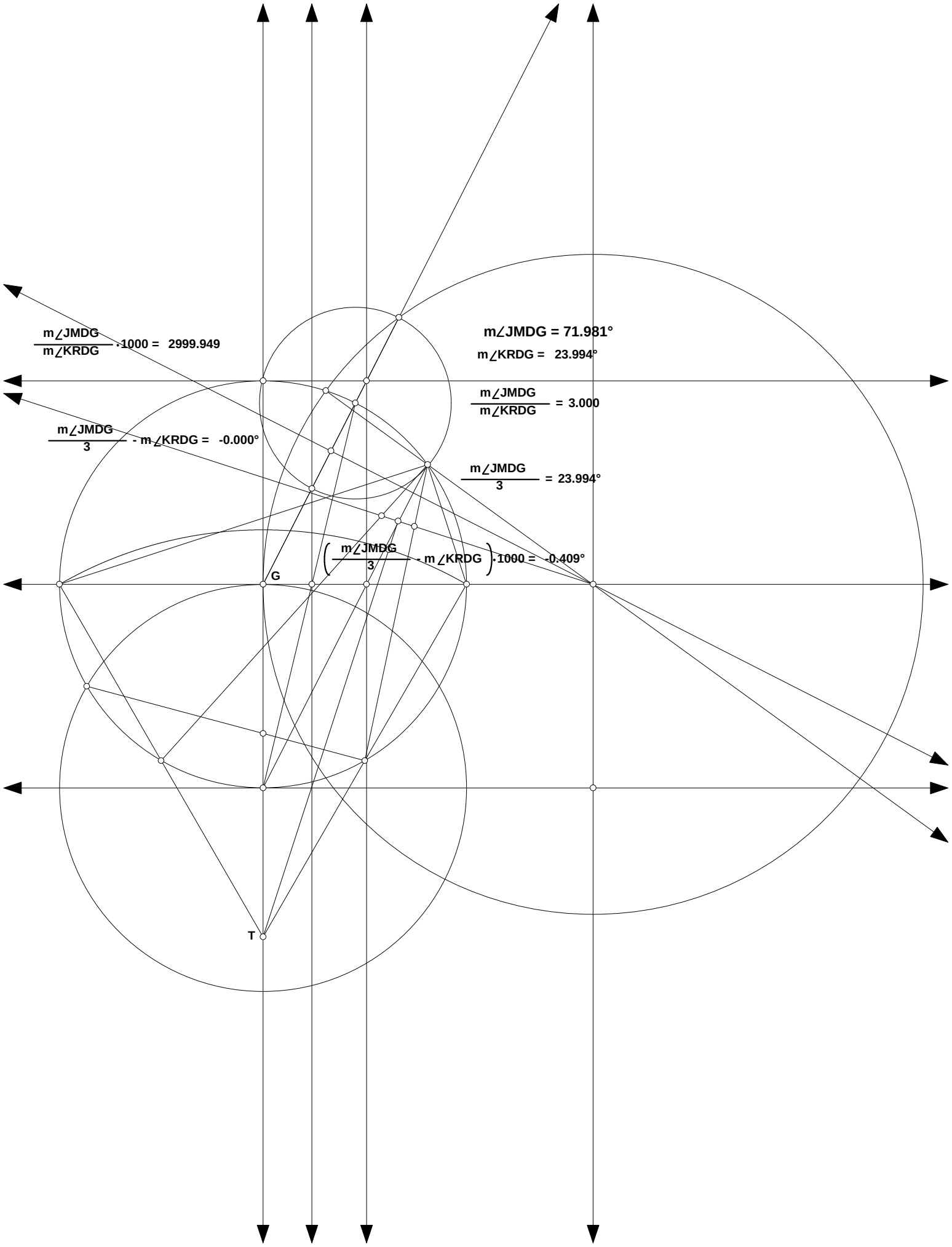
$\frac{m\angle ABCD}{m\angle ADCE} = 3.000$

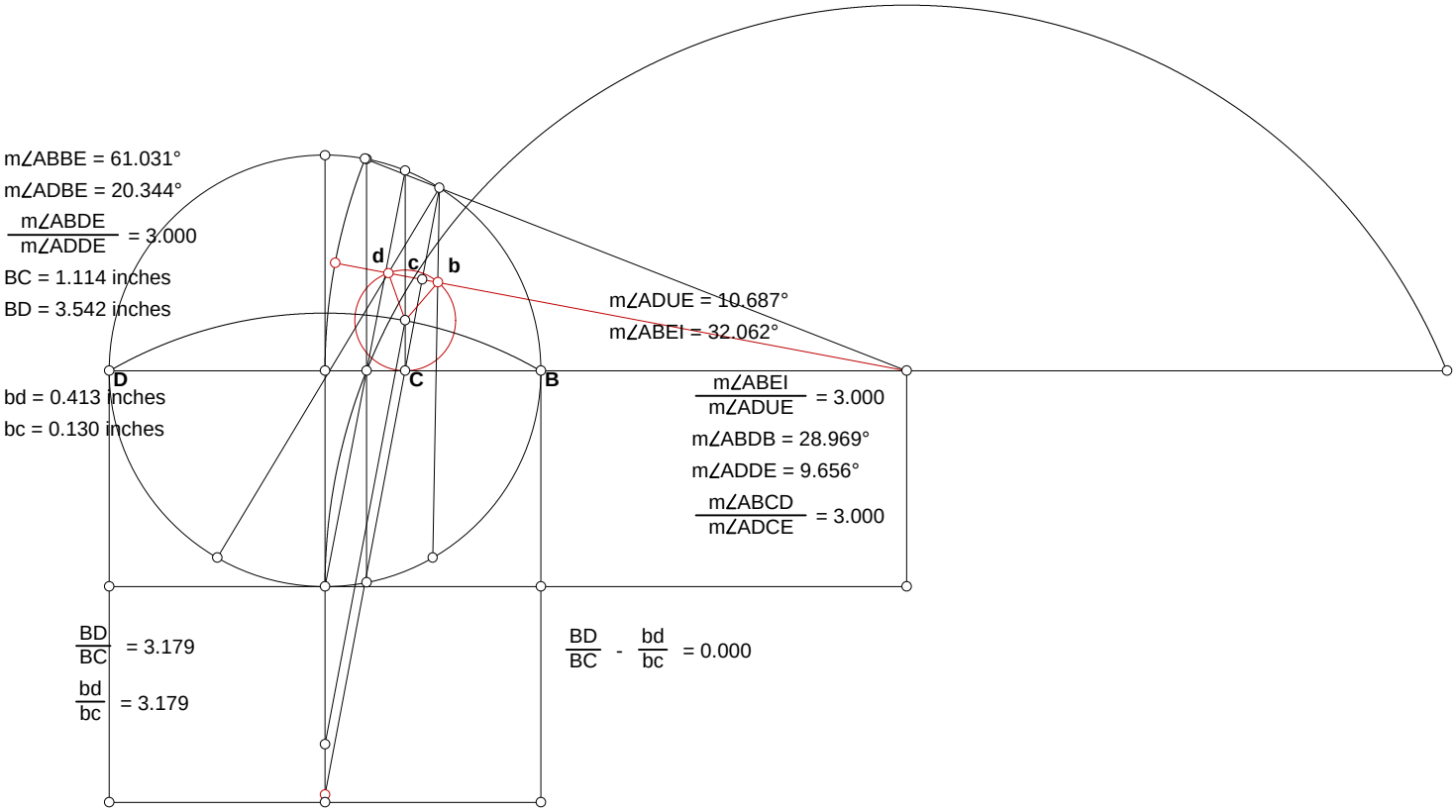
ABAN = 0.797 inches

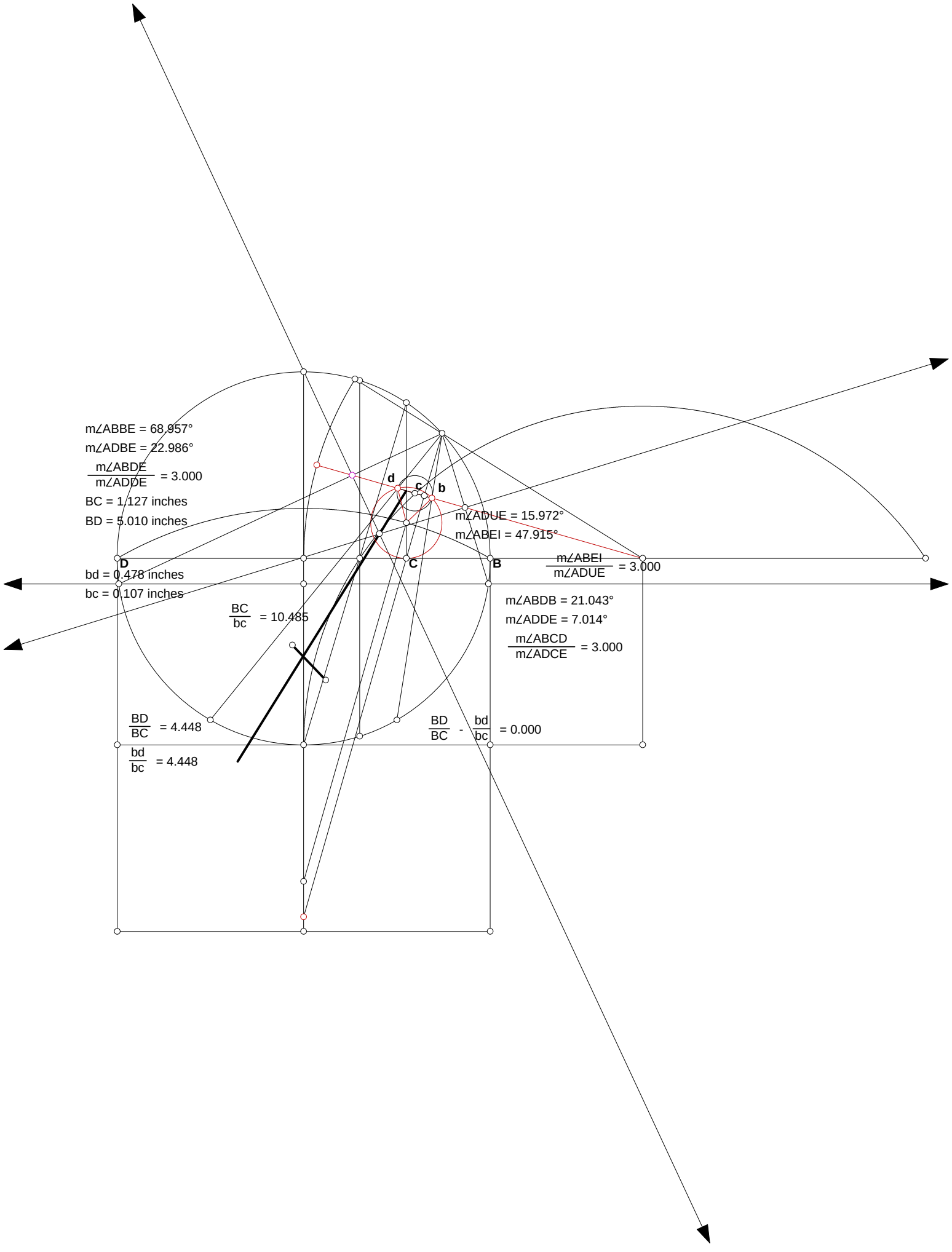
ADAC = 0.238 inches

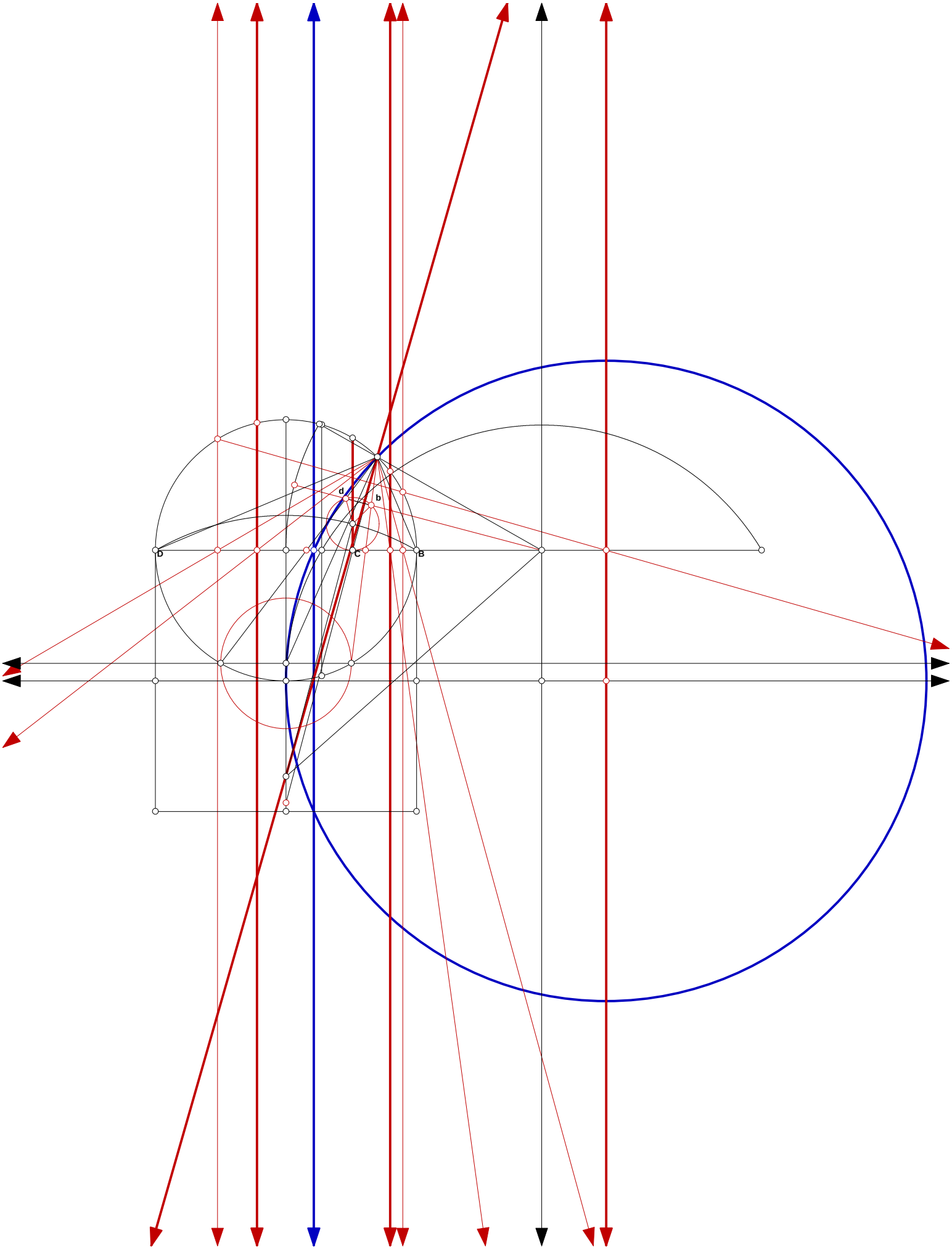
$\frac{ABAN}{ADAC} = 3.357$

A ratio between 3 and 3.732 for all angles.

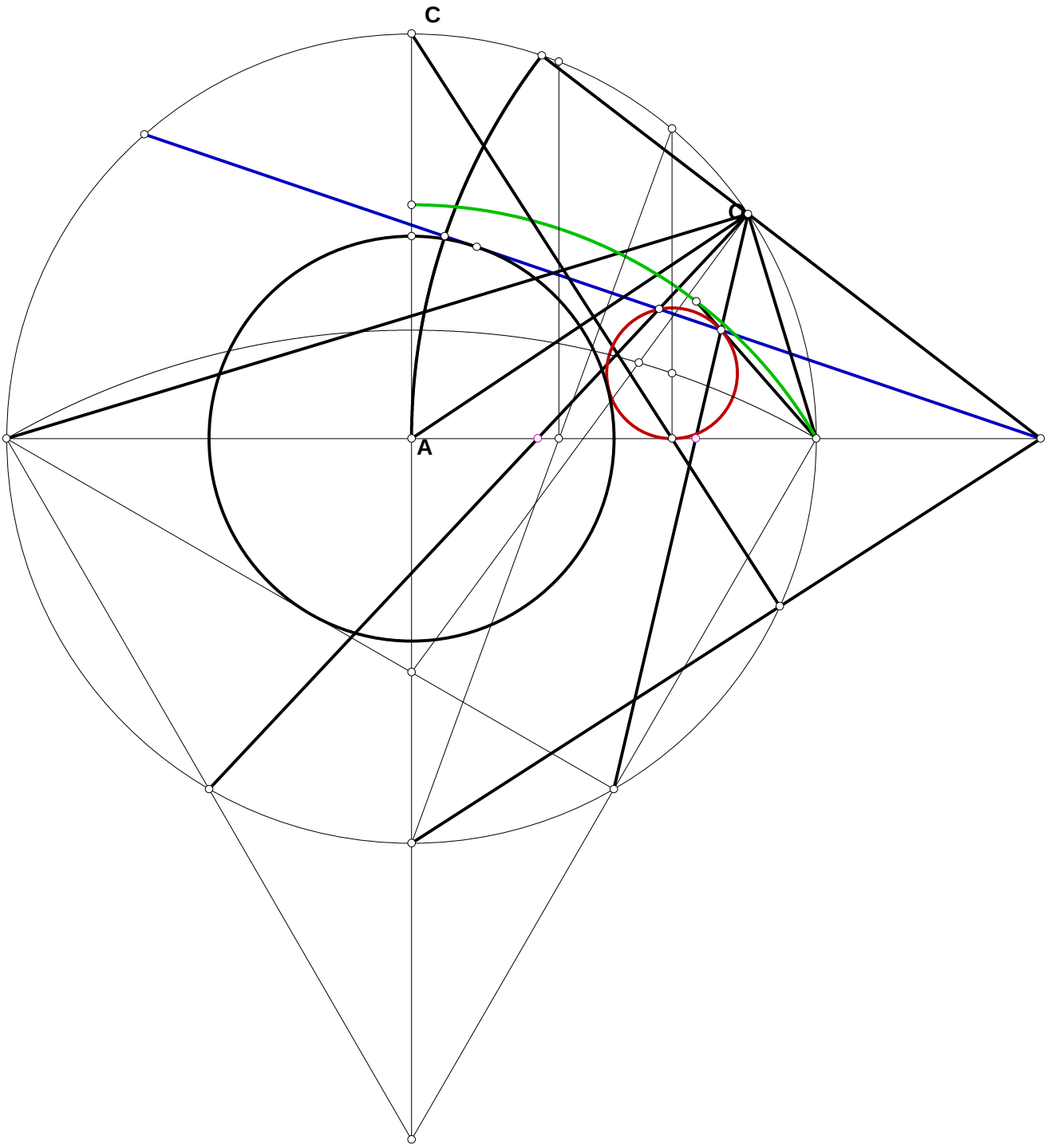

$\frac{m\angle JMDG}{m\angle KRDG} \cdot 1000 = 2999.949$
$m\angle JMDG = 71.981°$
$m\angle KRDG = 23.994°$
$\frac{m\angle JMDG}{m\angle KRDG} = 3.000$
$\frac{m\angle JMDG}{3} - m\angle KRDG = -0.000°$
$\frac{m\angle JMDG}{3} = 23.994°$
$\left(\frac{m\angle JMDG}{3} - m\angle KRDG\right) \cdot 1000 = -0.409°$
G
T


m∠ABBE = 61.031°
m∠ADBE = 20.344°
m∠ABDE / m∠ADDE = 3.000
BC = 1.114 inches
BD = 3.542 inches
bd = 0.413 inches
bc = 0.130 inches
d
c
b
D
C
B
m∠ADUE = 10.687°
m∠ABEI = 32.062°
m∠ABEI / m∠ADUE = 3.000
m∠ABDB = 28.969°
m∠ADDE = 9.656°
m∠ABCD / m∠ADCE = 3.000
BD / BC = 3.179
bd / bc = 3.179
BD / BC - bd / bc = 0.000


m∠ABBE = 68.957°
m∠ADBE = 22.986°
m∠ABDE / m∠ADDE = 3.000
BC = 1.127 inches
BD = 5.010 inches
bd = 0.478 inches
bc = 0.107 inches
BC / bc = 10.485
BD / BC = 4.448
bd / bc = 4.448
BD / BC - bd / bc = 0.000
m∠ADUE = 15.972°
m∠ABEI = 47.915°
m∠ABEI / m∠ADUE = 3.000
m∠ABDB = 21.043°
m∠ADDE = 7.014°
m∠ABCD / m∠ADCE = 3.000
D
C
B
d
c
b


d
b
D
C
B

m∠OKJ = 18.765°

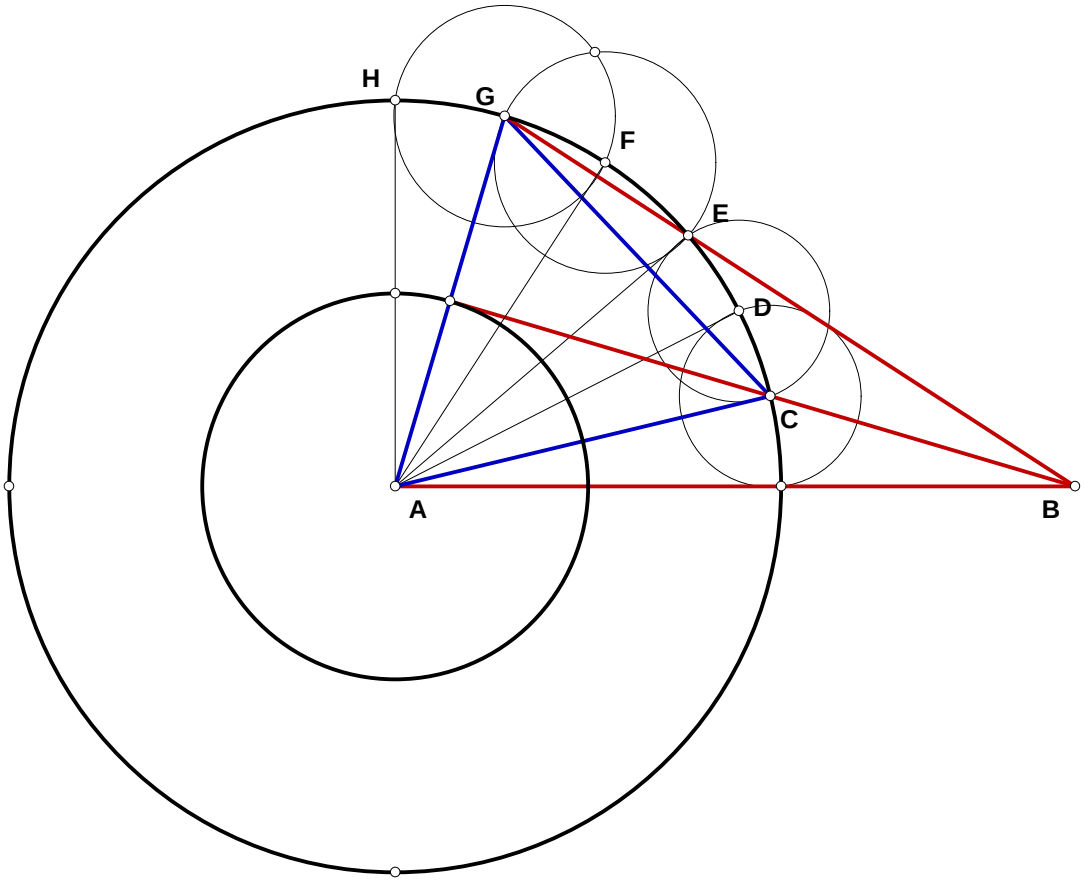


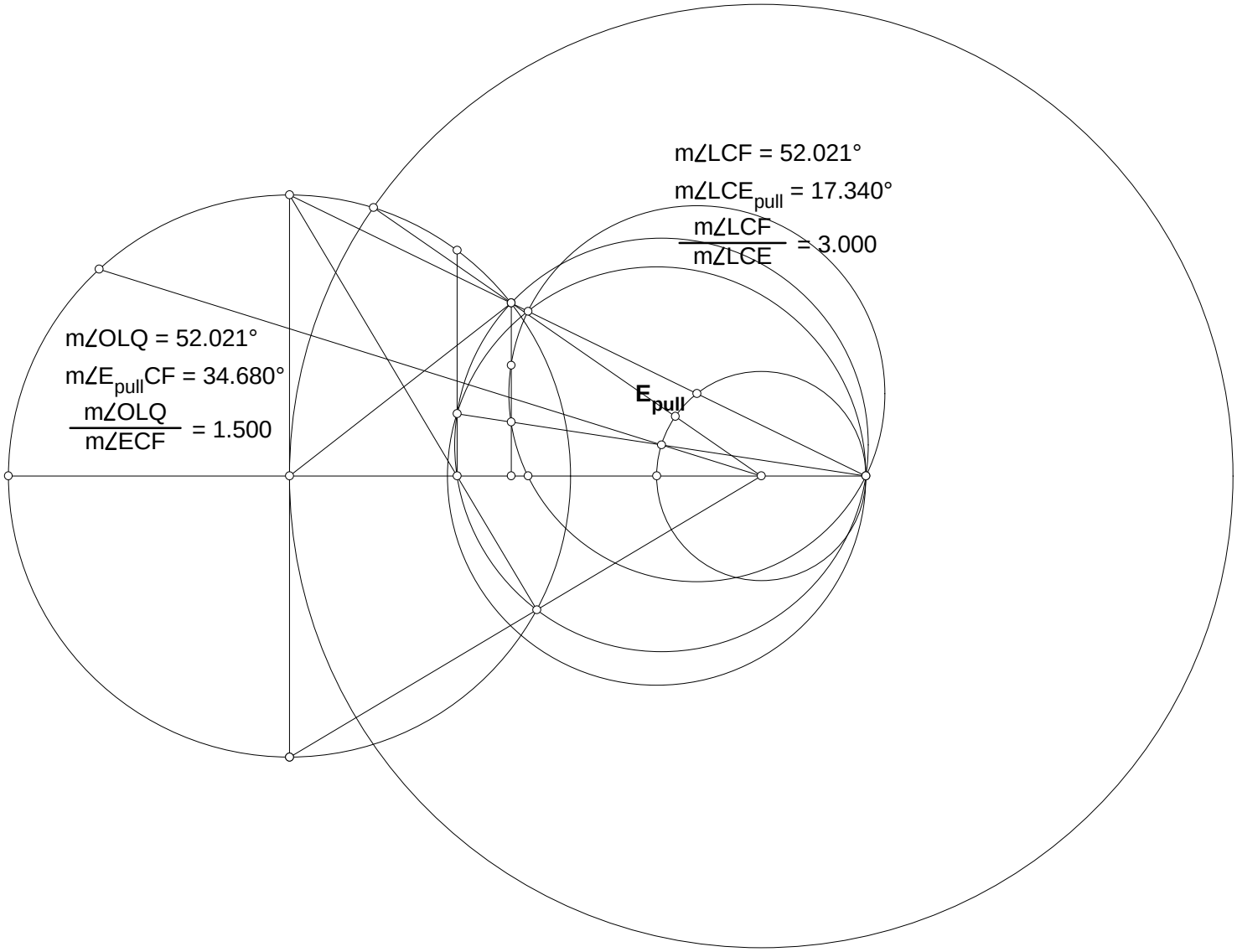
m∠EAH = 49.456°

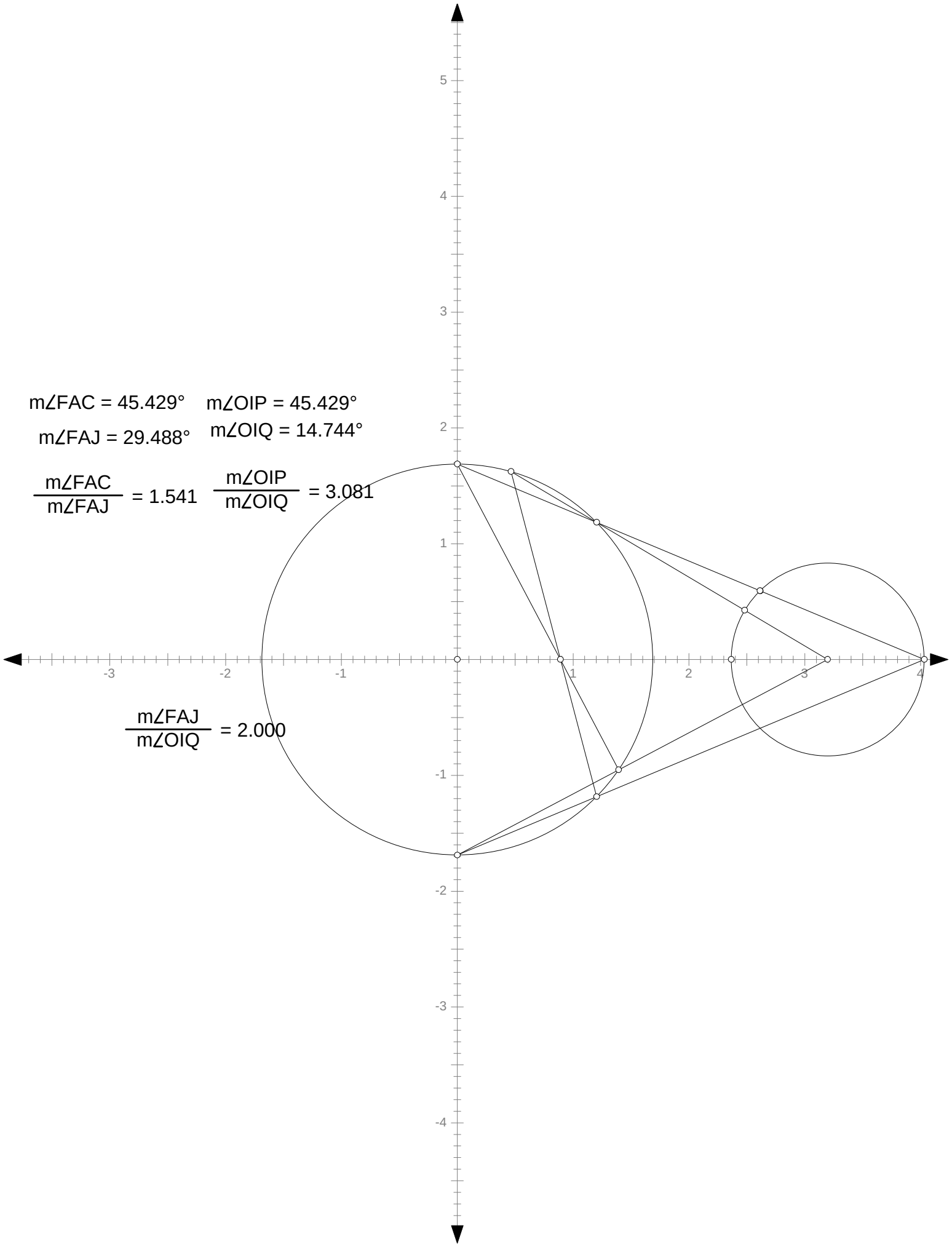
m∠EAH = 49.456°

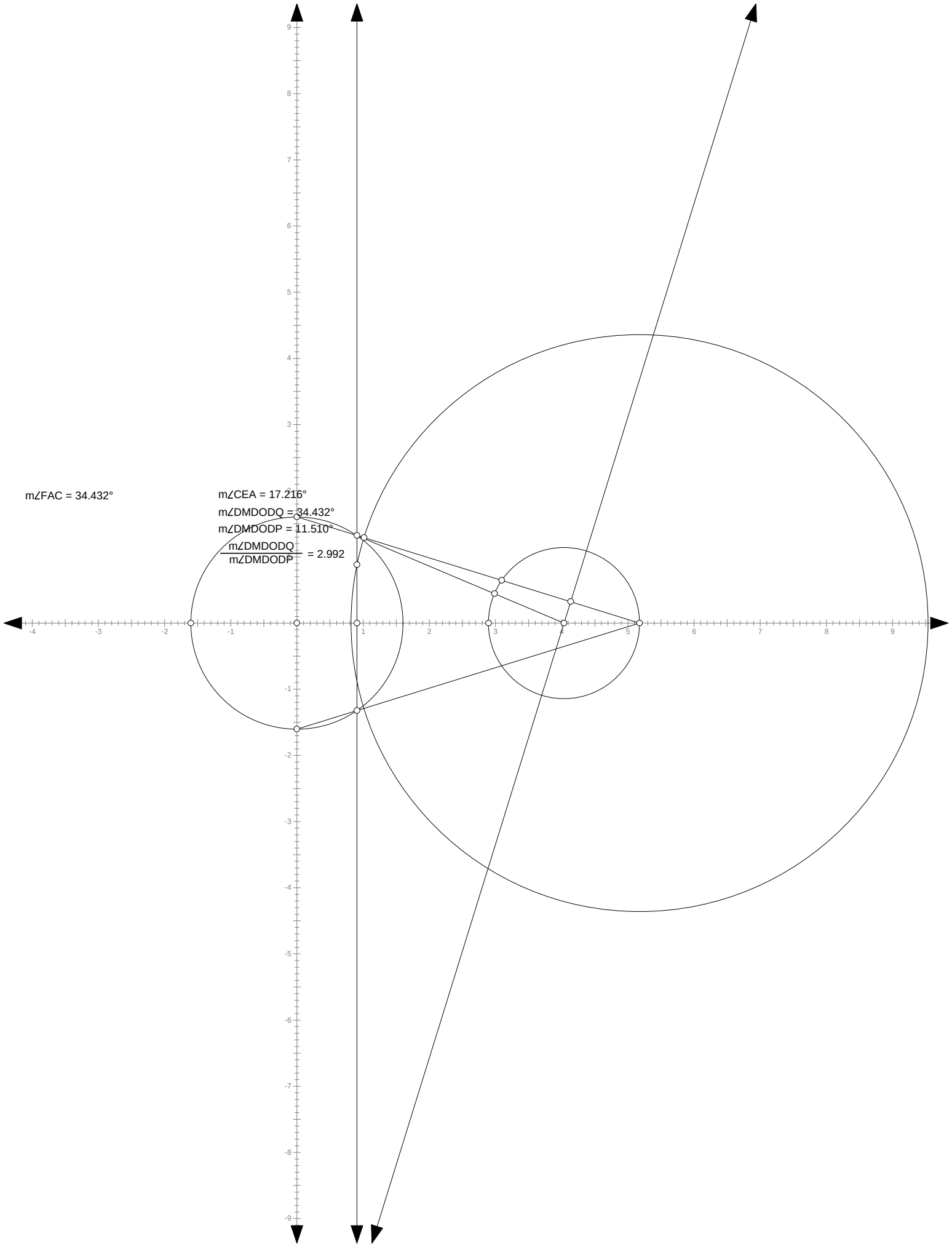
m∠EAH = 40.544°

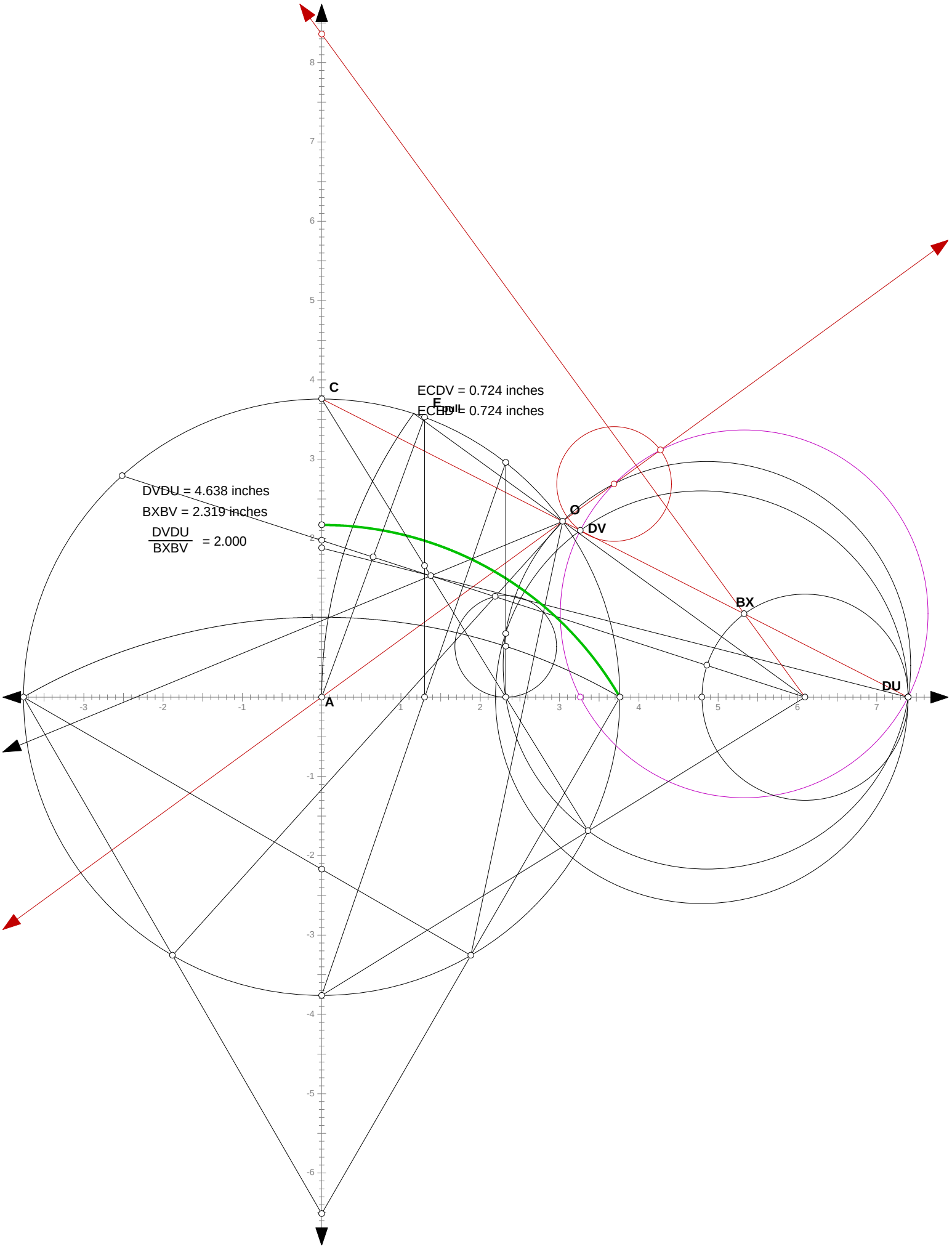
m∠EAH + m∠EAH = 90.000°

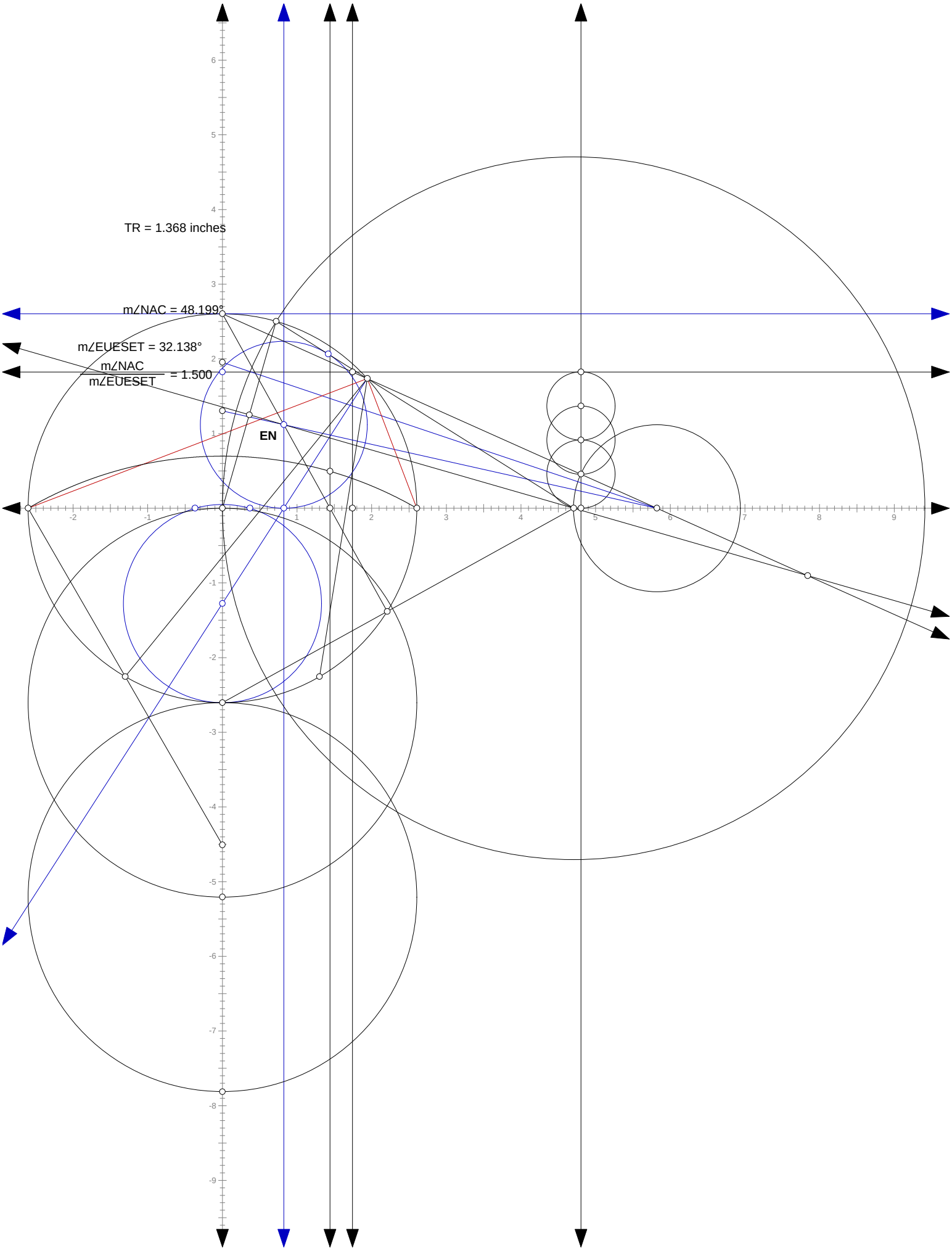


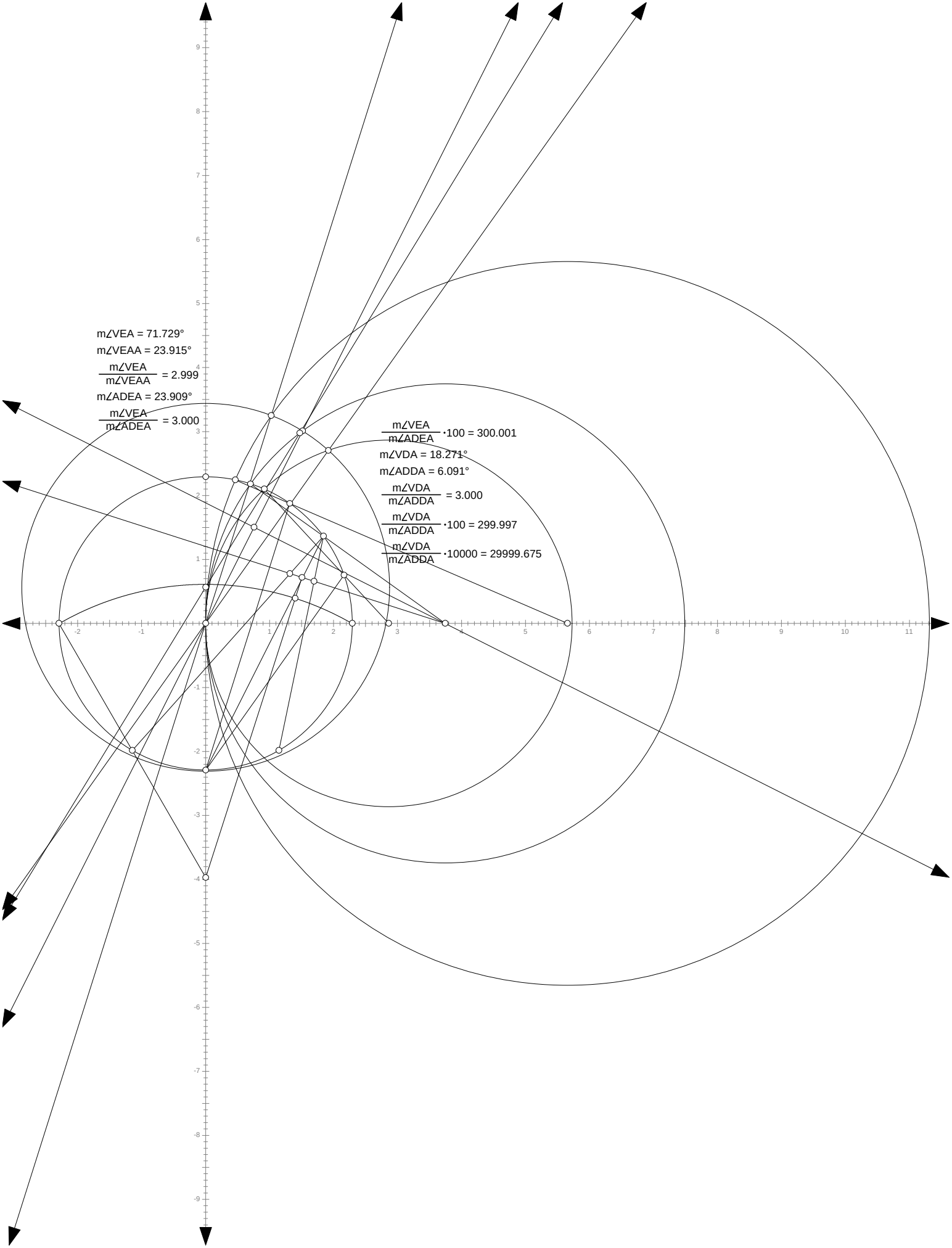

m∠LCF = 52.021°
m∠LCE$_{pull}$ = 17.340°
$\frac{m\angle LCF}{m\angle LCE}$ = 3.000
m∠OLQ = 52.021°
m∠E$_{pull}$CF = 34.680°
$\frac{m\angle OLQ}{m\angle ECF}$ = 1.500
E$_{pull}$

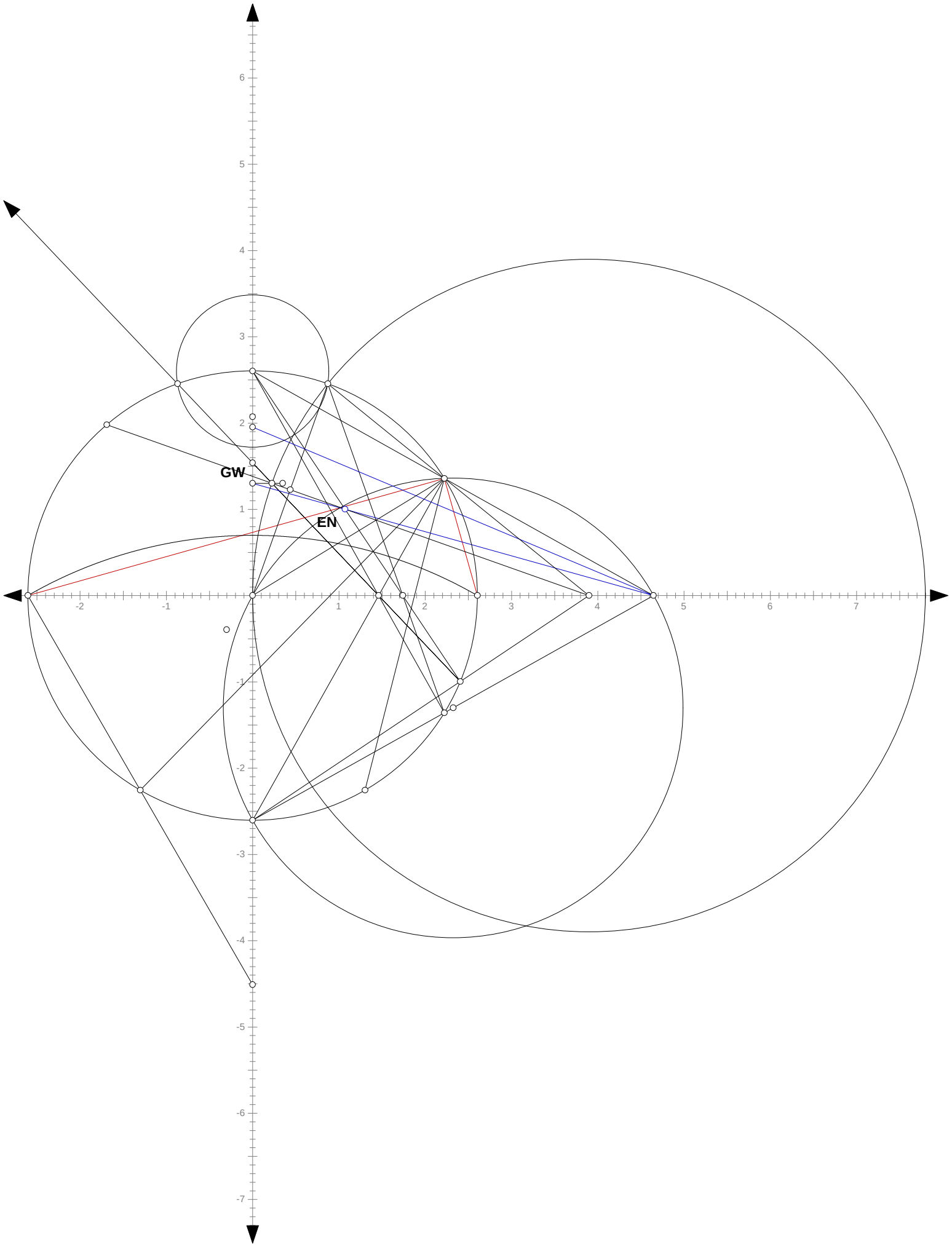

m∠FAC = 45.429°
m∠OIP = 45.429°
m∠FAJ = 29.488°
m∠OIQ = 14.744°
m∠FAC / m∠FAJ = 1.541
m∠OIP / m∠OIQ = 3.081
m∠FAJ / m∠OIQ = 2.000

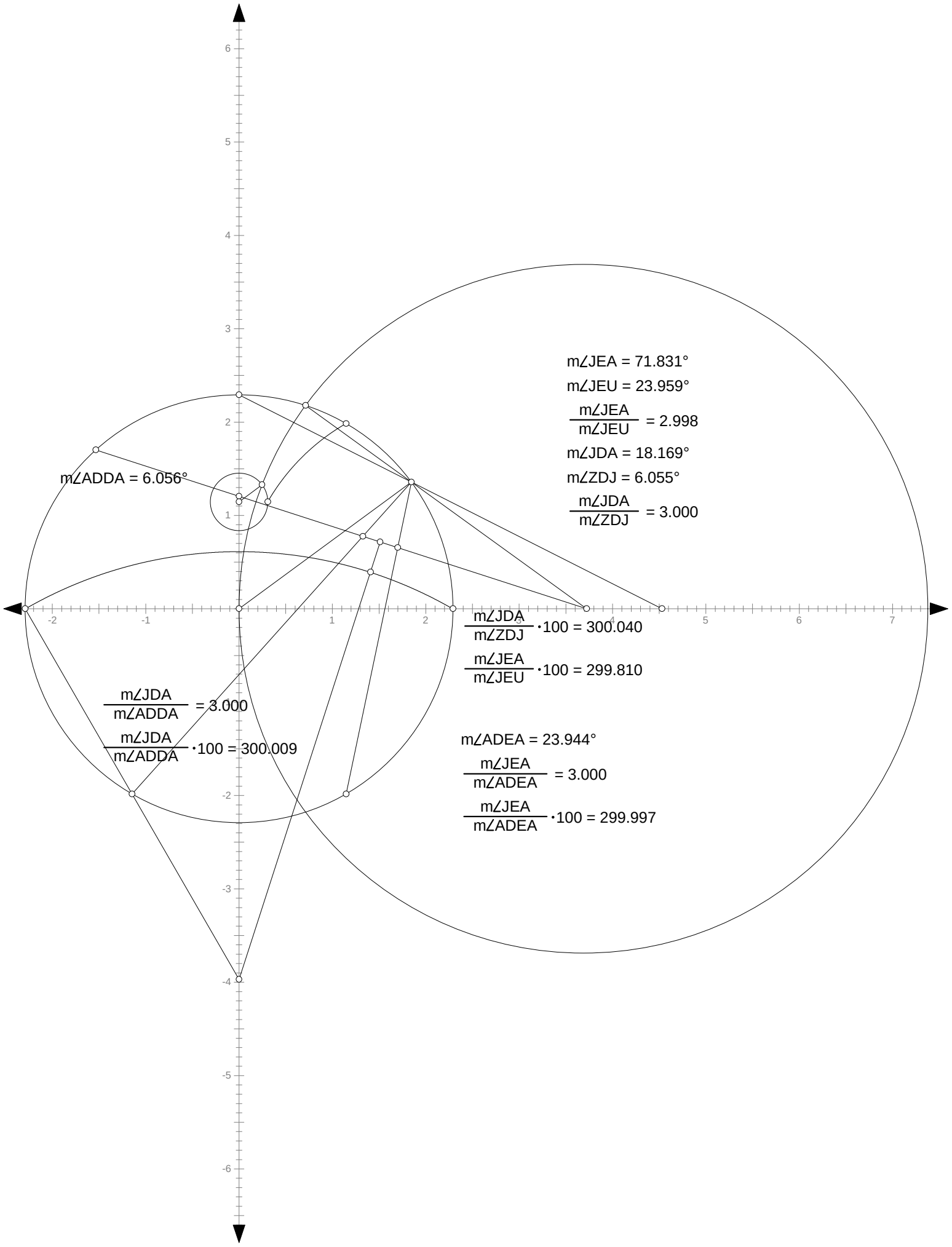

m∠FAC = 34.432°
m∠CEA = 17.216°
m∠DMDODQ = 34.432°
m∠DMDODP = 11.510°
m∠DMDODQ / m∠DMDODP = 2.992

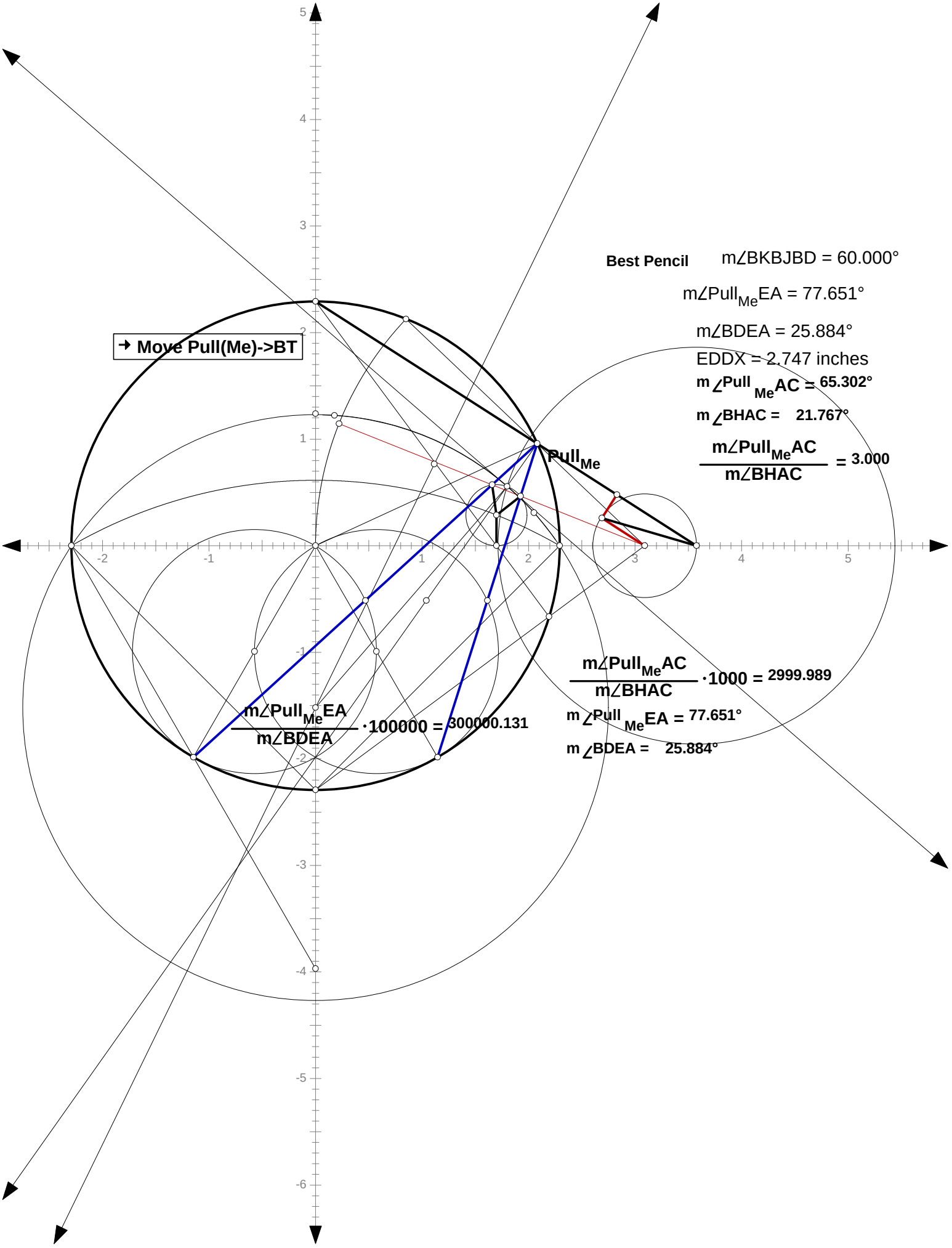

C
ECDV = 0.724 inches
ECBD = 0.724 inches
DVDU = 4.638 inches
BXBV = 2.319 inches
DVDU / BXBV = 2.000
O
DV
BX
DU
A

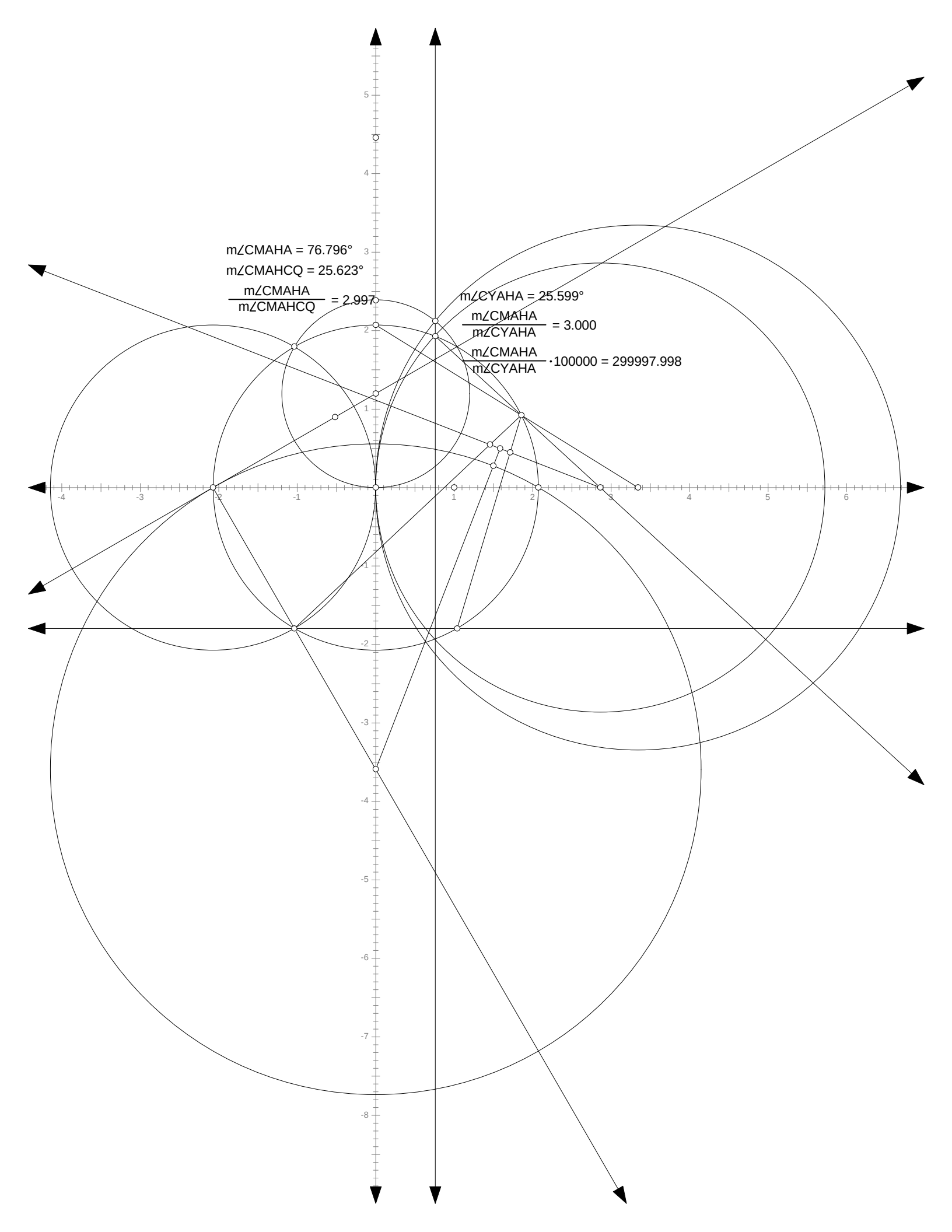

TR = 1.368 inches
m∠NAC = 48.199°
m∠EUESET = 32.138°
m∠NAC / m∠EUESET = 1.500
EN

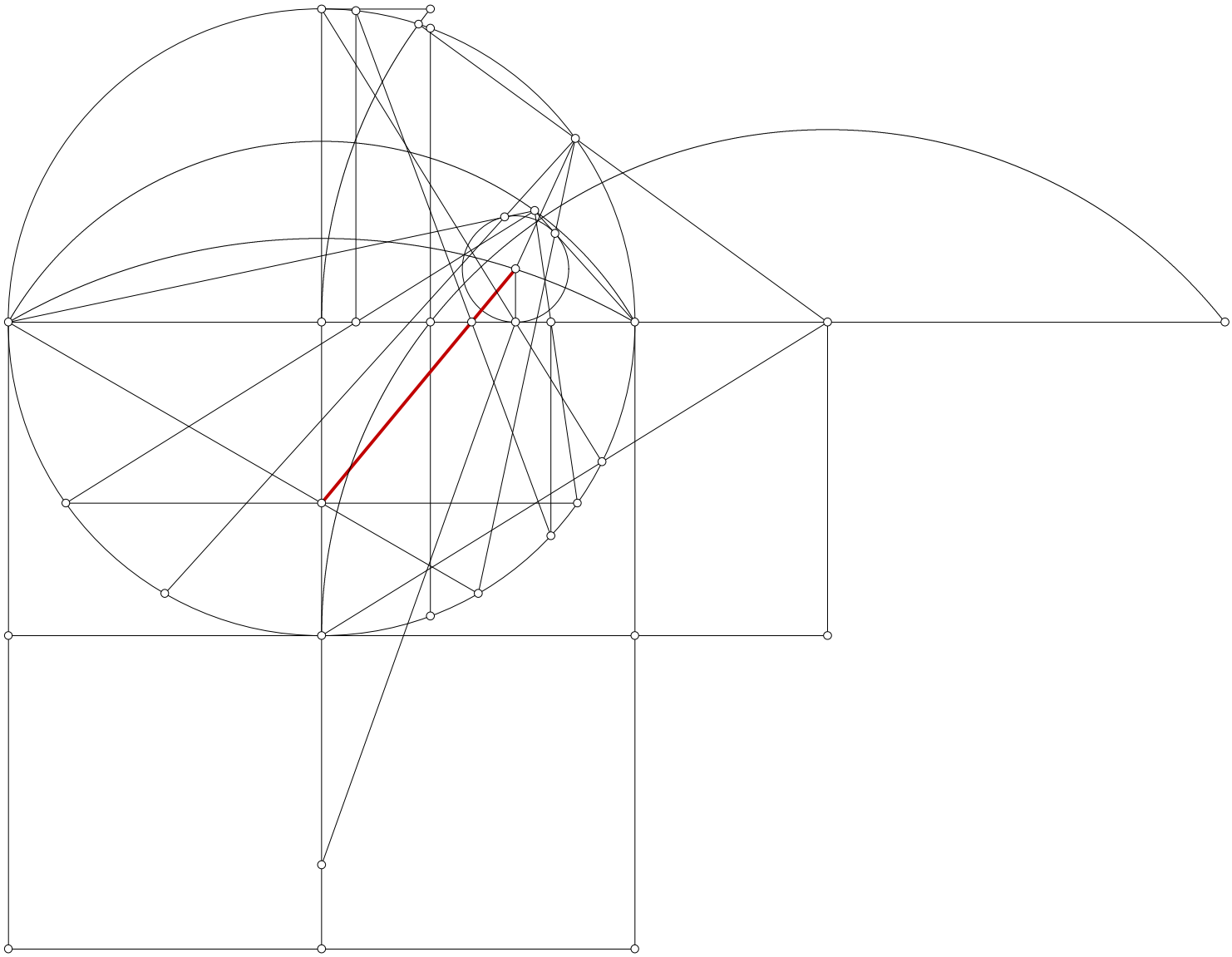

m∠VEA = 71.729°
m∠VEAA = 23.915°
m∠VEA / m∠VEAA = 2.999
m∠ADEA = 23.909°
m∠VEA / m∠ADEA = 3.000
m∠VEA / m∠ADEA ·100 = 300.001
m∠VDA = 18.271°
m∠ADDA = 6.091°
m∠VDA / m∠ADDA = 3.000
m∠VDA / m∠ADDA ·100 = 299.997
m∠VDA / m∠ADDA ·10000 = 29999.675


GW
EN
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-2
-1
1
2
3
4
5
6
7


m∠JEA = 71.831°
m∠JEU = 23.959°
m∠JEA / m∠JEU = 2.998
m∠JDA = 18.169°
m∠ZDJ = 6.055°
m∠JDA / m∠ZDJ = 3.000
m∠ADDA = 6.056°
m∠JDA / m∠ZDJ ·100 = 300.040
m∠JEA / m∠JEU ·100 = 299.810
m∠JDA / m∠ADDA = 3.000
m∠JDA / m∠ADDA ·100 = 300.009
m∠ADEA = 23.944°
m∠JEA / m∠ADEA = 3.000
m∠JEA / m∠ADEA ·100 = 299.997


Move Pull(Me)->BT
Best Pencil
m∠BKBJBD = 60.000°
m∠Pull_Me EA = 77.651°
m∠BDEA = 25.884°
EDDX = 2.747 inches
m∠Pull_Me AC = 65.302°
m∠BHAC = 21.767°
m∠Pull_Me AC / m∠BHAC = 3.000
Pull_Me
m∠Pull_Me EA / m∠BDEA ·100000 = 300000.131
m∠Pull_Me AC / m∠BHAC ·1000 = 2999.989
m∠Pull_Me EA = 77.651°
m∠BDEA = 25.884°


m∠CMAHA = 76.796°
m∠CMAHCQ = 25.623°
m∠CMAHA / m∠CMAHCQ = 2.997
m∠CYAHA = 25.599°
m∠CMAHA / m∠CYAHA = 3.000
m∠CMAHA / m∠CYAHA · 100000 = 299997.998



m∠FVFUBV = 18.895°

m∠OAC = 56.685°

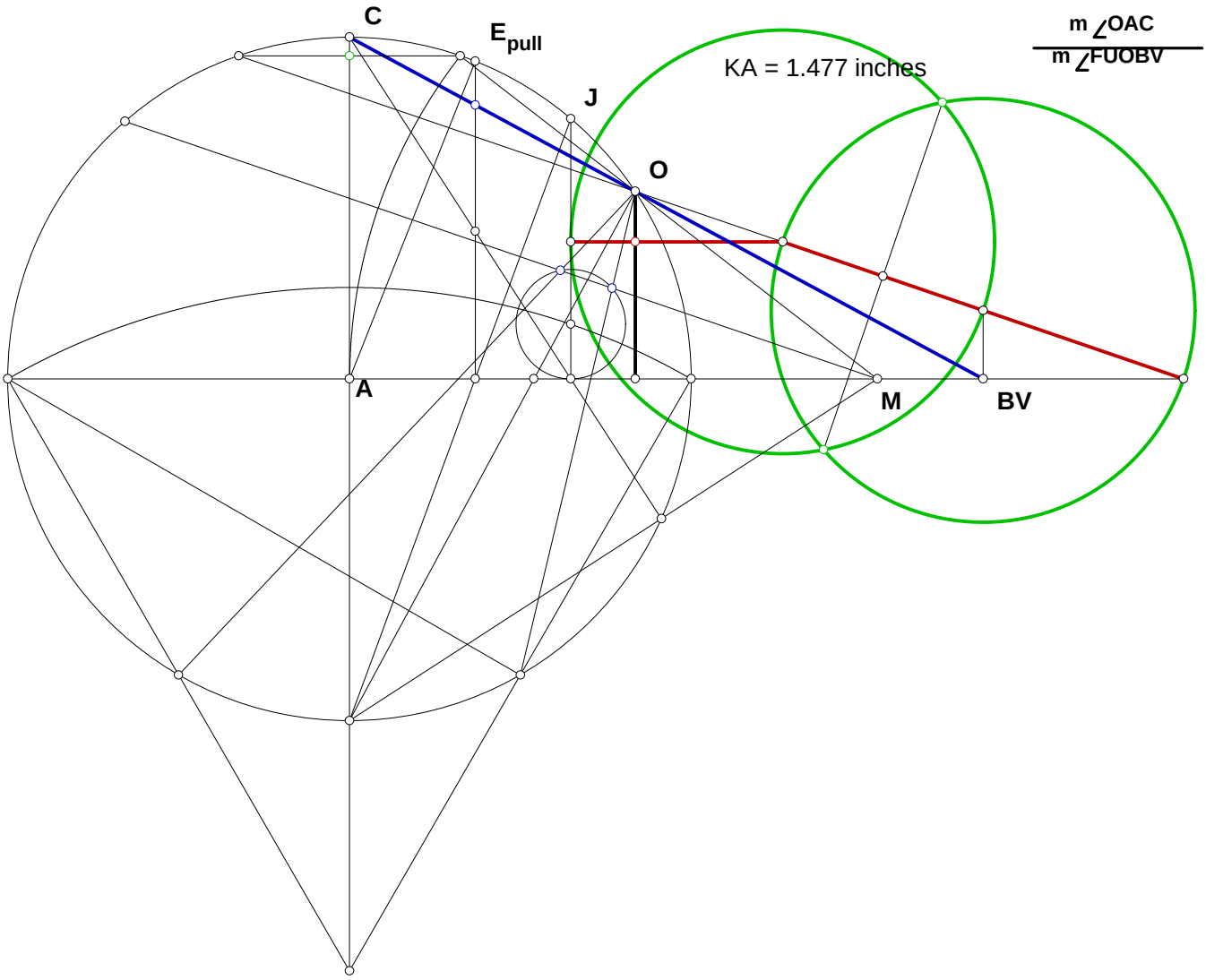
$\frac{m\angle OAC}{m\angle FVFUBV}$ = 3.000

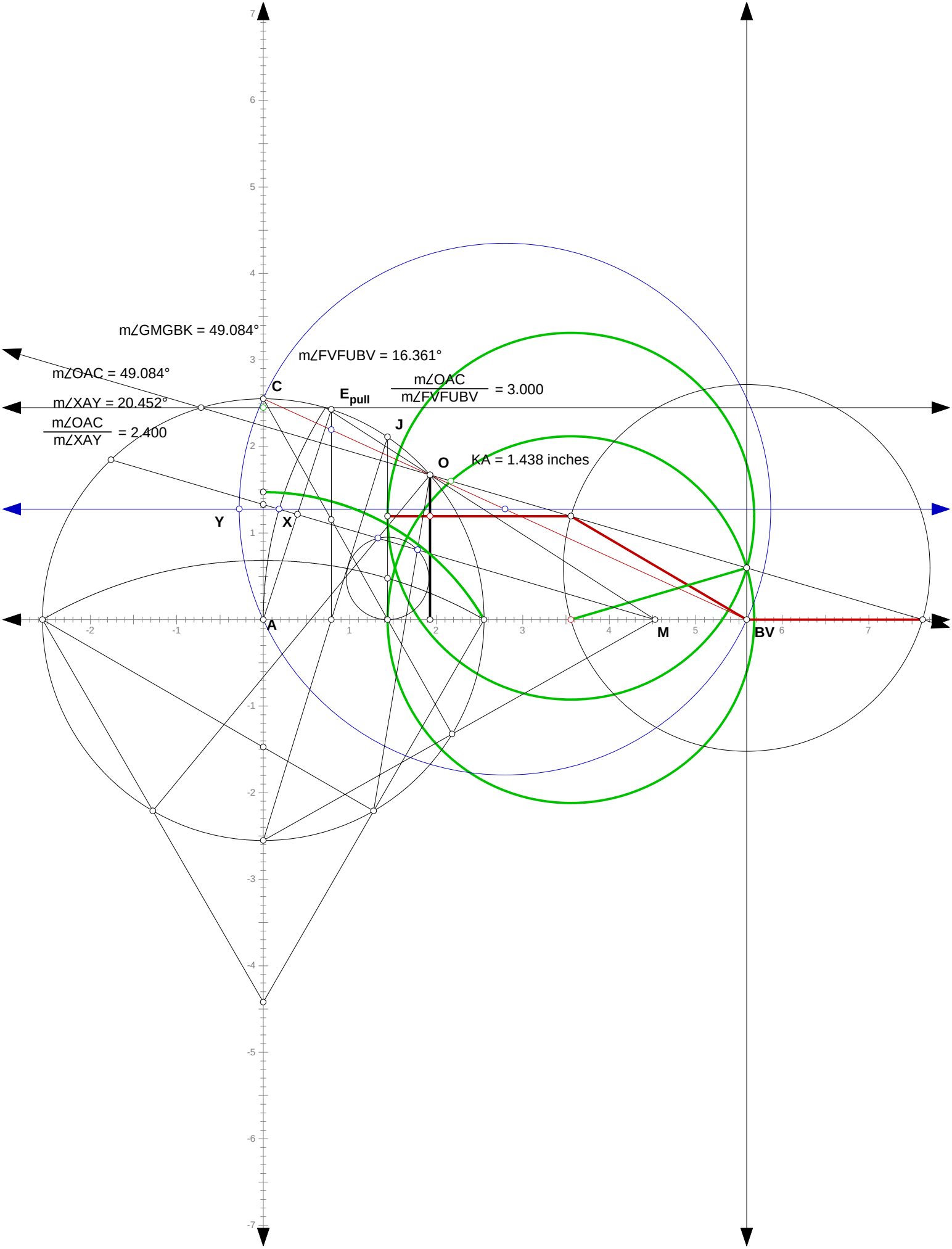
m ∠FUOBV = 9.447°

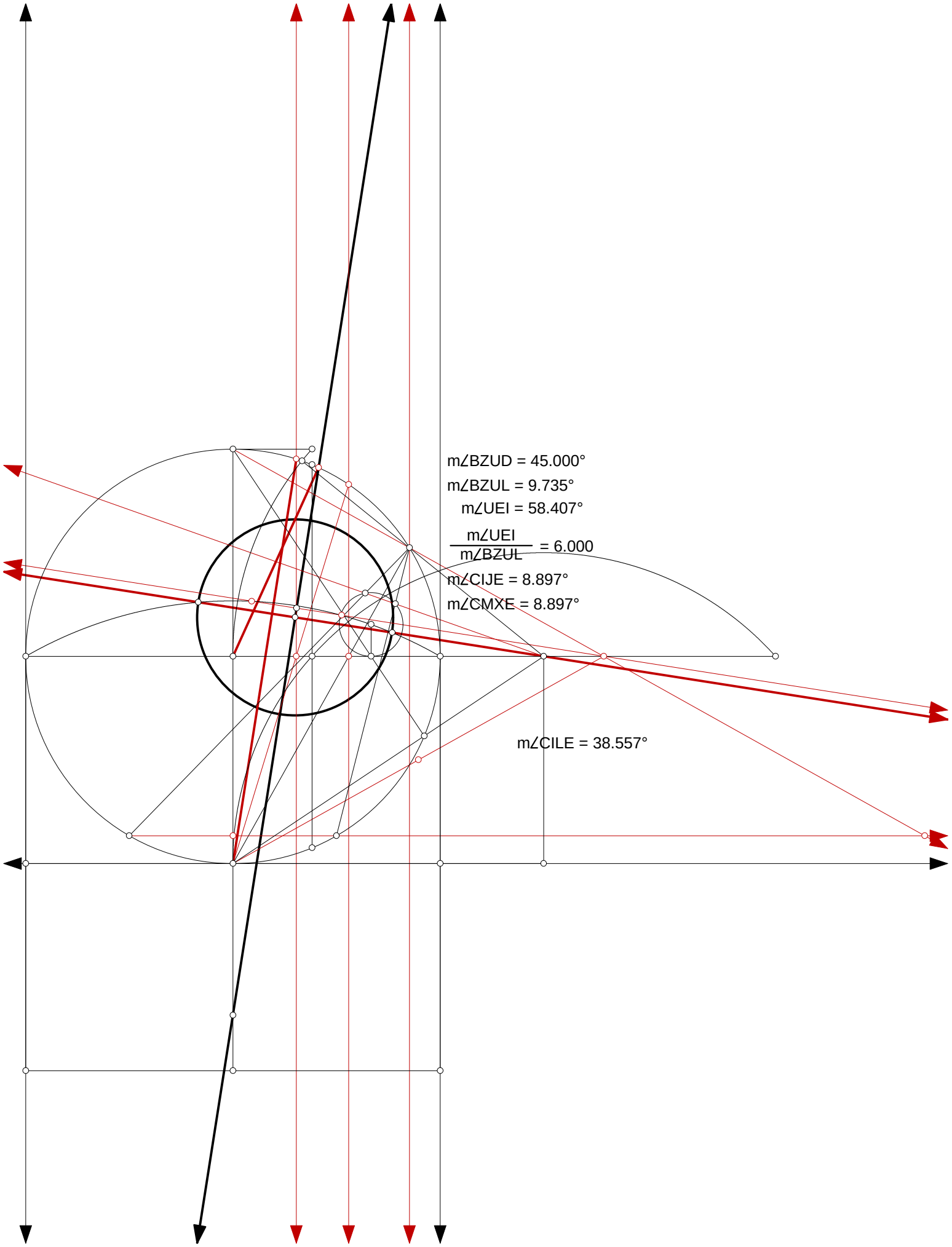
m ∠OAC = 56.685°

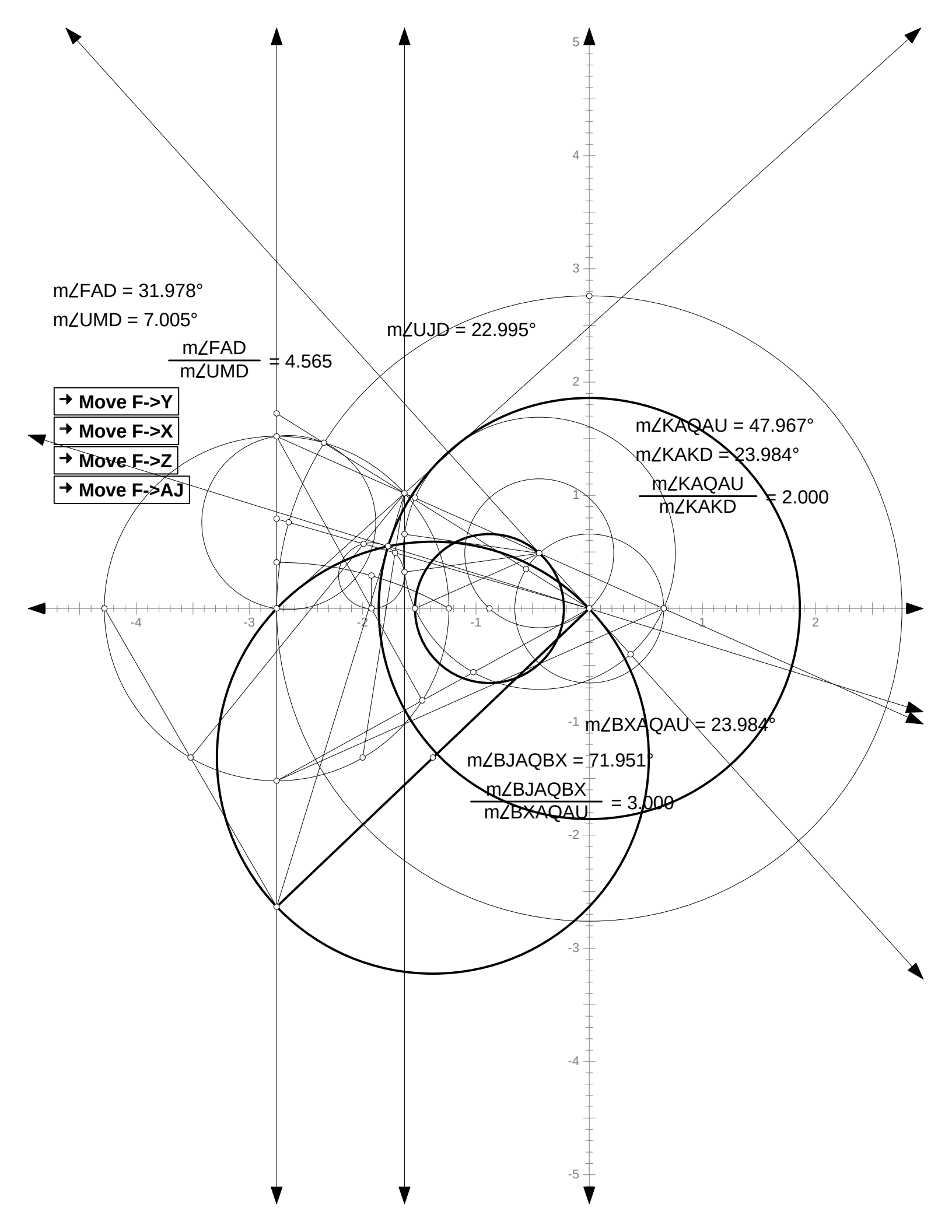
$\frac{\mathbf{m\angle OAC}}{\mathbf{m\angle FUOBV}}$ **= 6.000**

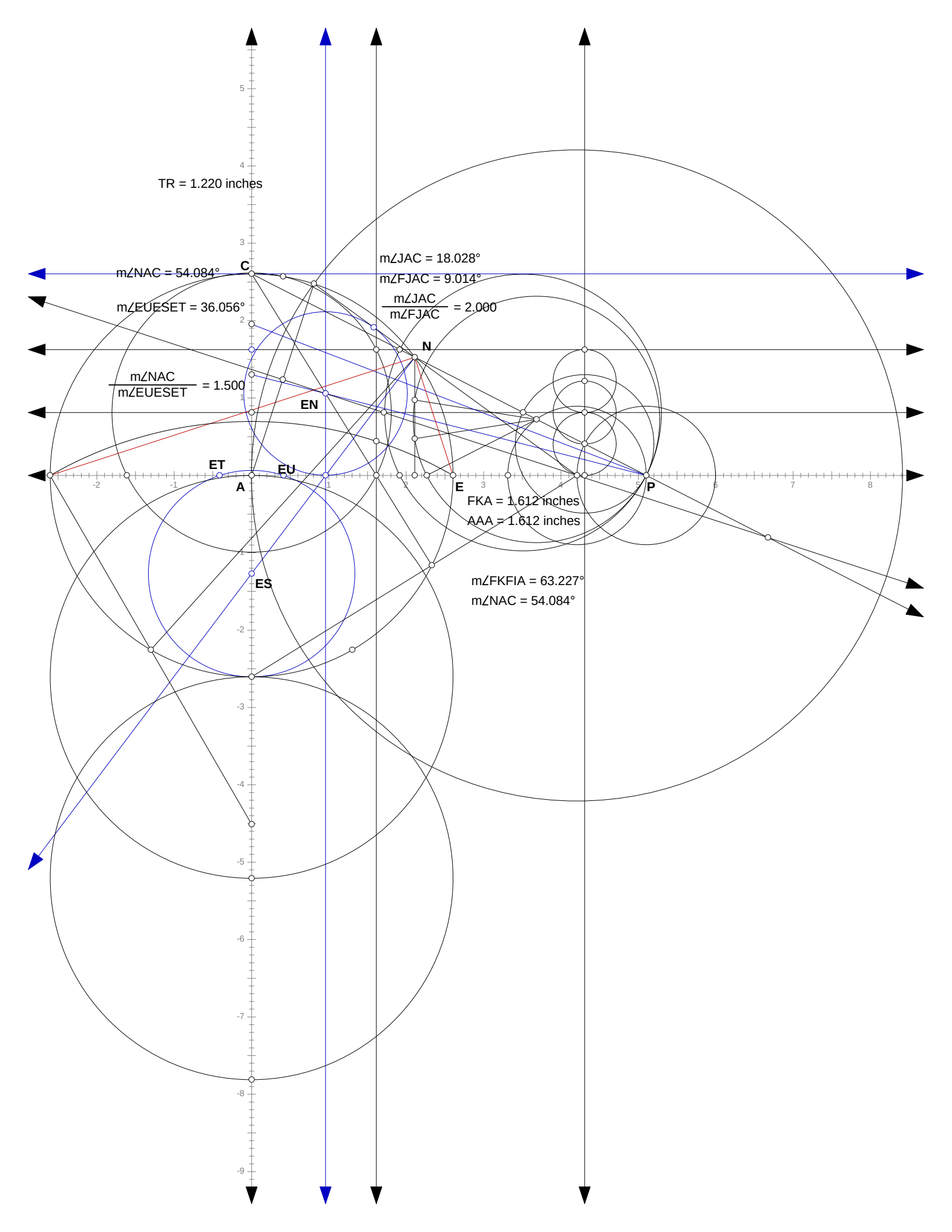
KA = 1.477 inches

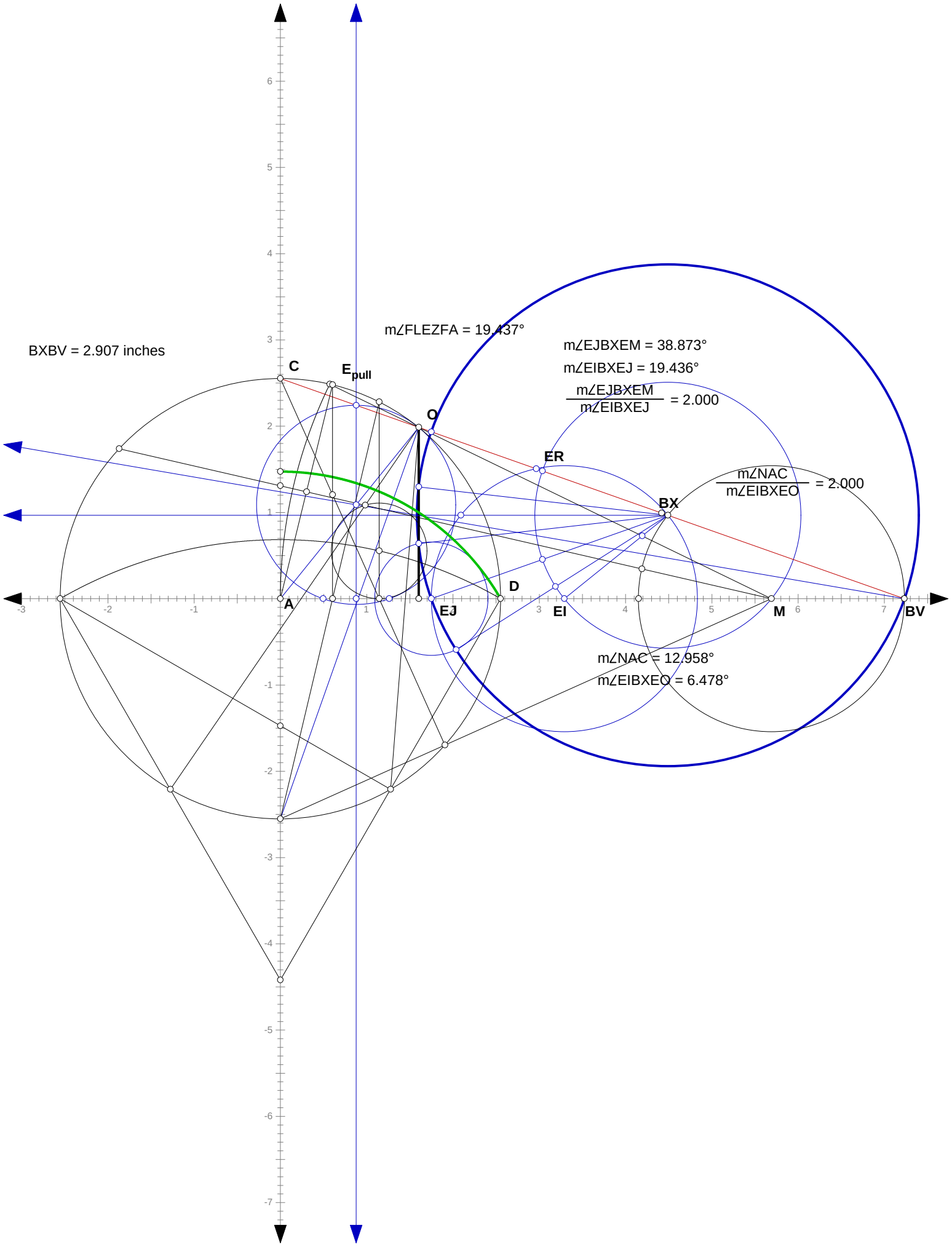


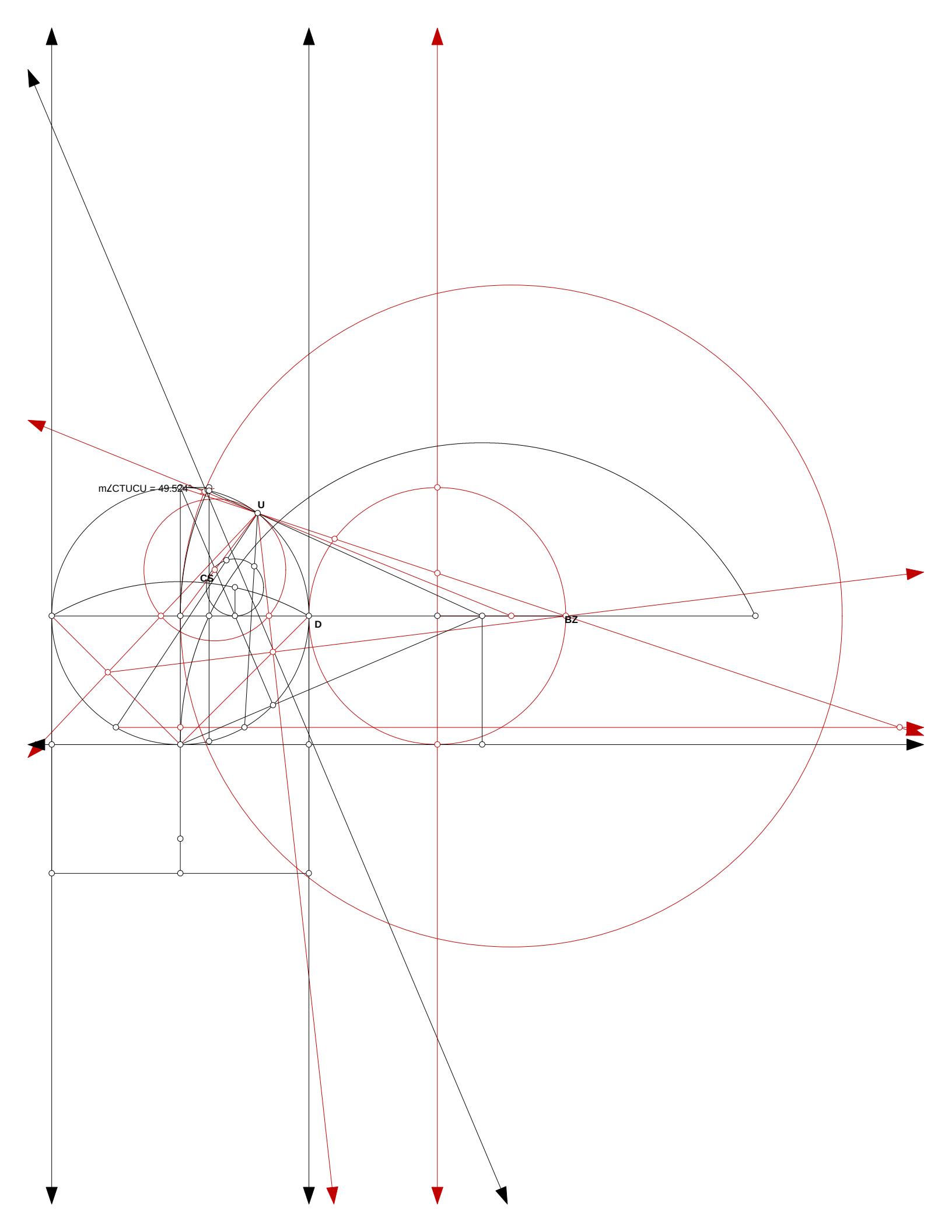

m∠GMGBK = 49.084°
m∠FVFUBV = 16.361°
m∠OAC = 49.084°
m∠XAY = 20.452°
m∠OAC / m∠XAY = 2.400
m∠OAC / m∠FVFUBV = 3.000
KA = 1.438 inches
C
E pull
J
O
Y
X
A
M
BV

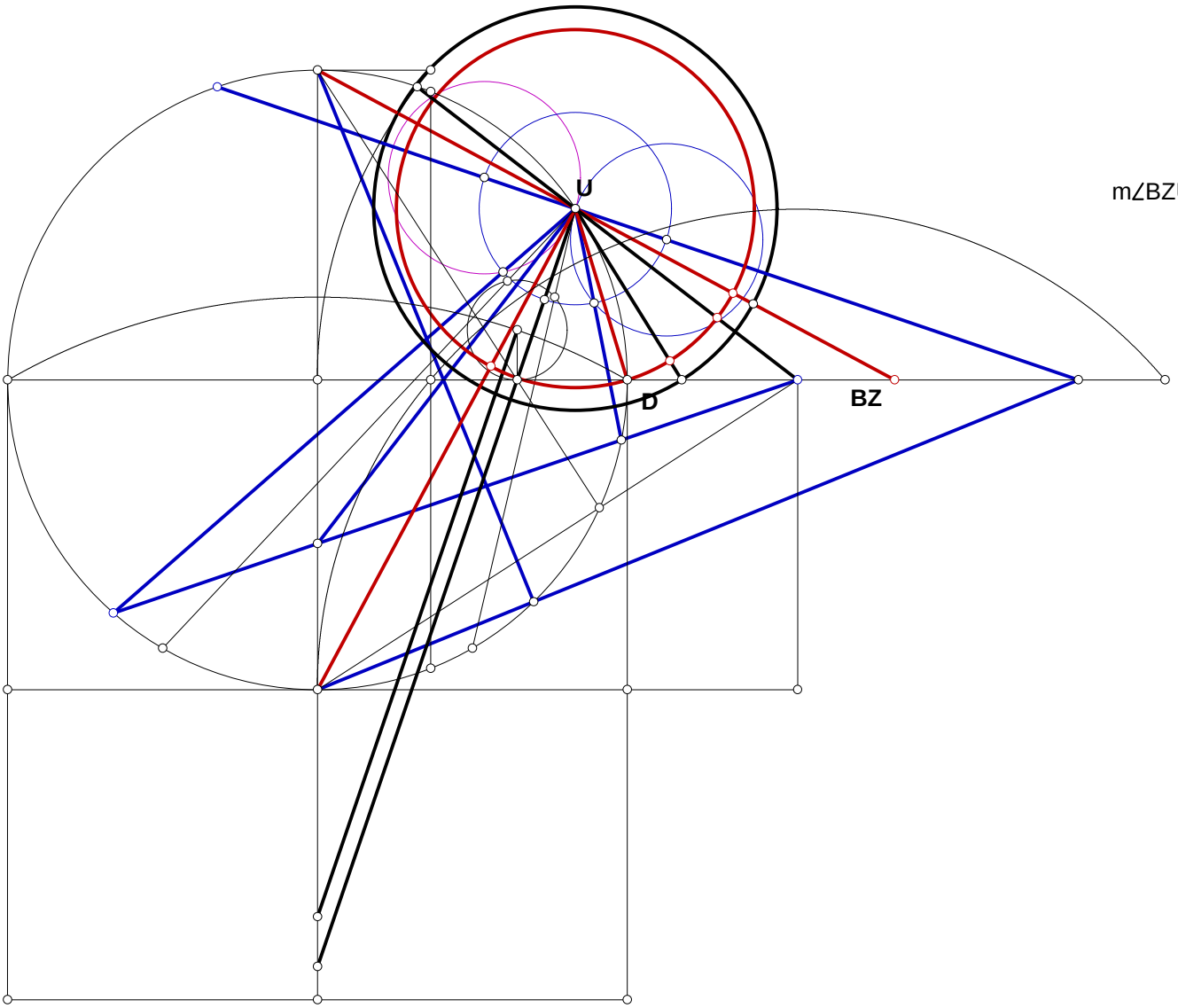

m∠BZUD = 45.000°
m∠BZUL = 9.735°
m∠UEI = 58.407°
m∠UEI / m∠BZUL = 6.000
m∠CIJE = 8.897°
m∠CMXE = 8.897°
m∠CILE = 38.557°


m∠FAD = 31.978°
m∠UMD = 7.005°
m∠FAD / m∠UMD = 4.565
m∠UJD = 22.995°
Move F->Y
Move F->X
Move F->Z
Move F->AJ
m∠KAQAU = 47.967°
m∠KAKD = 23.984°
m∠KAQAU / m∠KAKD = 2.000
m∠BXAQAU = 23.984°
m∠BJAQBX = 71.951°
m∠BJAQBX / m∠BXAQAU = 3.000

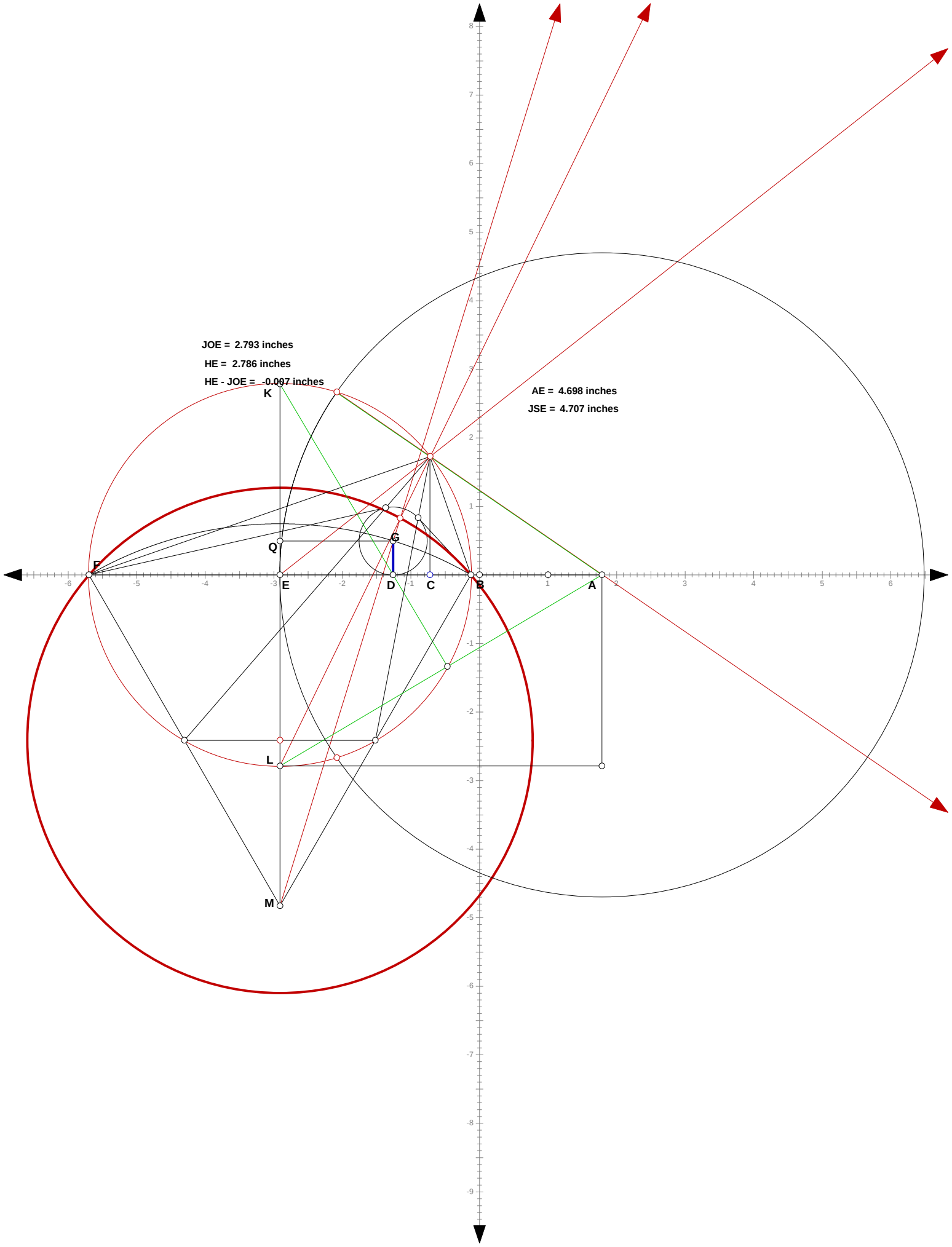

TR = 1.220 inches
C
m∠NAC = 54.084°
m∠EUESET = 36.056°
m∠NAC / m∠EUESET = 1.500
m∠JAC = 18.028°
m∠FJAC = 9.014°
m∠JAC / m∠FJAC = 2.000
N
EN
ET
EU
A
E
P
ES
FKA = 1.612 inches
AAA = 1.612 inches
m∠FKFIA = 63.227°
m∠NAC = 54.084°

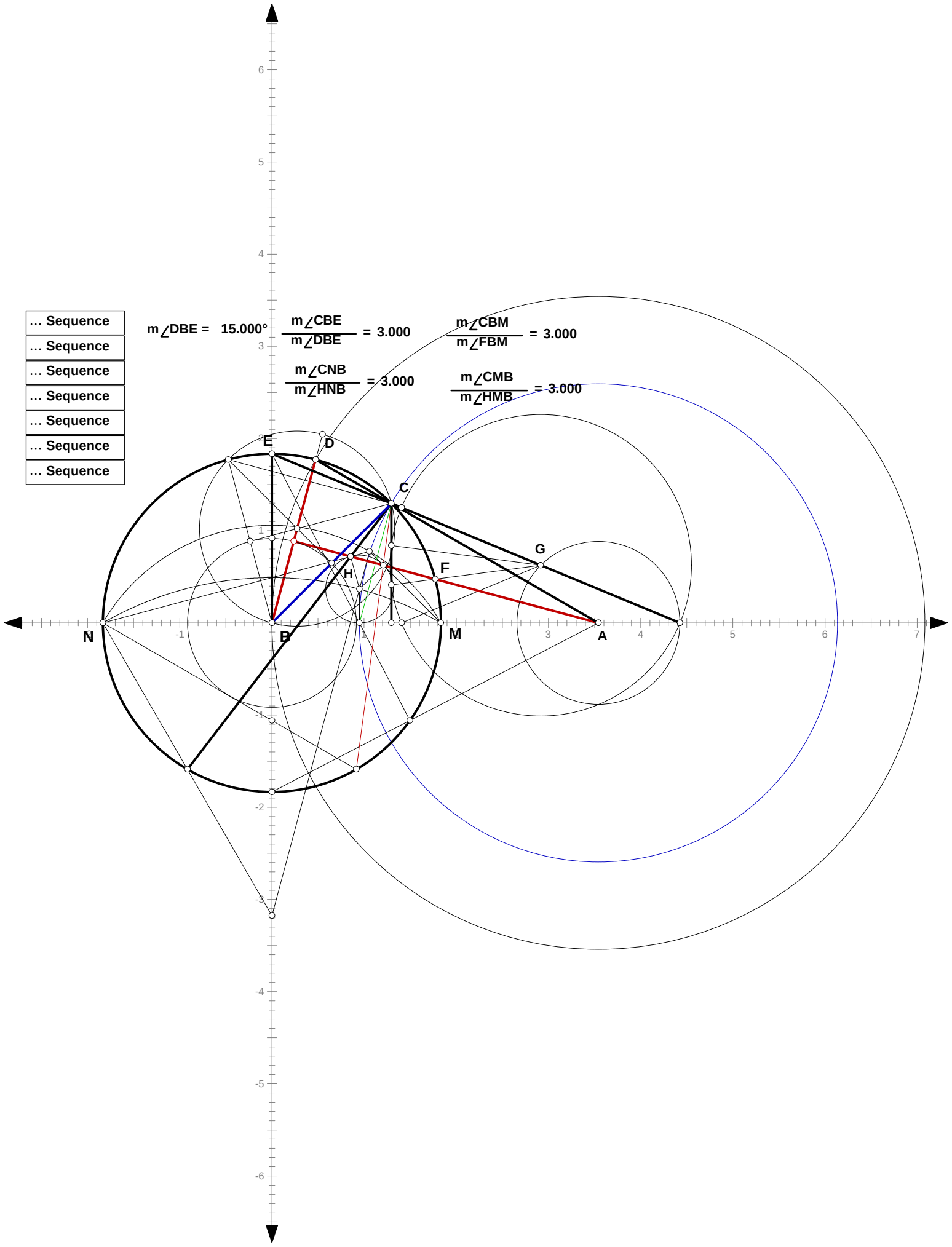

BXBV = 2.907 inches
m∠FLEZFA = 19.437°
m∠EJBXEM = 38.873°
m∠EIBXEJ = 19.436°
m∠EJBXEM / m∠EIBXEJ = 2.000
m∠NAC / m∠EIBXEO = 2.000
m∠NAC = 12.958°
m∠EIBXEO = 6.478°
C
E pull
O
ER
BX
D
A
EJ
EI
M
BV

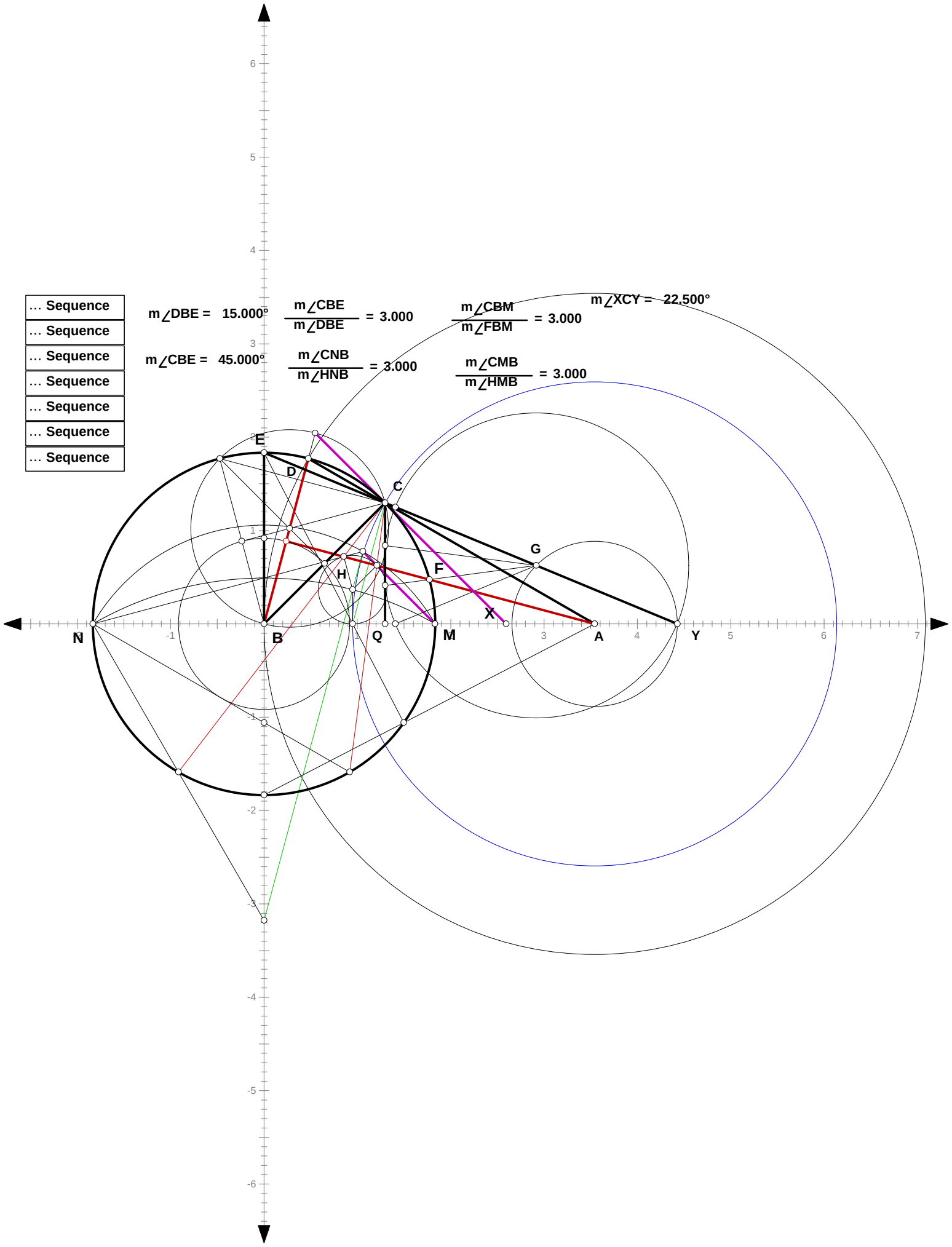

m∠CTUCU = 49.524°
U
CS
D
BZ

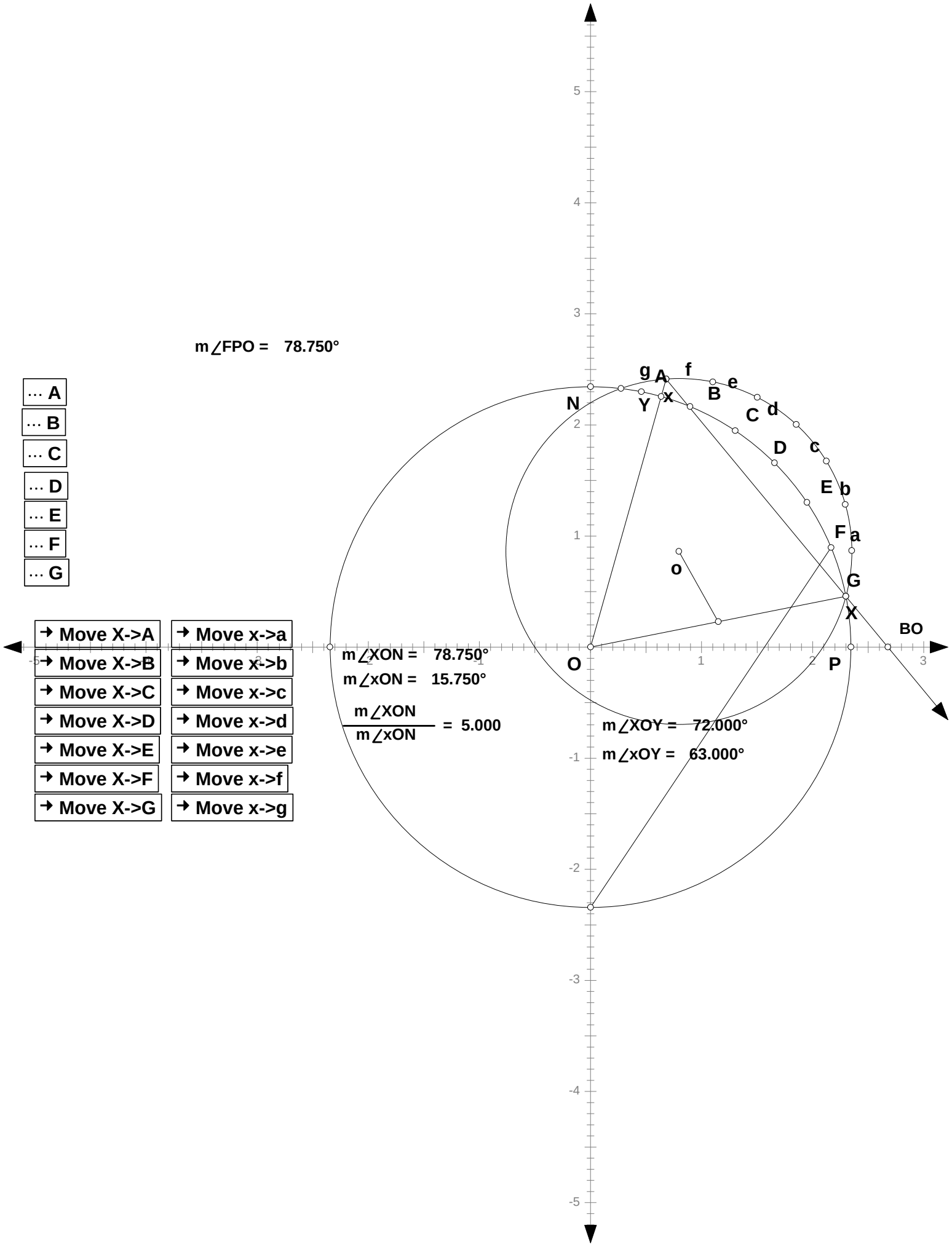


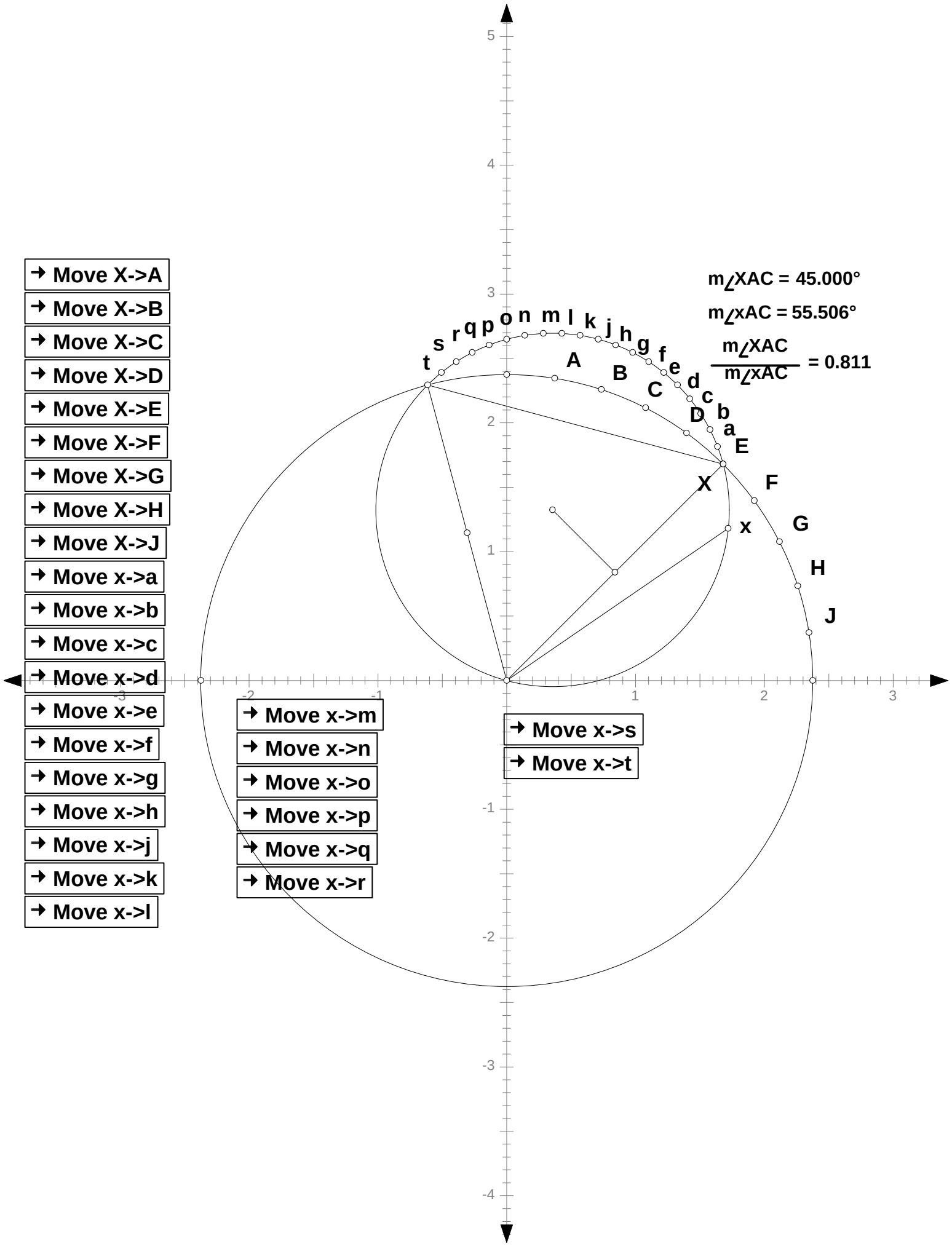
m∠BZUL = 9.404°

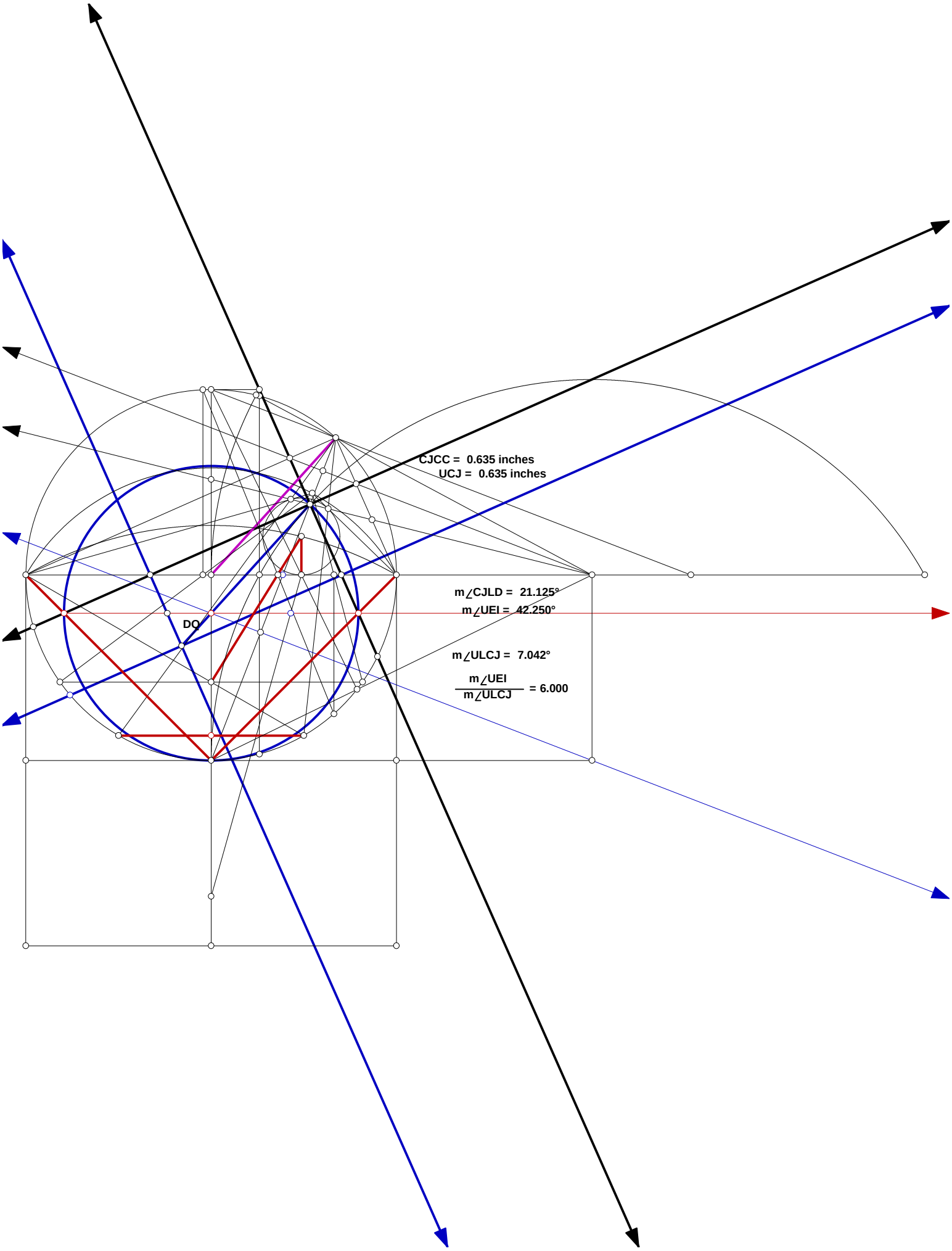

JOE = 2.793 inches
HE = 2.786 inches
HE - JOE = -0.007 inches
AE = 4.698 inches
JSE = 4.707 inches
K
Q
G
F
E
D
C
B
A
L
M

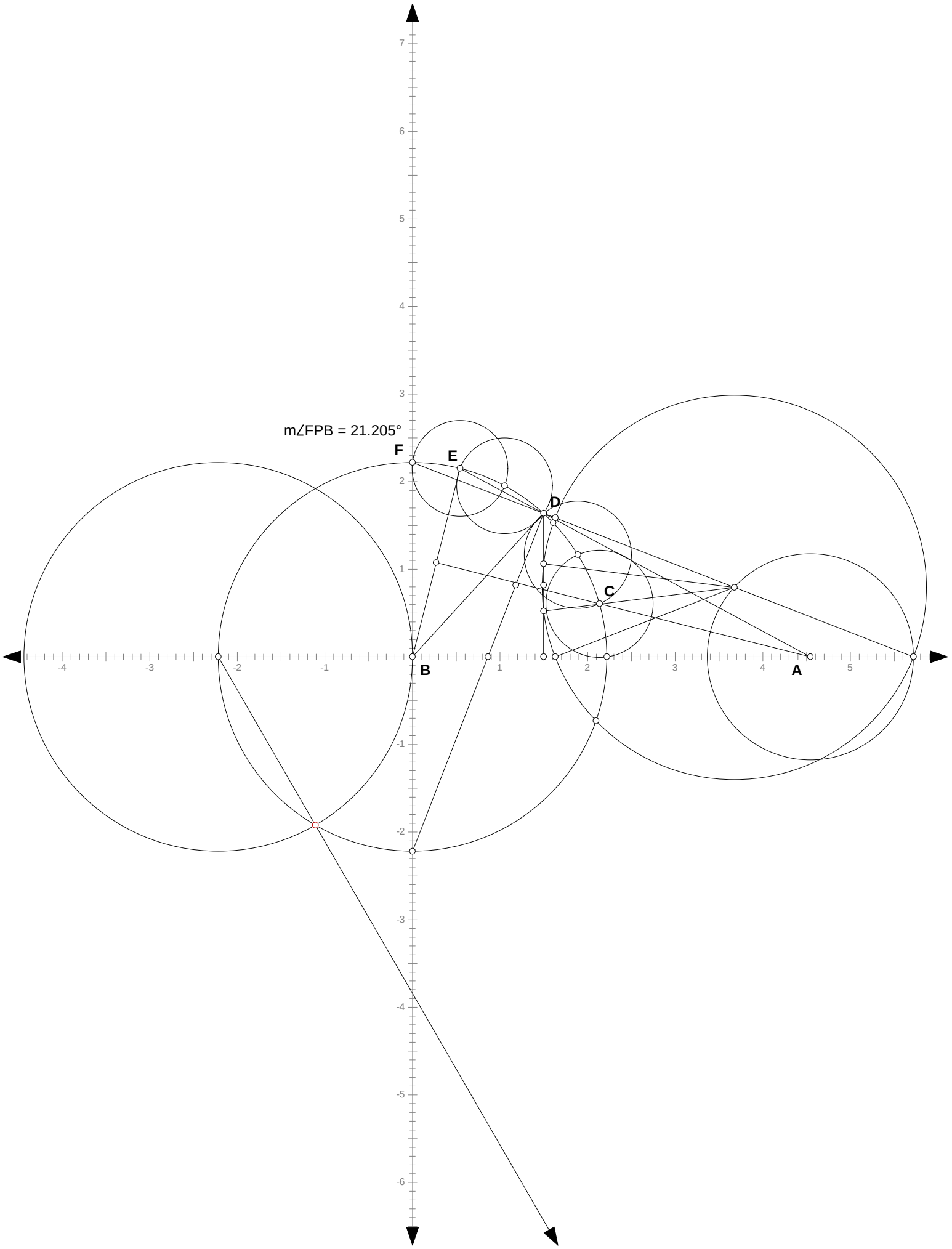

… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
m∠DBE = 15.000°
m∠CBE / m∠DBE = 3.000
m∠CBM / m∠FBM = 3.000
m∠CNB / m∠HNB = 3.000
m∠CMB / m∠HMB = 3.000
E
D
C
G
F
H
N
B
M
A

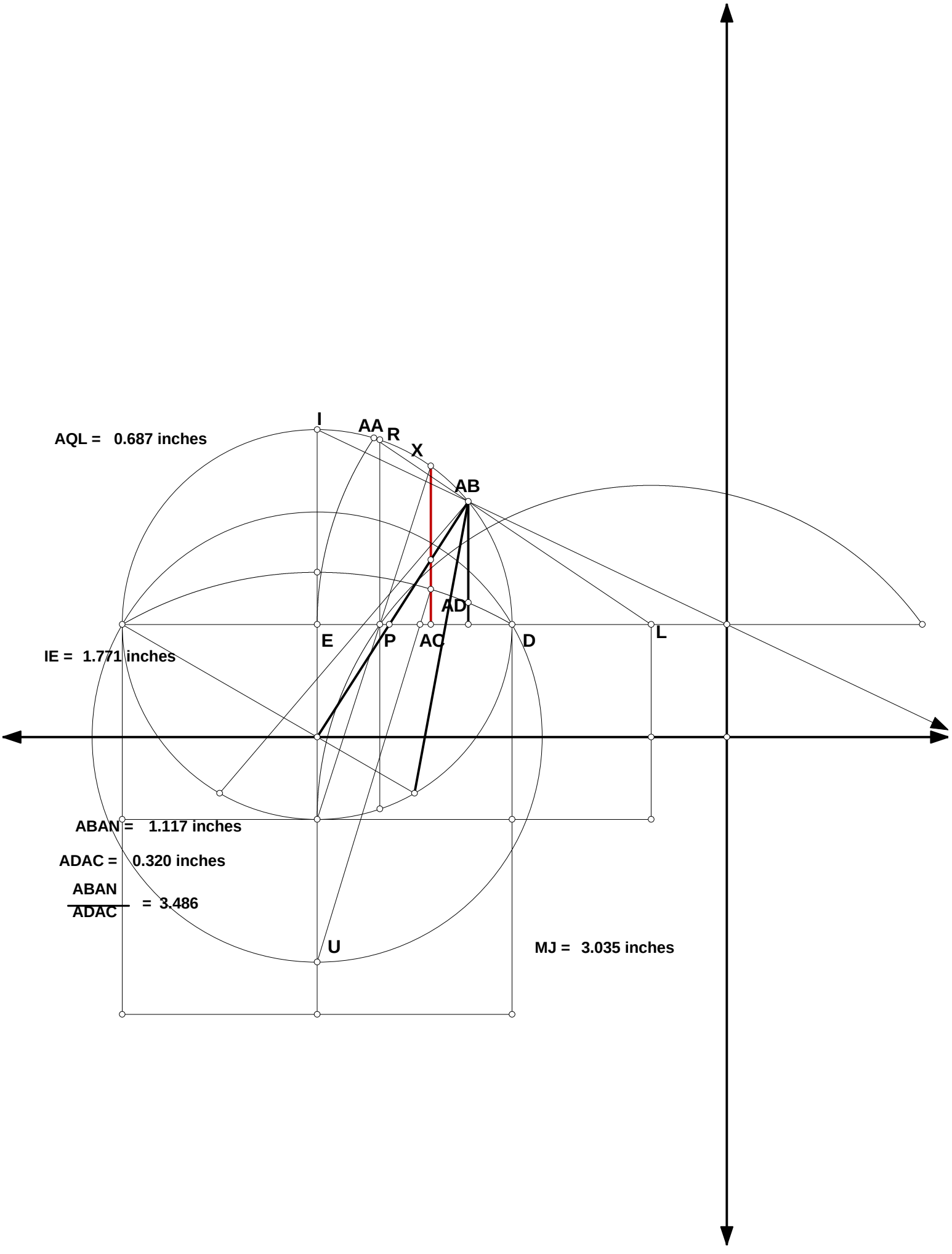

… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
m∠DBE = 15.000°
m∠CBE = 45.000°
m∠CBE / m∠DBE = 3.000
m∠CNB / m∠HNB = 3.000
m∠CBM / m∠FBM = 3.000
m∠CMB / m∠HMB = 3.000
m∠XCY = 22.500°
E
D
C
G
H
F
X
N
B
Q
M
A
Y

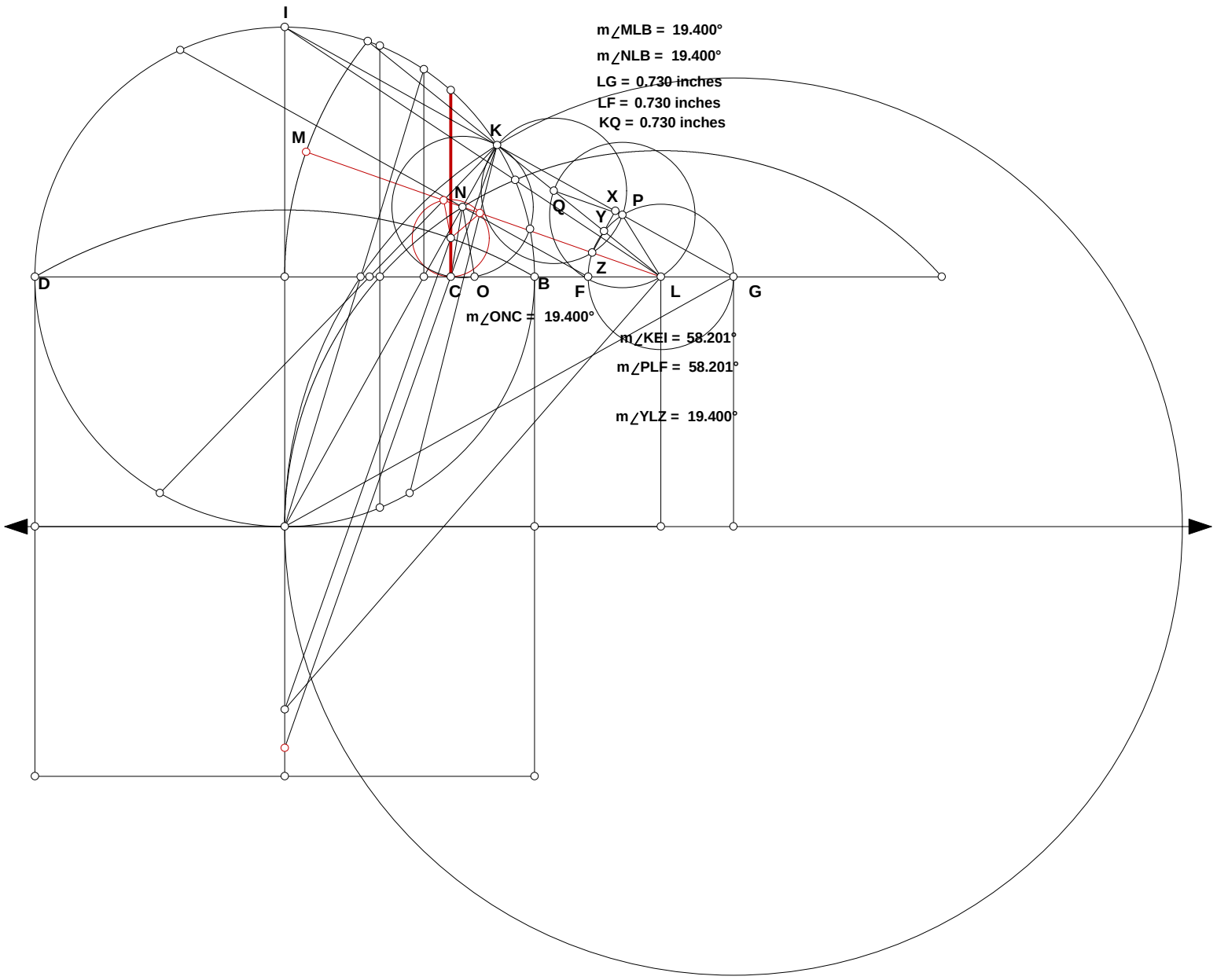

m∠FPO = 78.750°
… A
… B
… C
… D
… E
… F
… G
→ Move X->A
→ Move X->B
→ Move X->C
→ Move X->D
→ Move X->E
→ Move X->F
→ Move X->G
→ Move x->a
→ Move x->b
→ Move x->c
→ Move x->d
→ Move x->e
→ Move x->f
→ Move x->g
m∠XON = 78.750°
m∠xON = 15.750°
m∠XON / m∠xON = 5.000
m∠XOY = 72.000°
m∠xOY = 63.000°
N
Y
x
A
g
f
e
B
C
d
D
c
E
b
F
a
o
G
X
O
P
BO

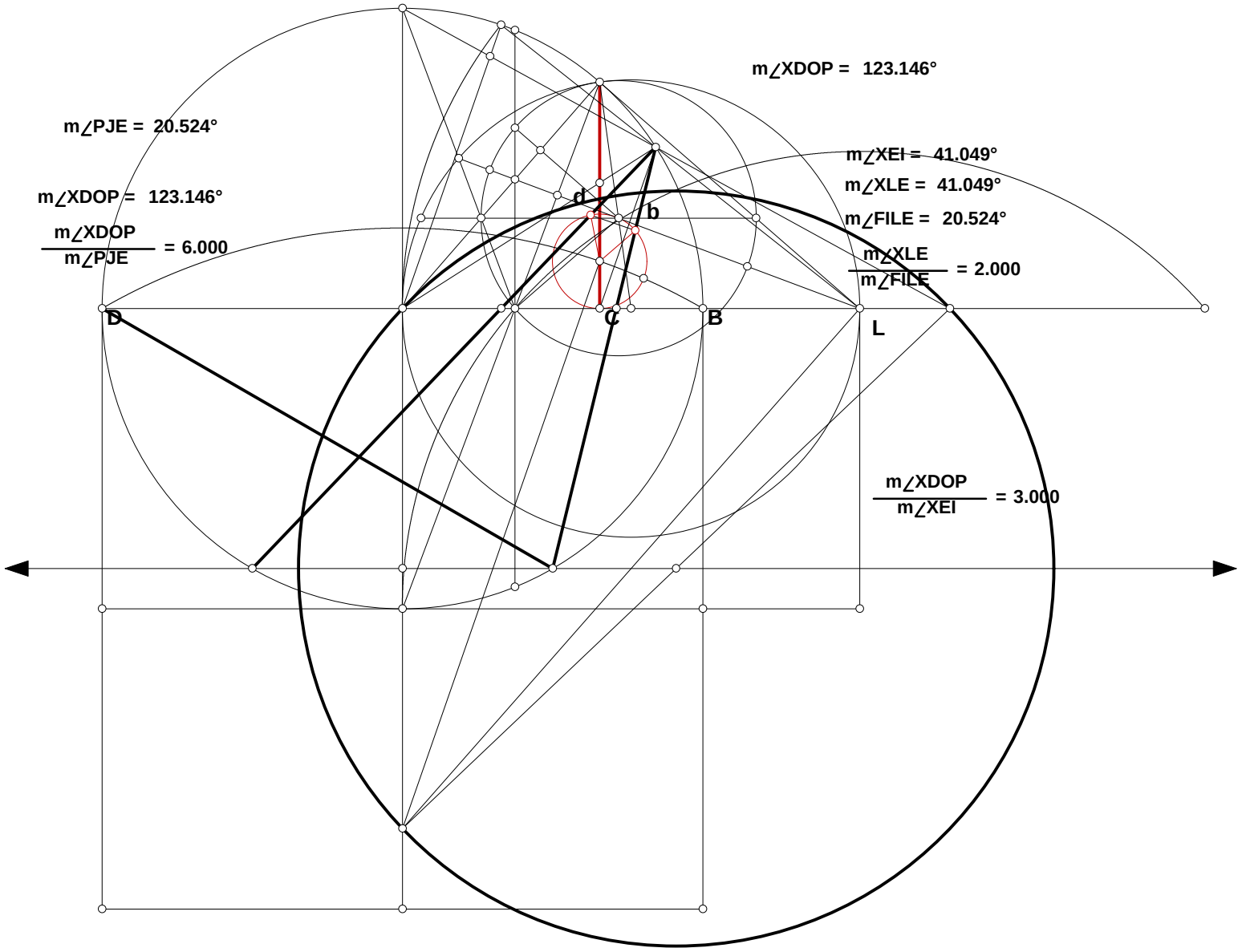

→ Move X->A
→ Move X->B
→ Move X->C
→ Move X->D
→ Move X->E
→ Move X->F
→ Move X->G
→ Move X->H
→ Move X->J
→ Move x->a
→ Move x->b
→ Move x->c
→ Move x->d
→ Move x->e
→ Move x->f
→ Move x->g
→ Move x->h
→ Move x->j
→ Move x->k
→ Move x->l
→ Move x->m
→ Move x->n
→ Move x->o
→ Move x->p
→ Move x->q
→ Move x->r
→ Move x->s
→ Move x->t
m∠XAC = 45.000°
m∠xAC = 55.506°
m∠XAC / m∠xAC = 0.811
t s r q p o n m l k j h g f e d c b a
A B C D E F G H J
X x
5 4 3 2 1 -1 -2 -3 -4
-3 -2 -1 1 2 3

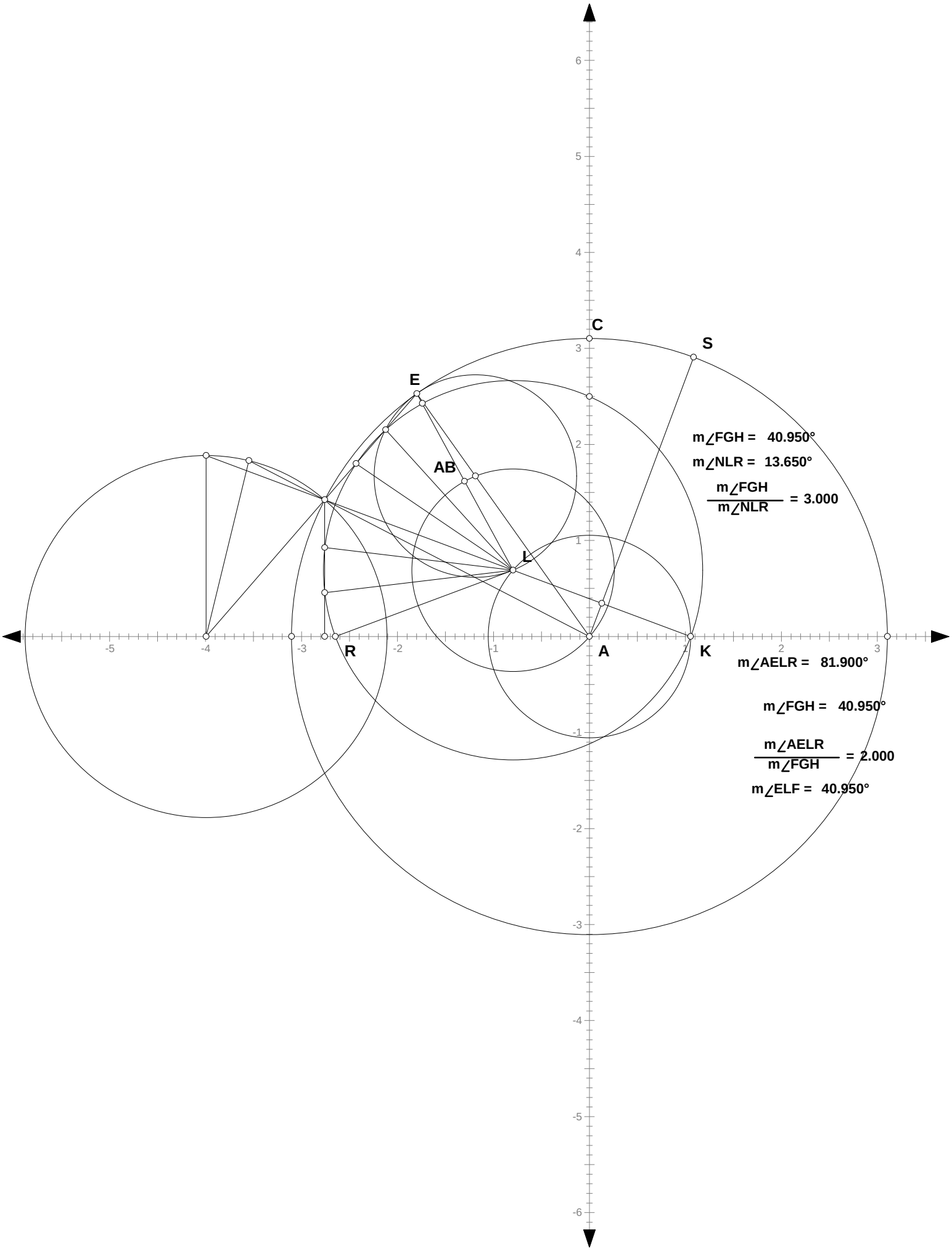

CJCC = 0.635 inches
UCJ = 0.635 inches
m∠CJLD = 21.125°
m∠UEI = 42.250°
DO
m∠ULCJ = 7.042°
m∠UEI / m∠ULCJ = 6.000

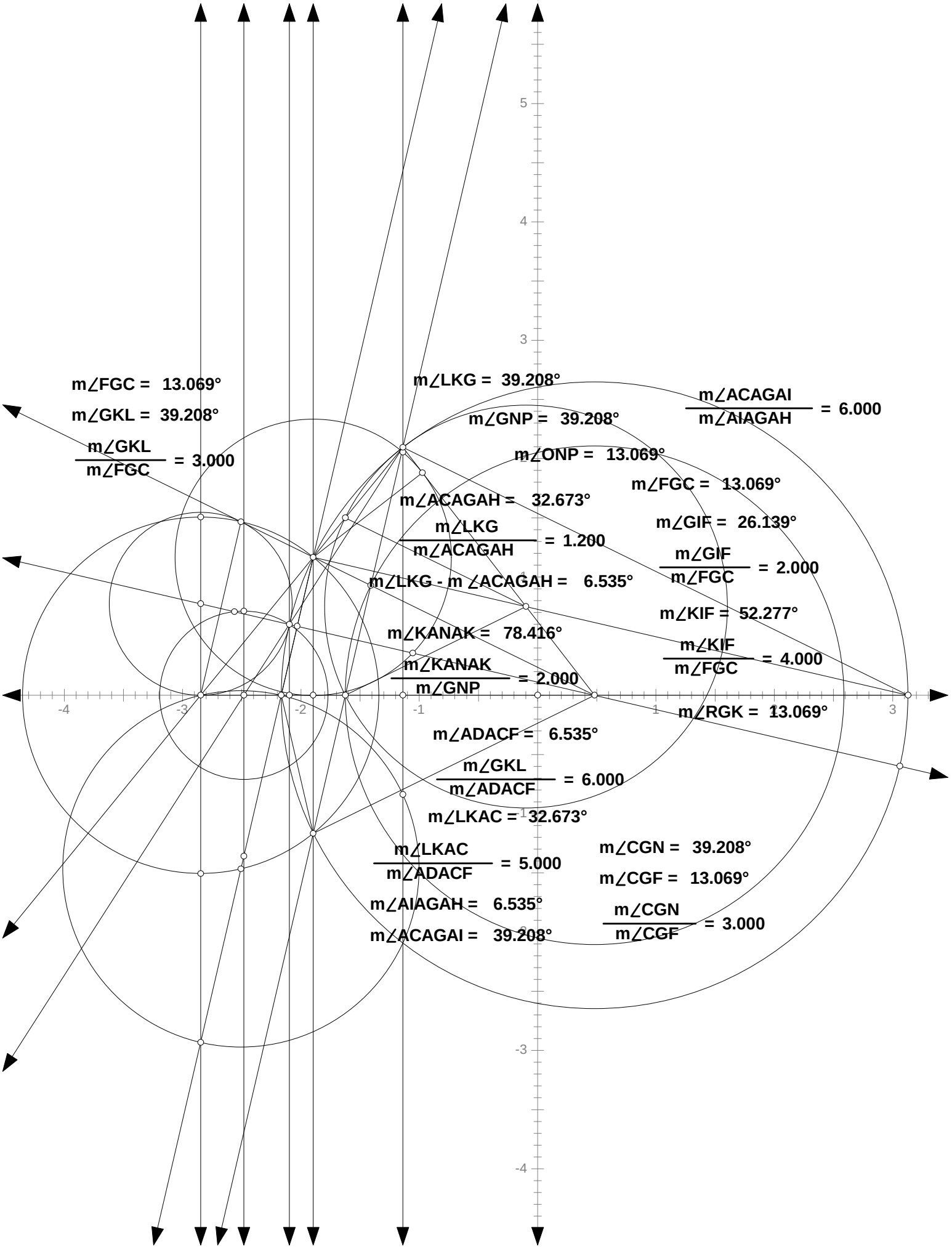

m∠FPB = 21.205°
F
E
D
C
B
A

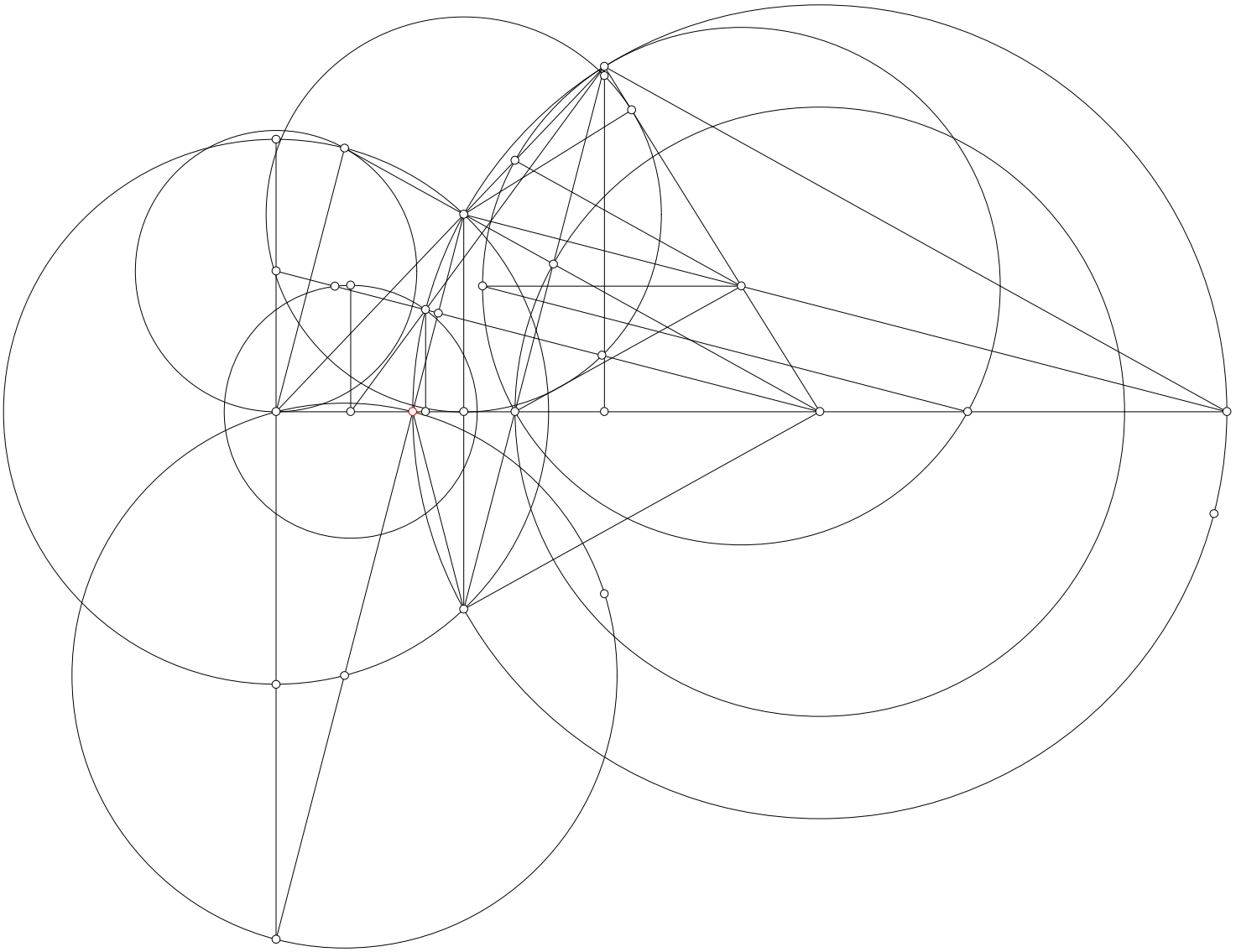

I
AA
R
X
AB
AD
E
P
AC
D
L
U
AQL = 0.687 inches
IE = 1.771 inches
ABAN = 1.117 inches
ADAC = 0.320 inches
ABAN / ADAC = 3.486
MJ = 3.035 inches

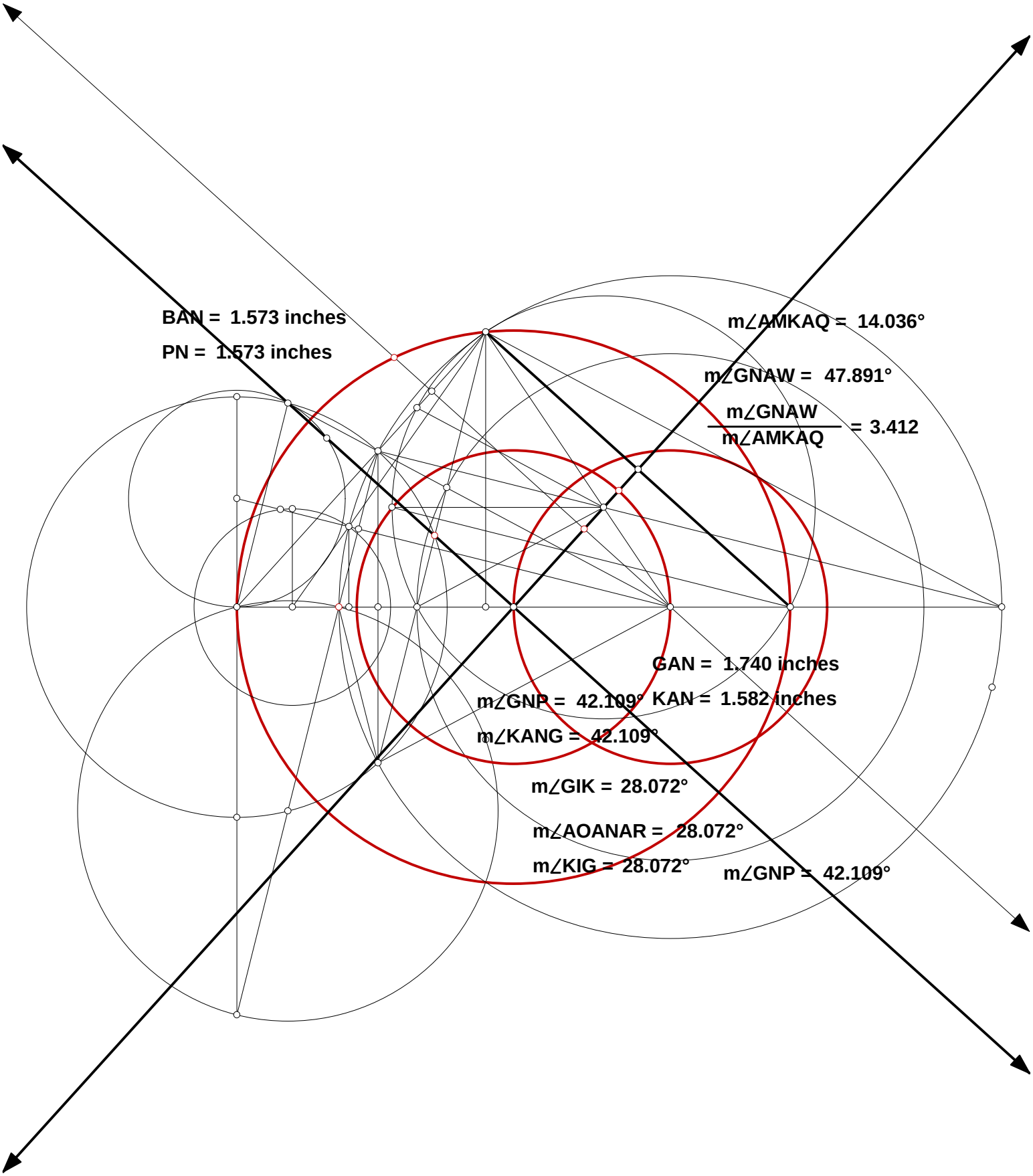

I
M
K
N
Q
X
P
Y
Z
D
C
O
B
F
L
G
m∠MLB = 19.400°
m∠NLB = 19.400°
LG = 0.730 inches
LF = 0.730 inches
KQ = 0.730 inches
m∠ONC = 19.400°
m∠KEI = 58.201°
m∠PLF = 58.201°
m∠YLZ = 19.400°

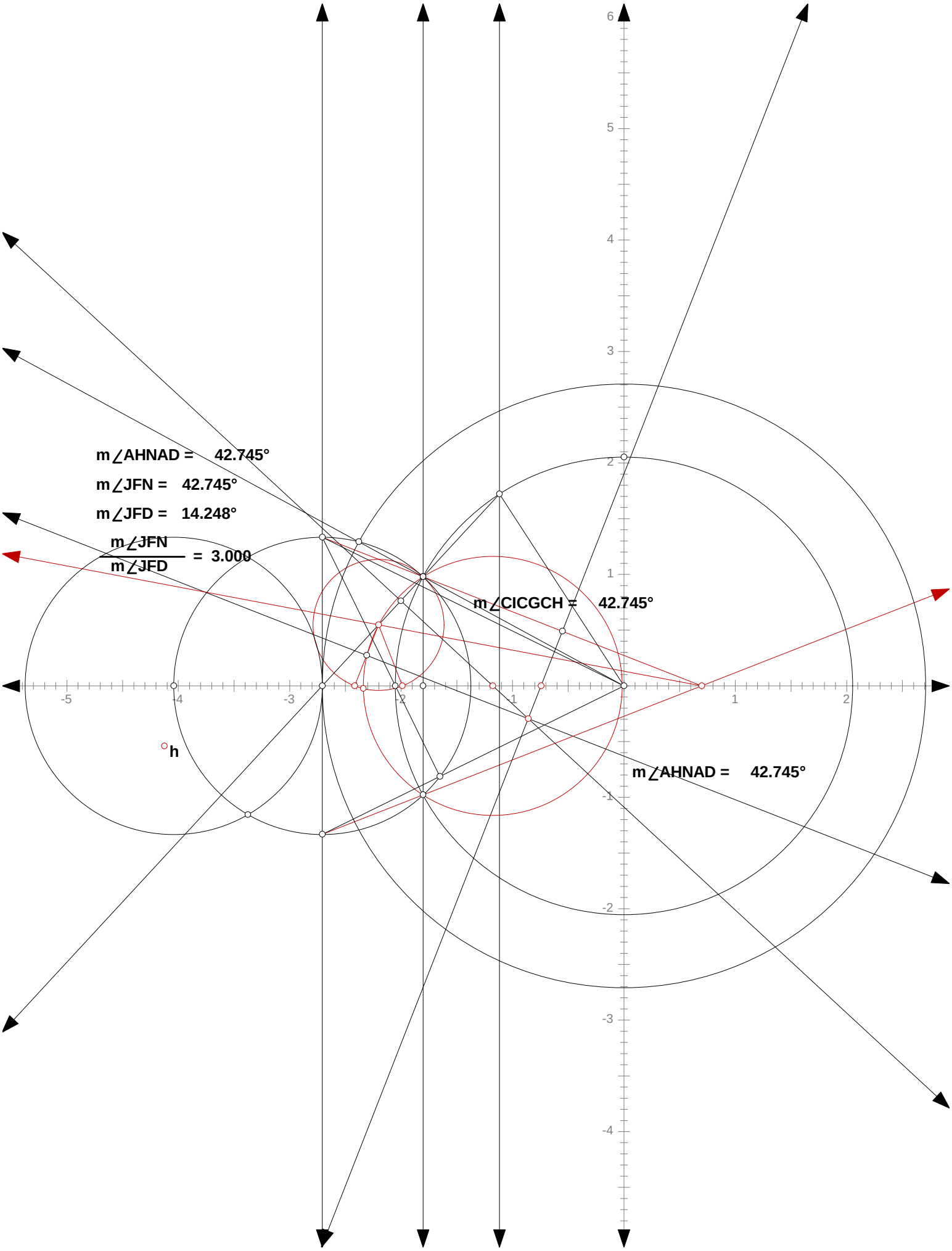

m∠XDOP = 123.146°
m∠PJE = 20.524°
m∠XEI = 41.049°
m∠XLE = 41.049°
m∠XDOP = 123.146°
m∠FILE = 20.524°
m∠XDOP / m∠PJE = 6.000
m∠XLE / m∠FILE = 2.000
d
b
D
C
B
L
m∠XDOP / m∠XEI = 3.000

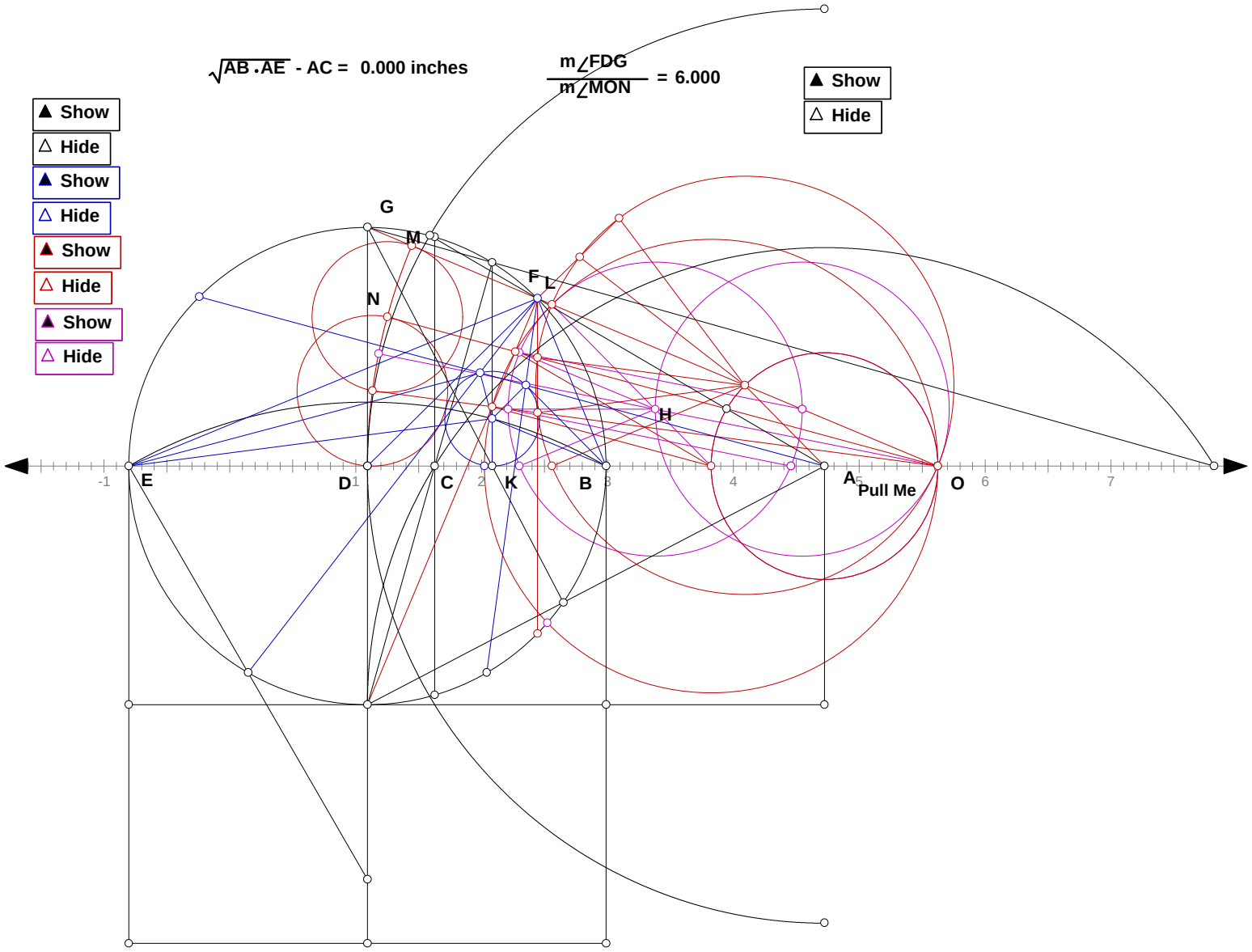

C
S
E
AB
L
R
A
K
m∠FGH = 40.950°
m∠NLR = 13.650°
m∠FGH / m∠NLR = 3.000
m∠AELR = 81.900°
m∠FGH = 40.950°
m∠AELR / m∠FGH = 2.000
m∠ELF = 40.950°

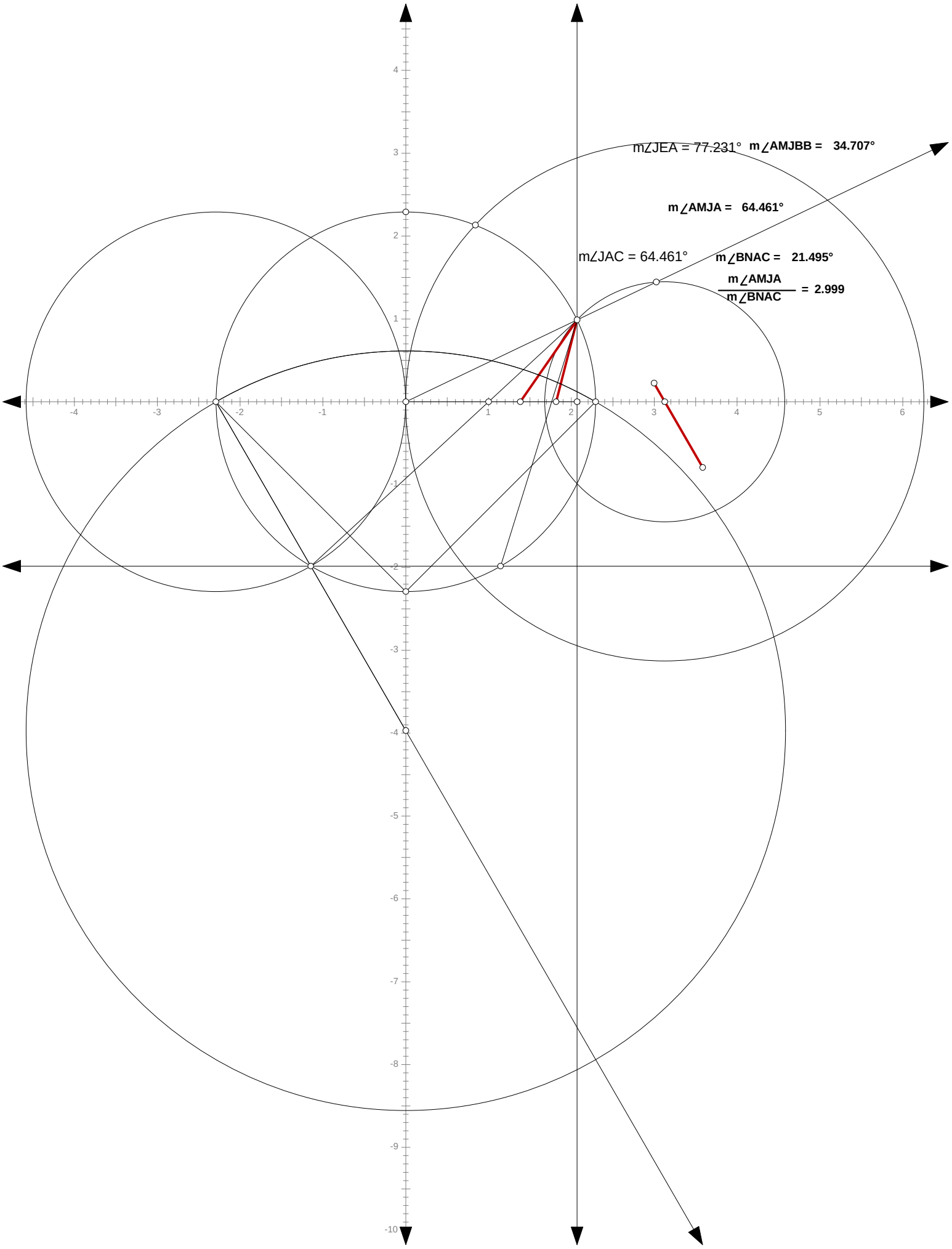

m∠FGC = 13.069°
m∠GKL = 39.208°
m∠GKL / m∠FGC = 3.000
m∠LKG = 39.208°
m∠GNP = 39.208°
m∠ONP = 13.069°
m∠FGC = 13.069°
m∠ACAGAH = 32.673°
m∠LKG / m∠ACAGAH = 1.200
m∠LKG - m ∠ACAGAH = 6.535°
m∠KANAK = 78.416°
m∠KANAK / m∠GNP = 2.000
m∠ADACF = 6.535°
m∠GKL / m∠ADACF = 6.000
m∠LKAC = 32.673°
m∠LKAC / m∠ADACF = 5.000
m∠AIAGAH = 6.535°
m∠ACAGAI = 39.208°
m∠ACAGAI / m∠AIAGAH = 6.000
m∠GIF = 26.139°
m∠GIF / m∠FGC = 2.000
m∠KIF = 52.277°
m∠KIF / m∠FGC = 4.000
m∠RGK = 13.069°
m∠CGN = 39.208°
m∠CGF = 13.069°
m∠CGN / m∠CGF = 3.000

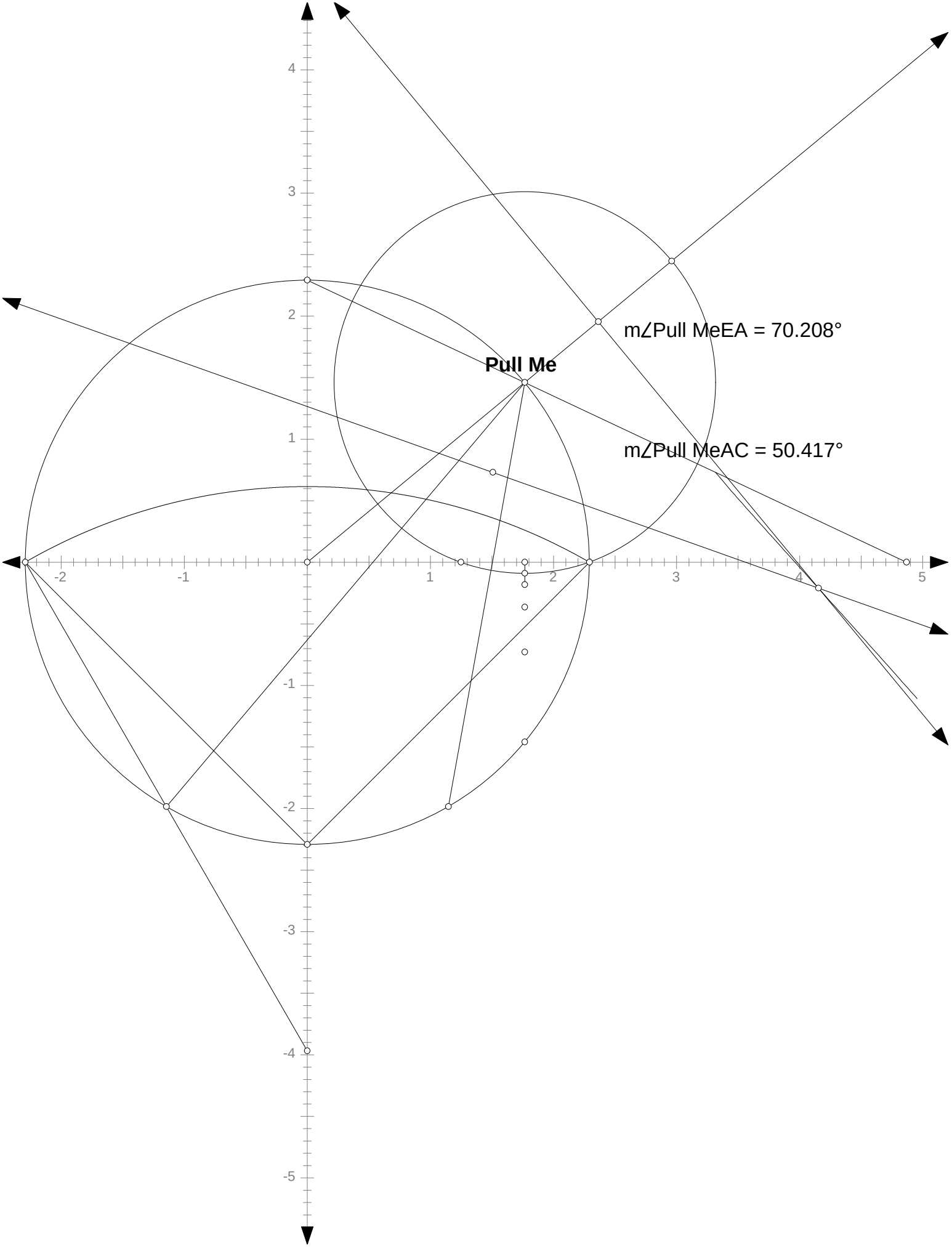


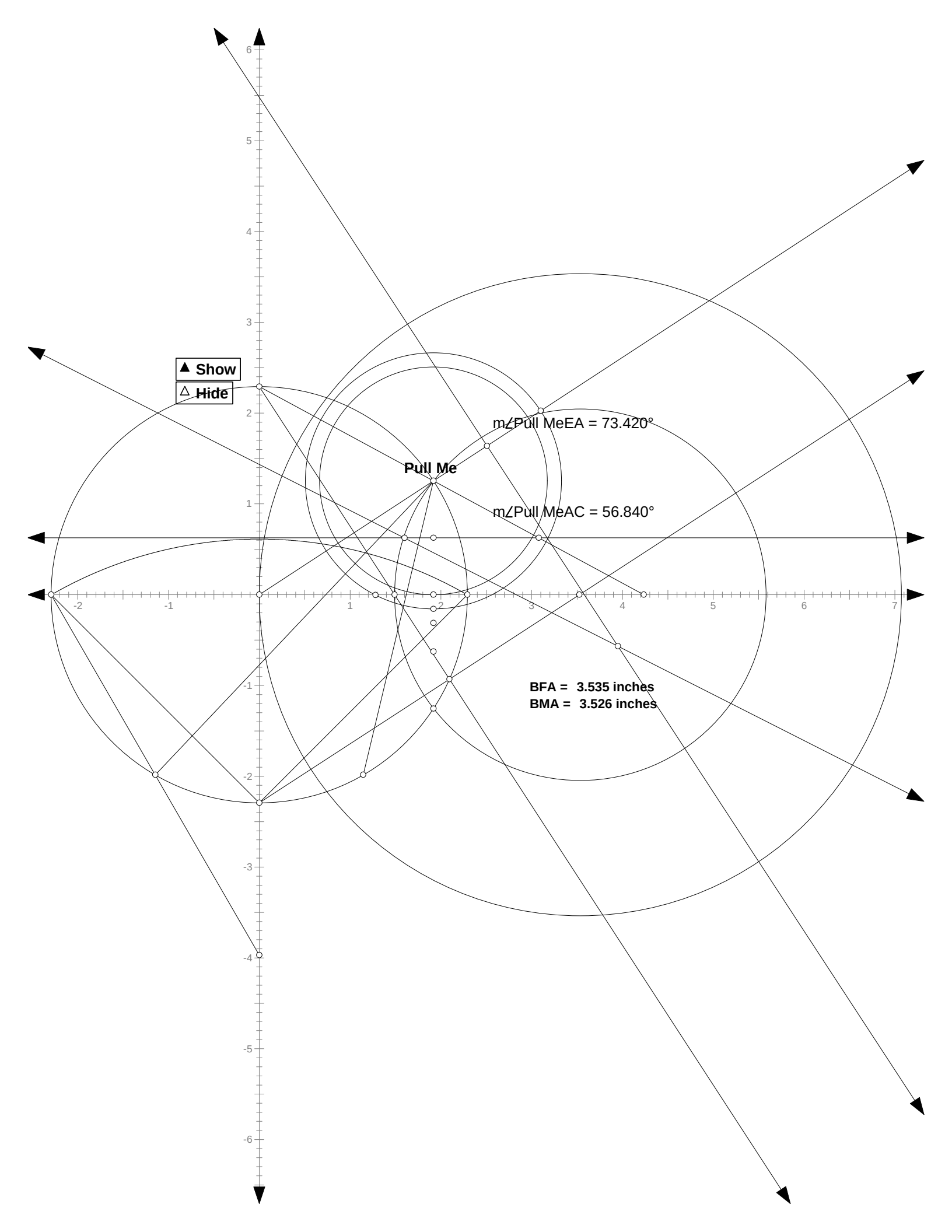

BAN = 1.573 inches
PN = 1.573 inches
m∠AMKAQ = 14.036°
m∠GNAW = 47.891°
m∠GNAW / m∠AMKAQ = 3.412
GAN = 1.740 inches
m∠GNP = 42.109°
KAN = 1.582 inches
m∠KANG = 42.109°
m∠GIK = 28.072°
m∠AOANAR = 28.072°
m∠KIG = 28.072°
m∠GNP = 42.109°

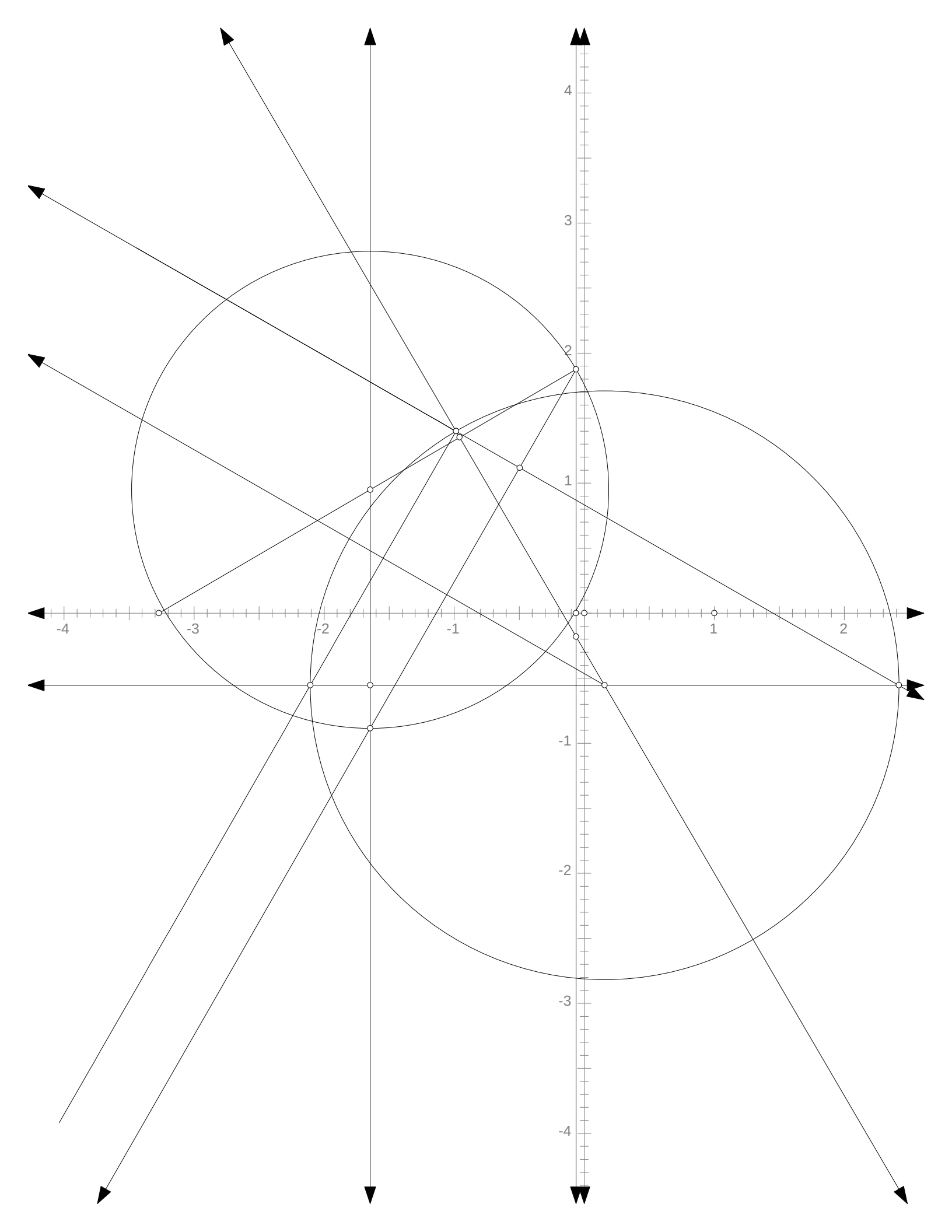

m∠AHNAD = 42.745°
m∠JFN = 42.745°
m∠JFD = 14.248°
m∠JFN / m∠JFD = 3.000
m∠CICGCH = 42.745°
h
m∠AHNAD = 42.745°

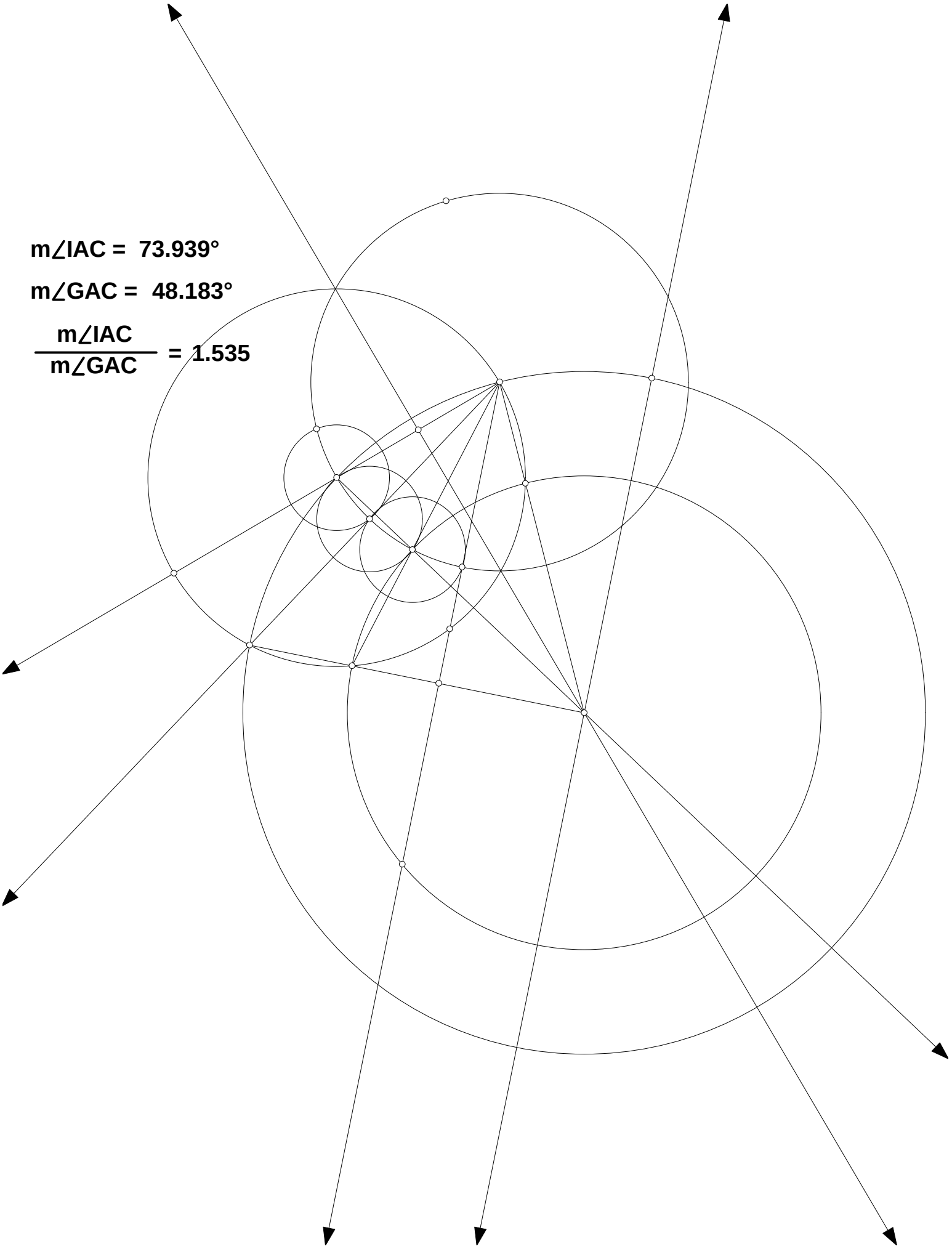

√AB • AE - AC = 0.000 inches
m∠FDG / m∠MON = 6.000
Show
Hide
Show
Hide
Show
Hide
Show
Hide
Show
Hide
G
M
N
F
L
H
E
D
C
K
B
A
Pull Me
O
-1
1
2
3
4
5
6
7

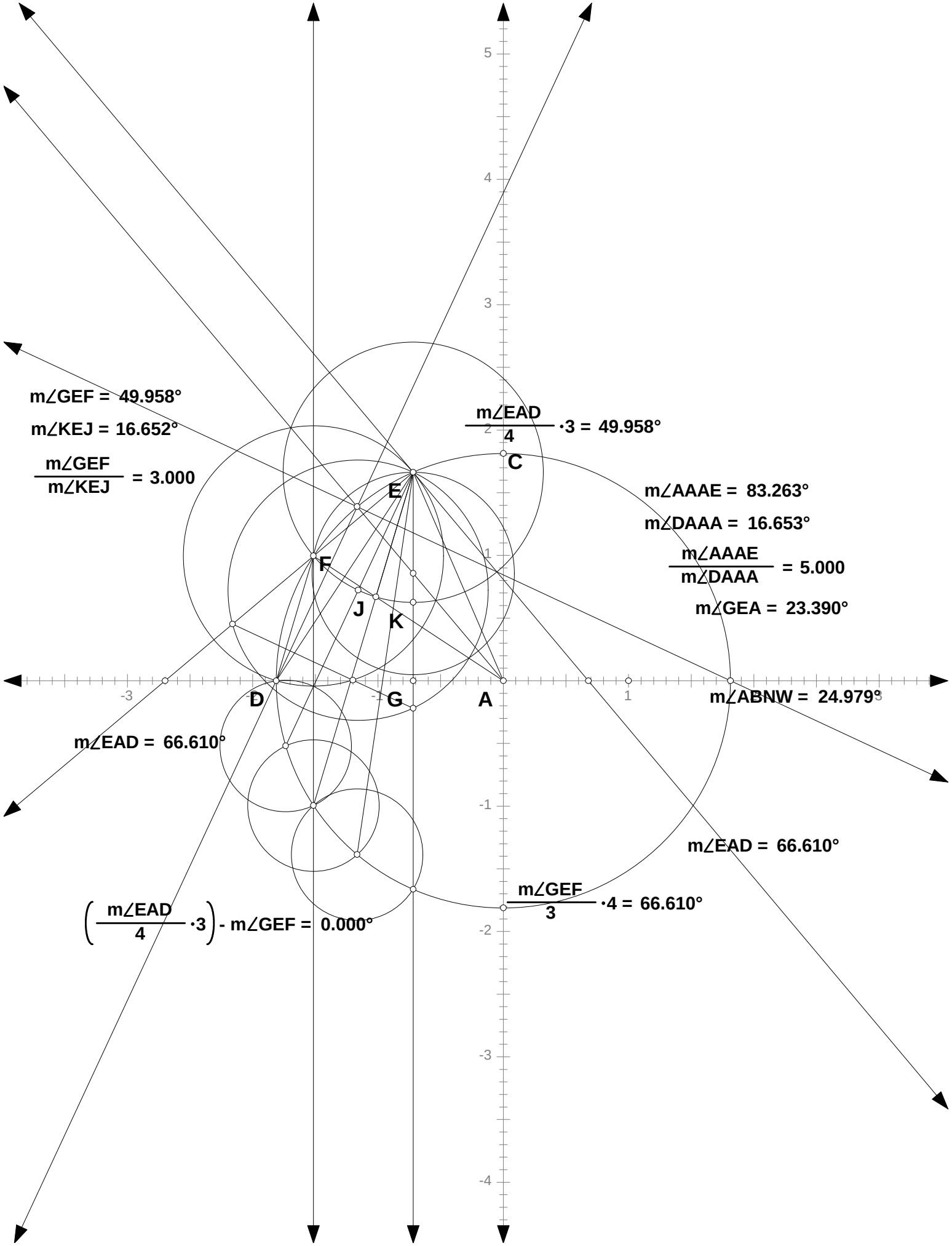

m∠JEA = 77.231°
m∠AMJBB = 34.707°
m∠AMJA = 64.461°
m∠JAC = 64.461°
m∠BNAC = 21.495°
m∠AMJA / m∠BNAC = 2.999

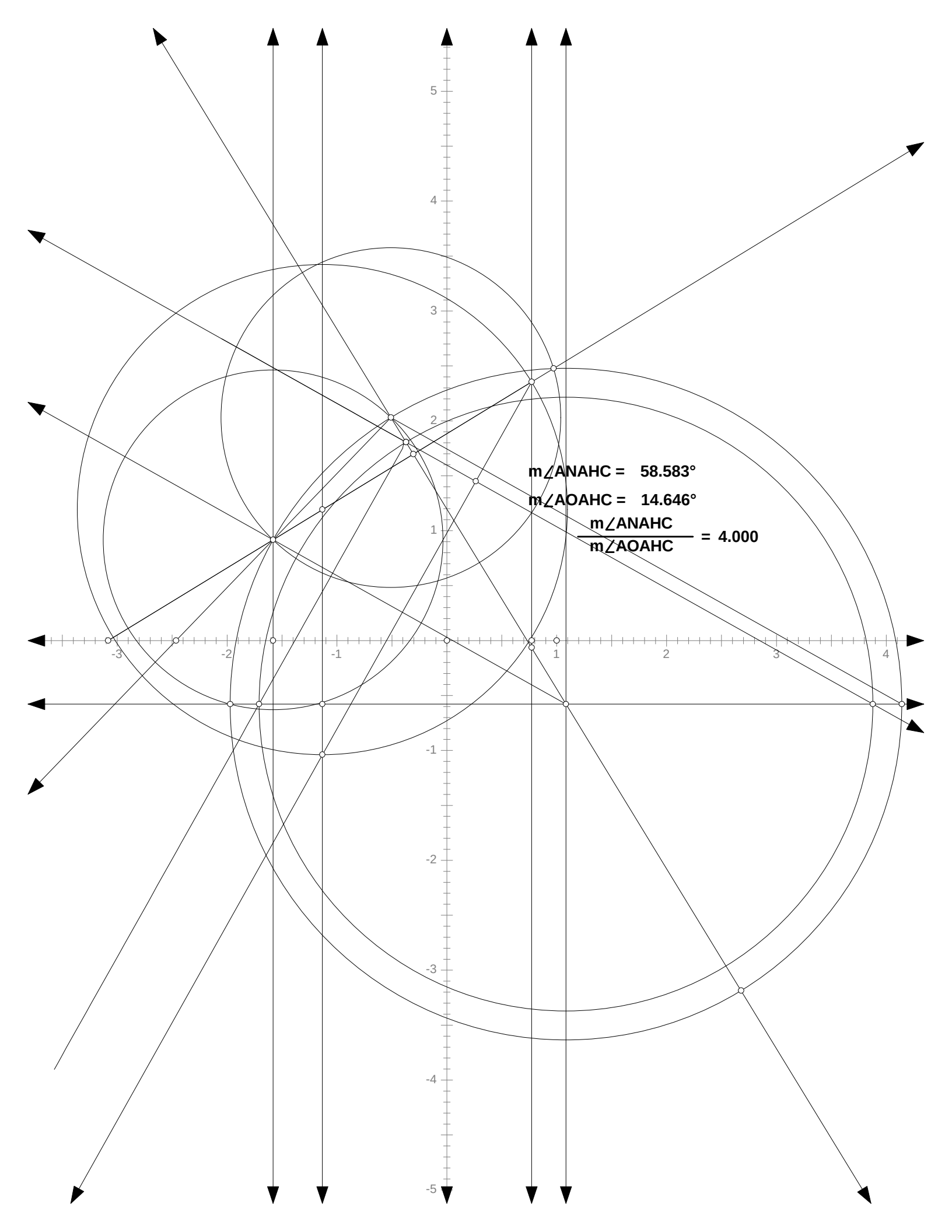

m∠Pull MeEA = 70.208°
Pull Me
m∠Pull MeAC = 50.417°
4
3
2
1
-1
-2
-3
-4
-5
-2
-1
1
2
3
4
5

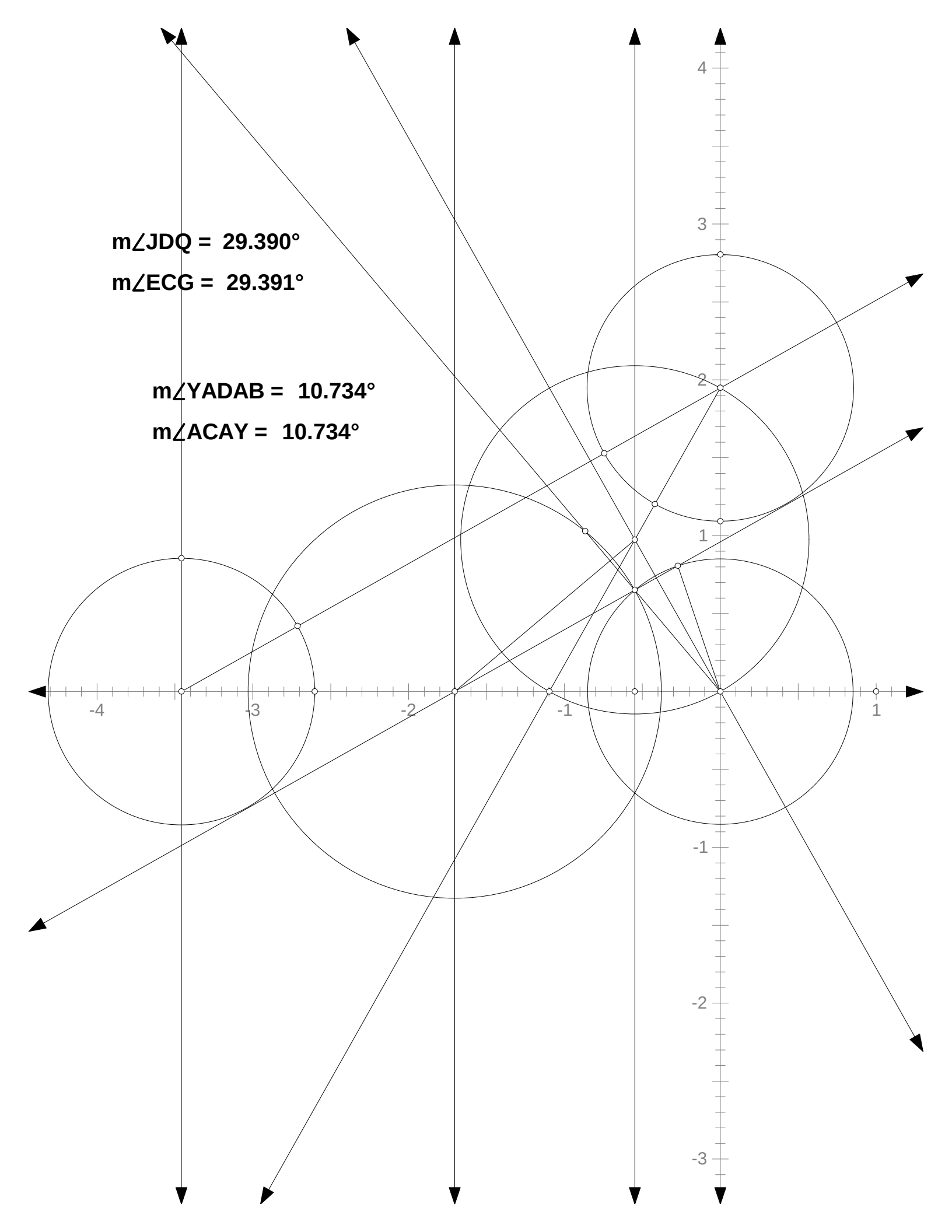

Show
Hide
m∠Pull MeEA = 73.420°
Pull Me
m∠Pull MeAC = 56.840°
BFA = 3.535 inches
BMA = 3.526 inches

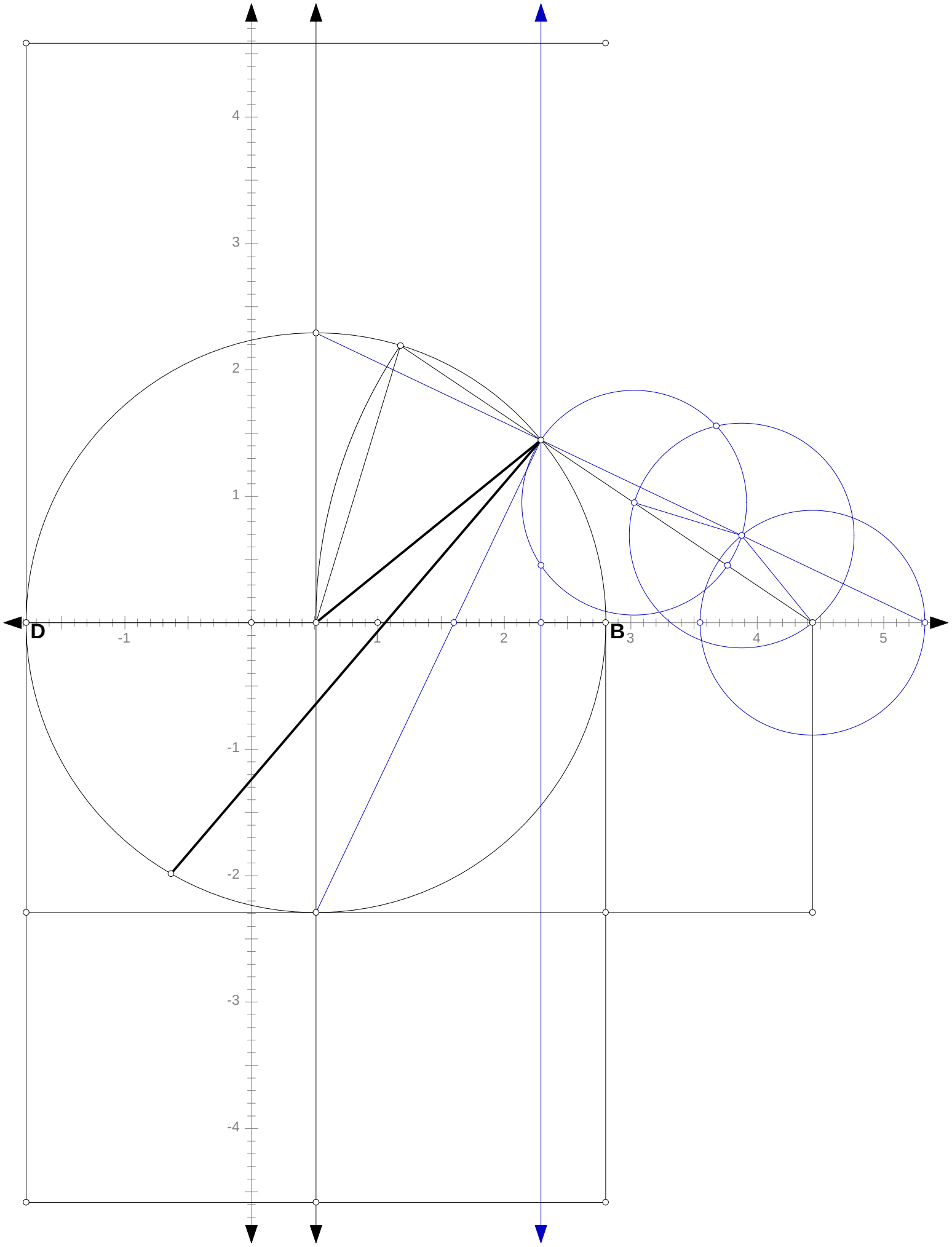

4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2

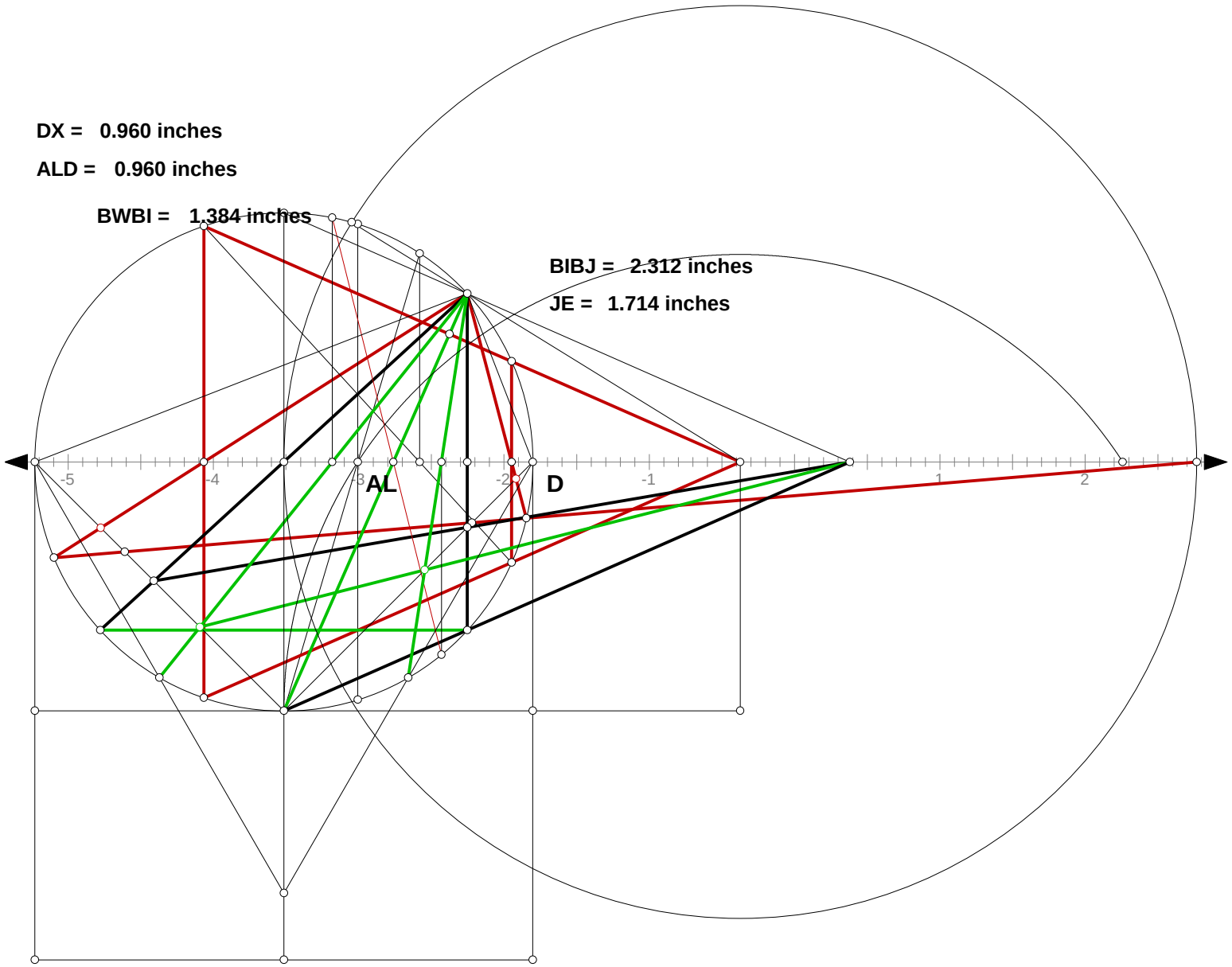

m∠IAC = 73.939°
m∠GAC = 48.183°
m∠IAC / m∠GAC = 1.535


m∠GEF = 49.958°
m∠KEJ = 16.652°
m∠GEF / m∠KEJ = 3.000
m∠EAD / 4 ·3 = 49.958°
m∠AAAE = 83.263°
m∠DAAA = 16.653°
m∠AAAE / m∠DAAA = 5.000
m∠GEA = 23.390°
m∠ABNW = 24.979°
m∠EAD = 66.610°
m∠EAD = 66.610°
m∠GEF / 3 ·4 = 66.610°
(m∠EAD / 4 ·3) - m∠GEF = 0.000°
C
E
F
J
K
D
G
A


m∠ANAHC = 58.583°
m∠AOAHC = 14.646°
m∠ANAHC / m∠AOAHC = 4.000

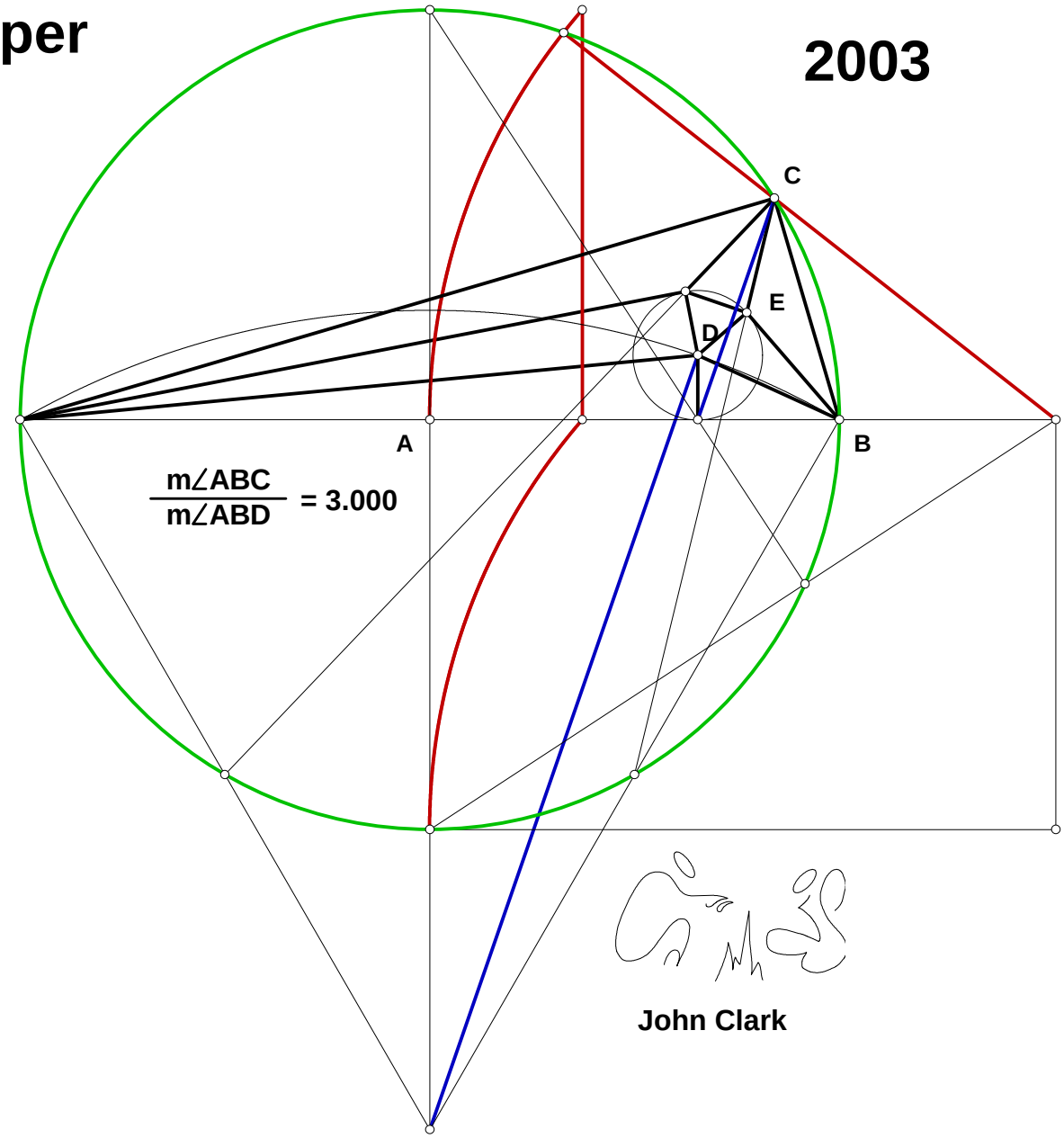

m∠JDQ = 29.390°
m∠ECG = 29.391°
m∠YADAB = 10.734°
m∠ACAY = 10.734°
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
1


4
3
2
1
-1
-2
-3
-4
D
-1
1
2
B
3
4
5

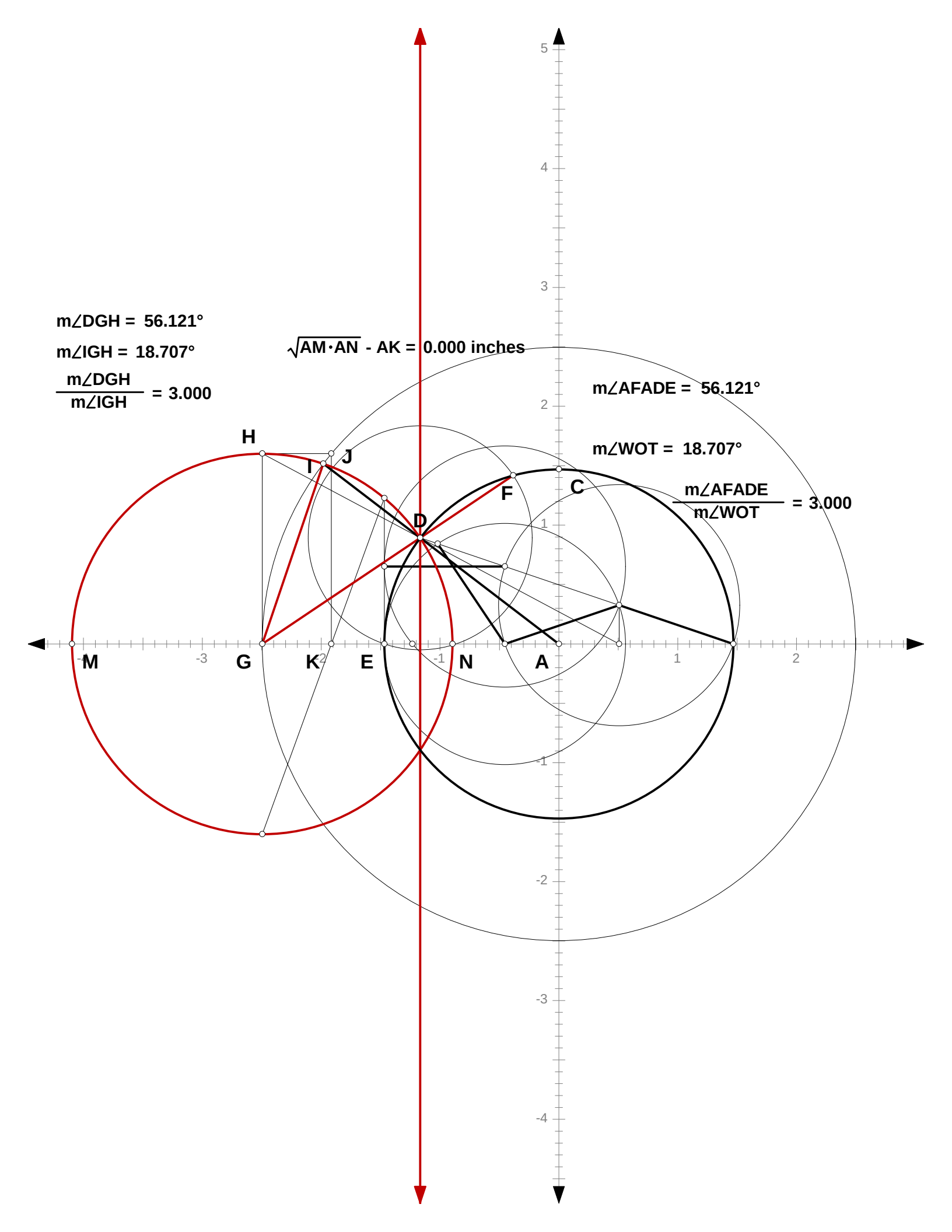

DX = 0.960 inches
ALD = 0.960 inches
BWBI = 1.384 inches
BIBJ = 2.312 inches
JE = 1.714 inches
-5
-4
-3
AL
-2
D
-1
1
2

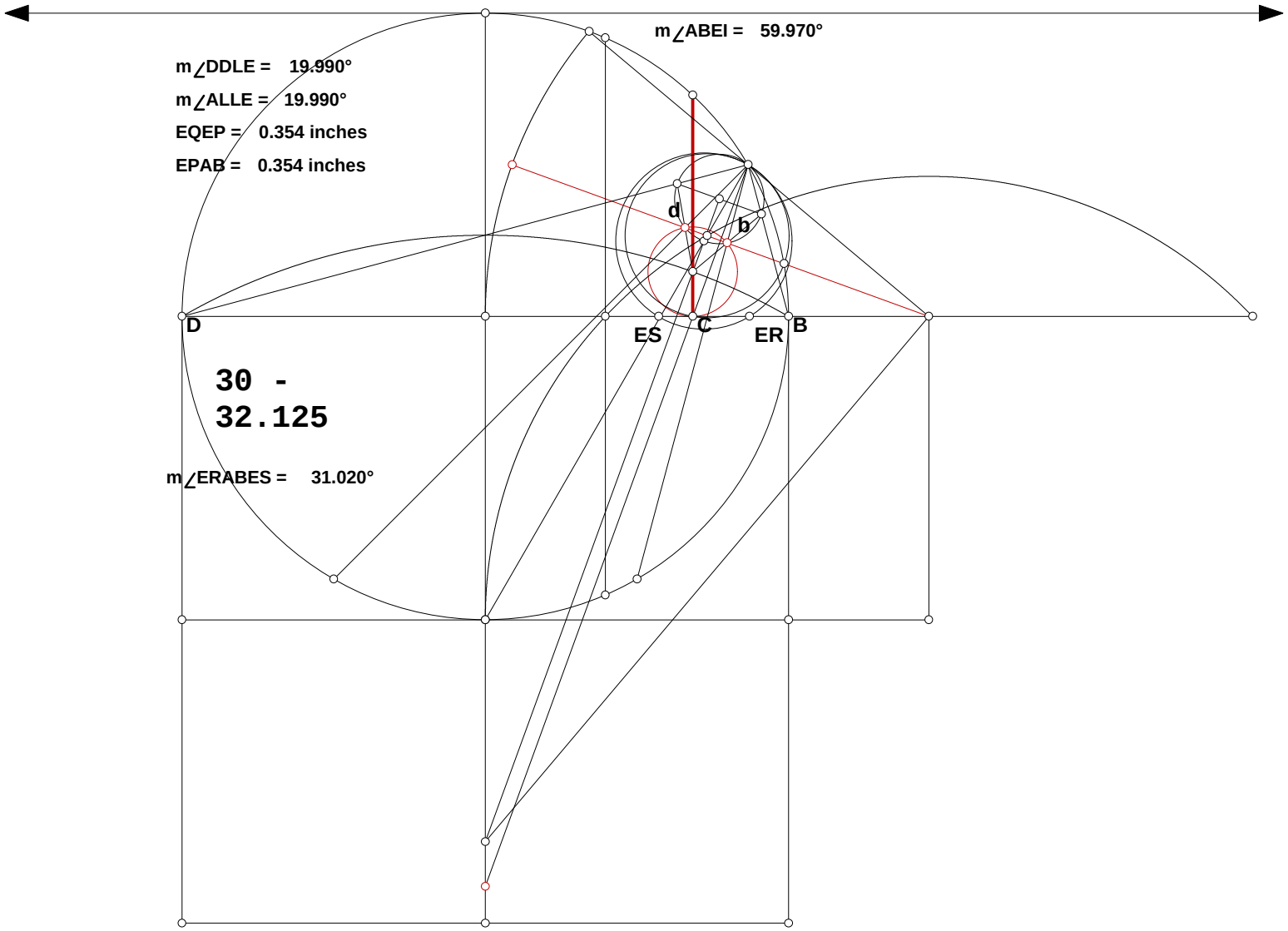
Three Pieces Of Paper

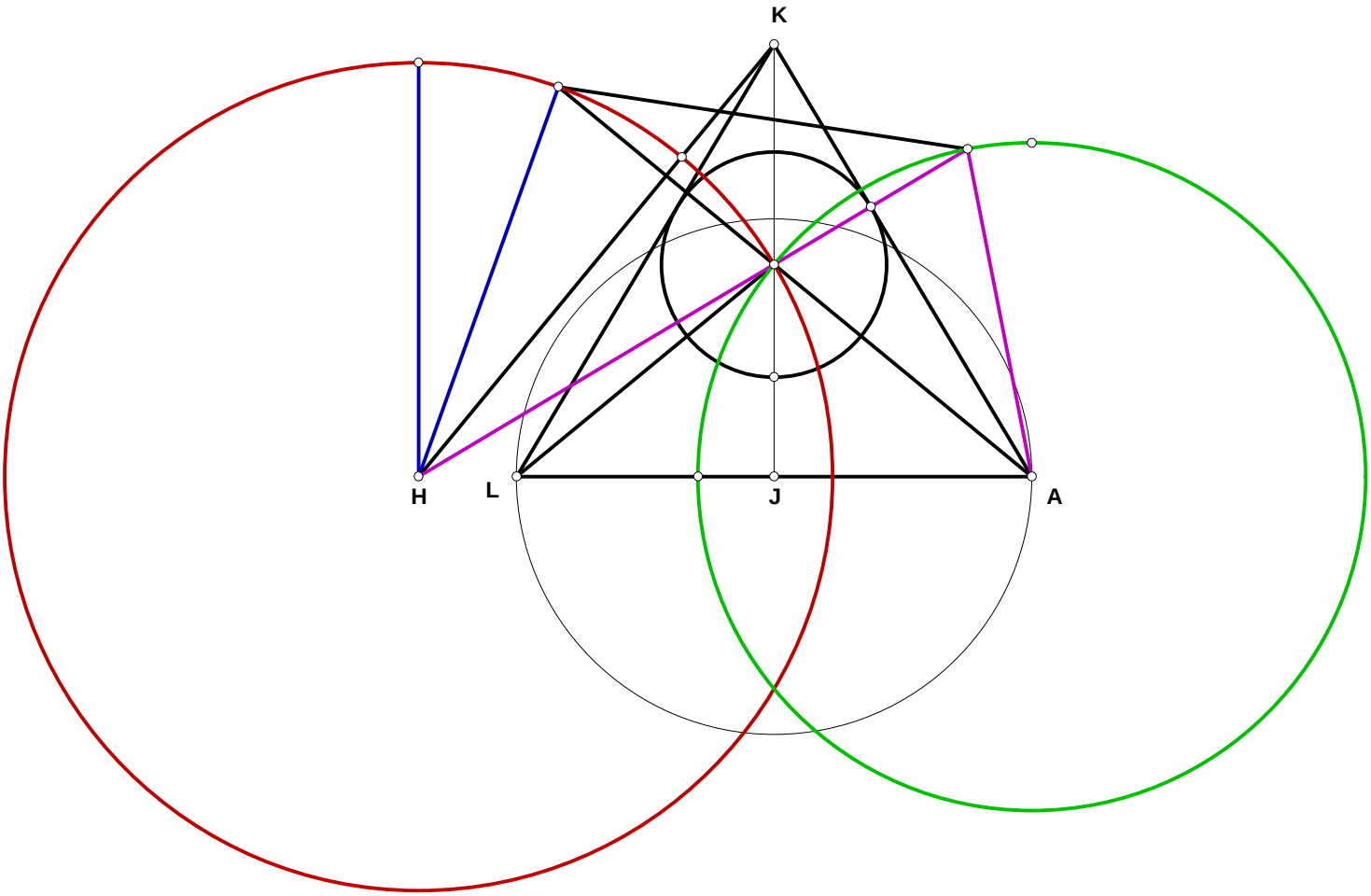
2003

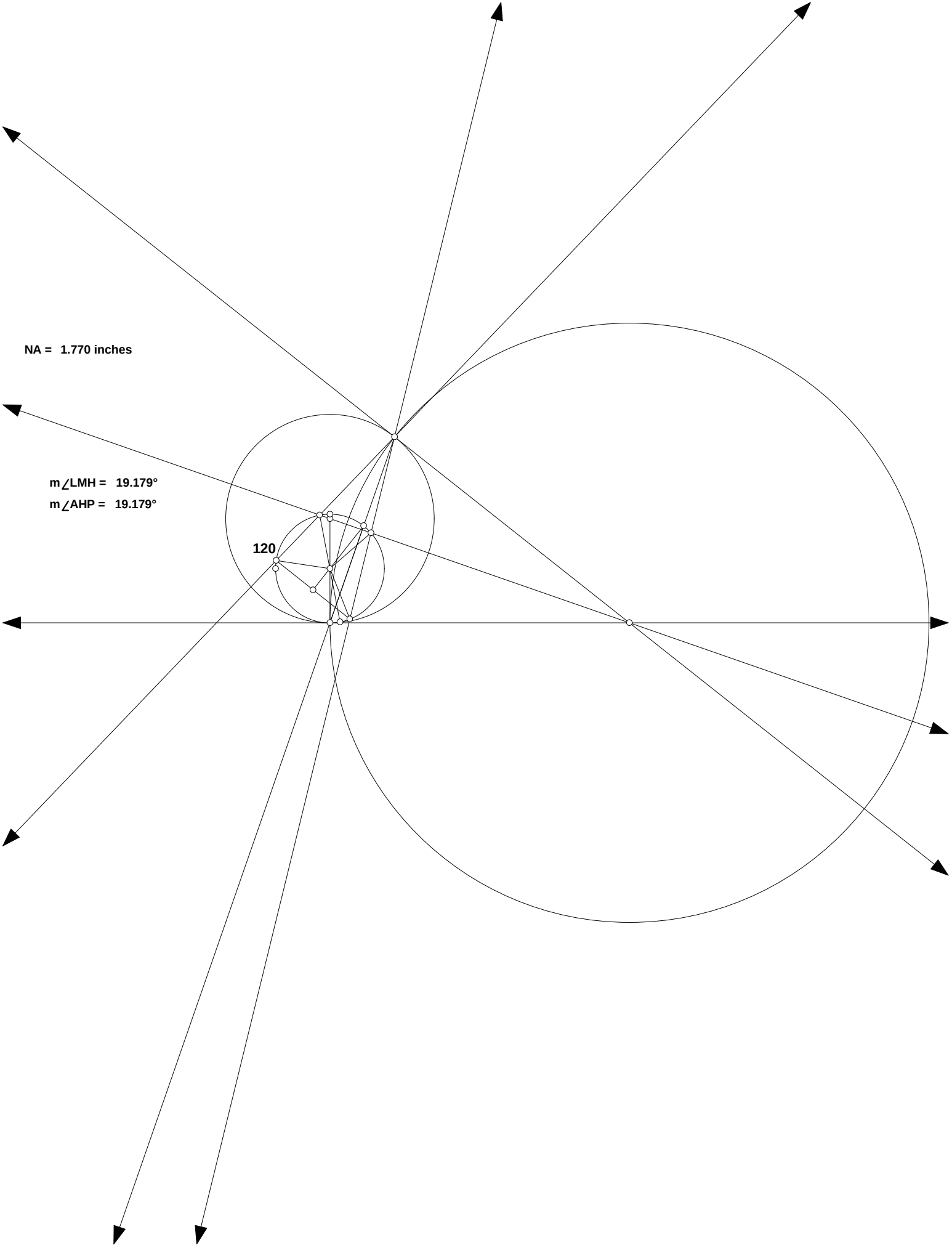


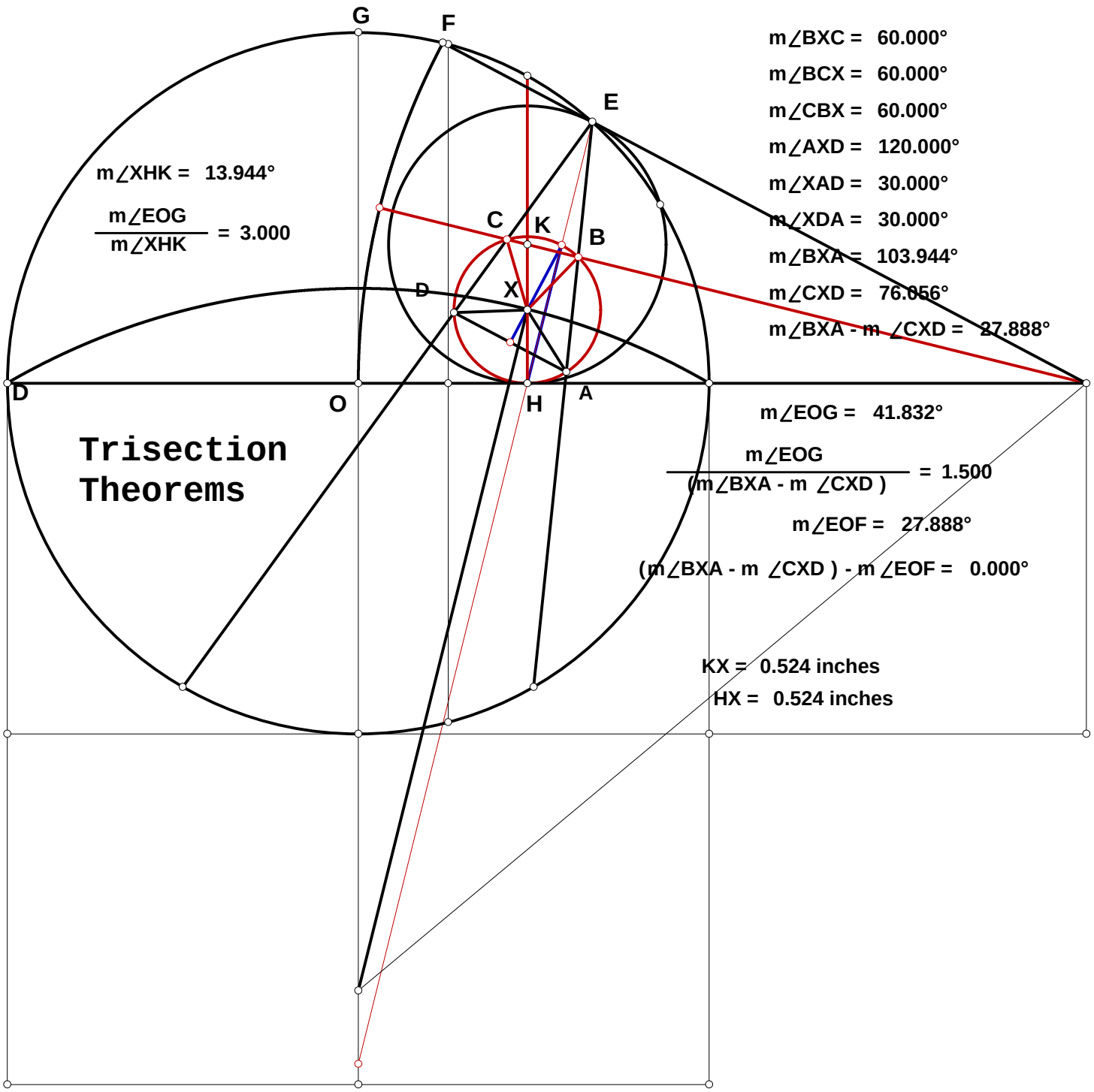
John Clark

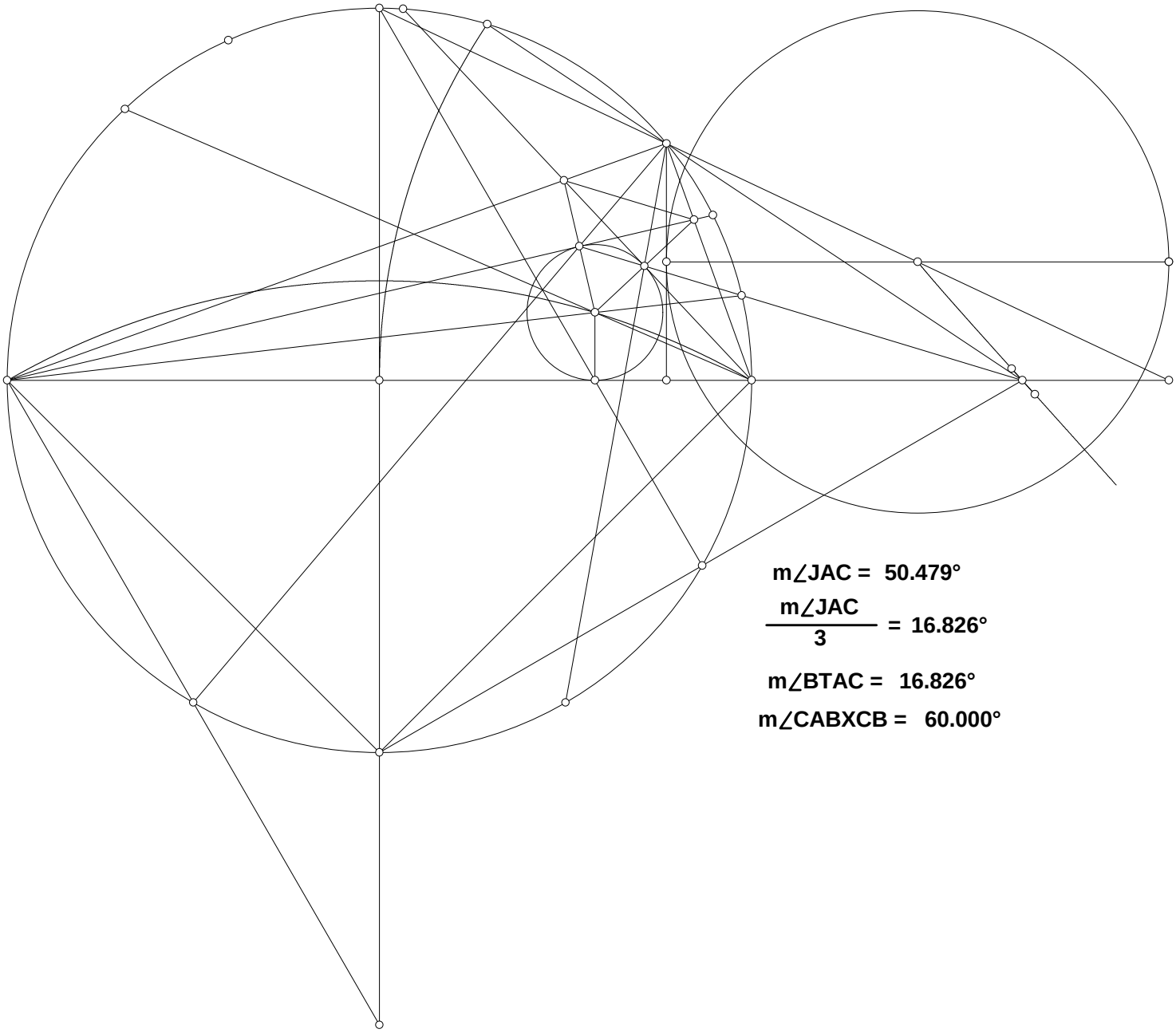

m∠DGH = 56.121°
m∠IGH = 18.707°
m∠DGH / m∠IGH = 3.000
√AM·AN - AK = 0.000 inches
m∠AFADE = 56.121°
m∠WOT = 18.707°
m∠AFADE / m∠WOT = 3.000
H
J
I
F
C
D
M
G
K
E
N
A
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
1
2


m∠ABEI = 59.970°
m∠DDLE = 19.990°
m∠ALLE = 19.990°
EQEP = 0.354 inches
EPAB = 0.354 inches
d
b
D
ES
C
ER
B
30 -
32.125
m∠ERABES = 31.020°


K
H
L
J
A


NA = 1.770 inches
m∠LMH = 19.179°
m∠AHP = 19.179°
120


G
F
E
C
K
B
D
X
D
O
H
A
m∠XHK = 13.944°
m∠EOG / m∠XHK = 3.000
m∠BXC = 60.000°
m∠BCX = 60.000°
m∠CBX = 60.000°
m∠AXD = 120.000°
m∠XAD = 30.000°
m∠XDA = 30.000°
m∠BXA = 103.944°
m∠CXD = 76.056°
m∠BXA - m ∠CXD = 27.888°
m∠EOG = 41.832°
m∠EOG / (m∠BXA - m ∠CXD) = 1.500
m∠EOF = 27.888°
(m∠BXA - m ∠CXD) - m∠EOF = 0.000°
KX = 0.524 inches
HX = 0.524 inches
Trisection Theorems

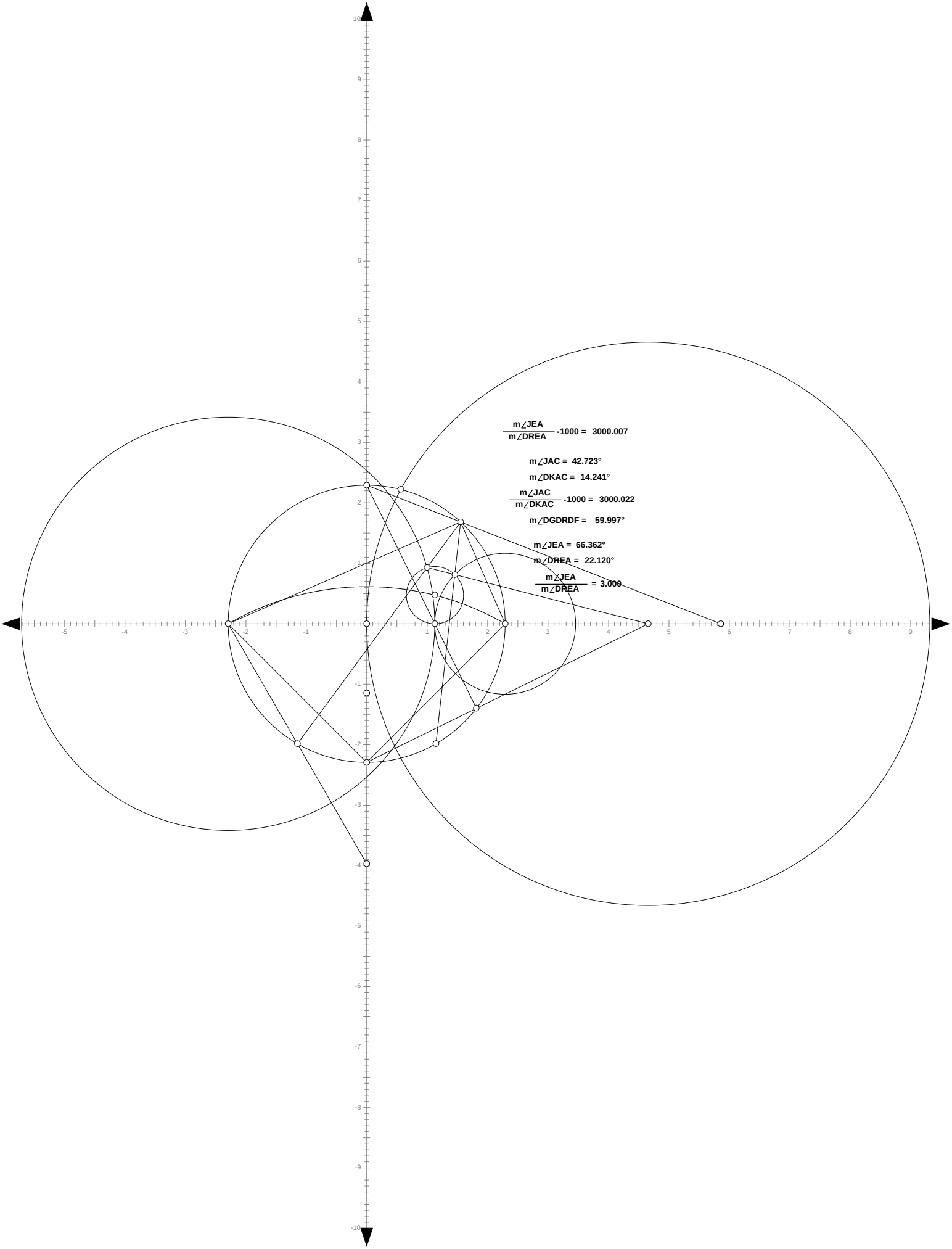


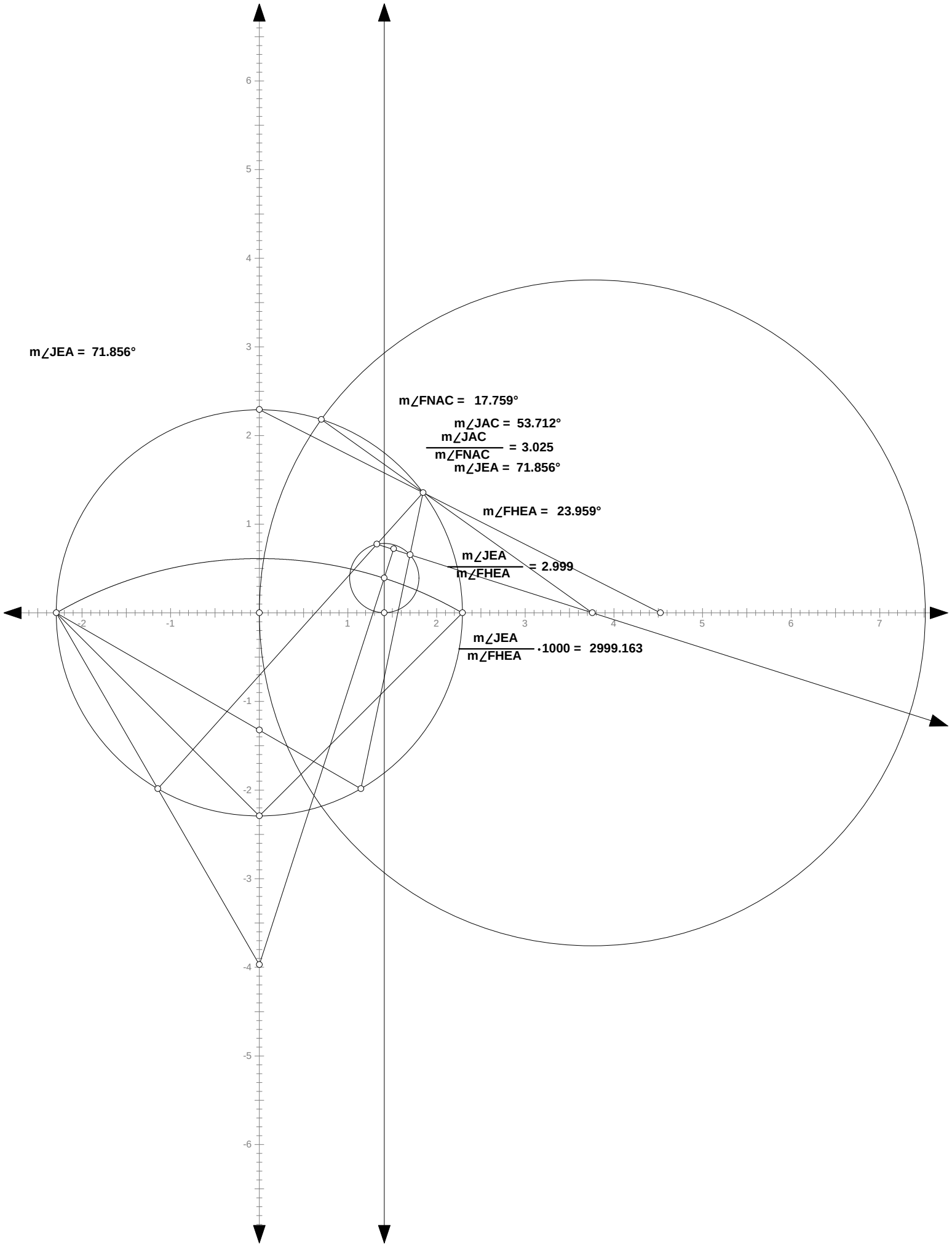
$m\angle JAC = 50.479°$

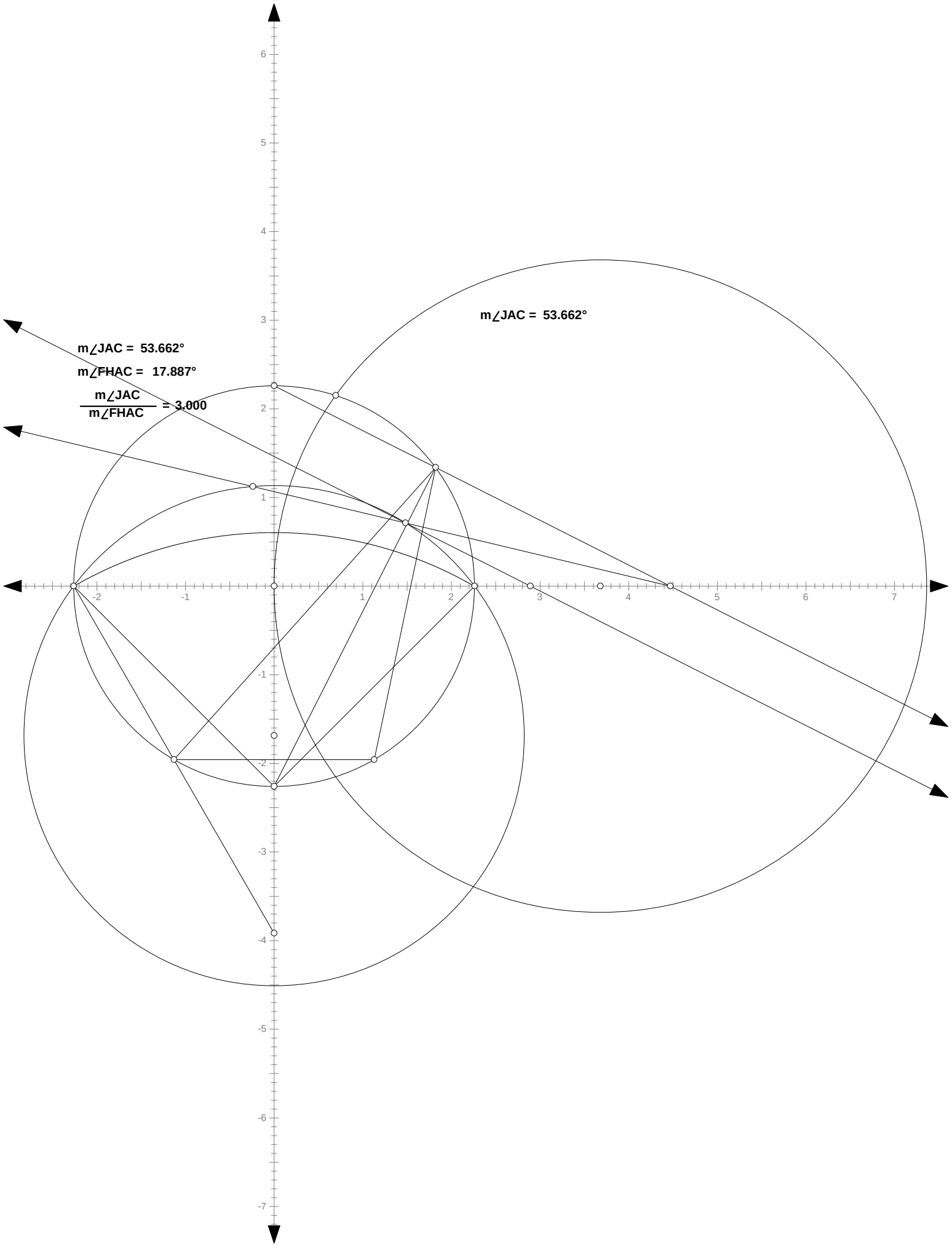
$\frac{m\angle JAC}{3} = 16.826°$

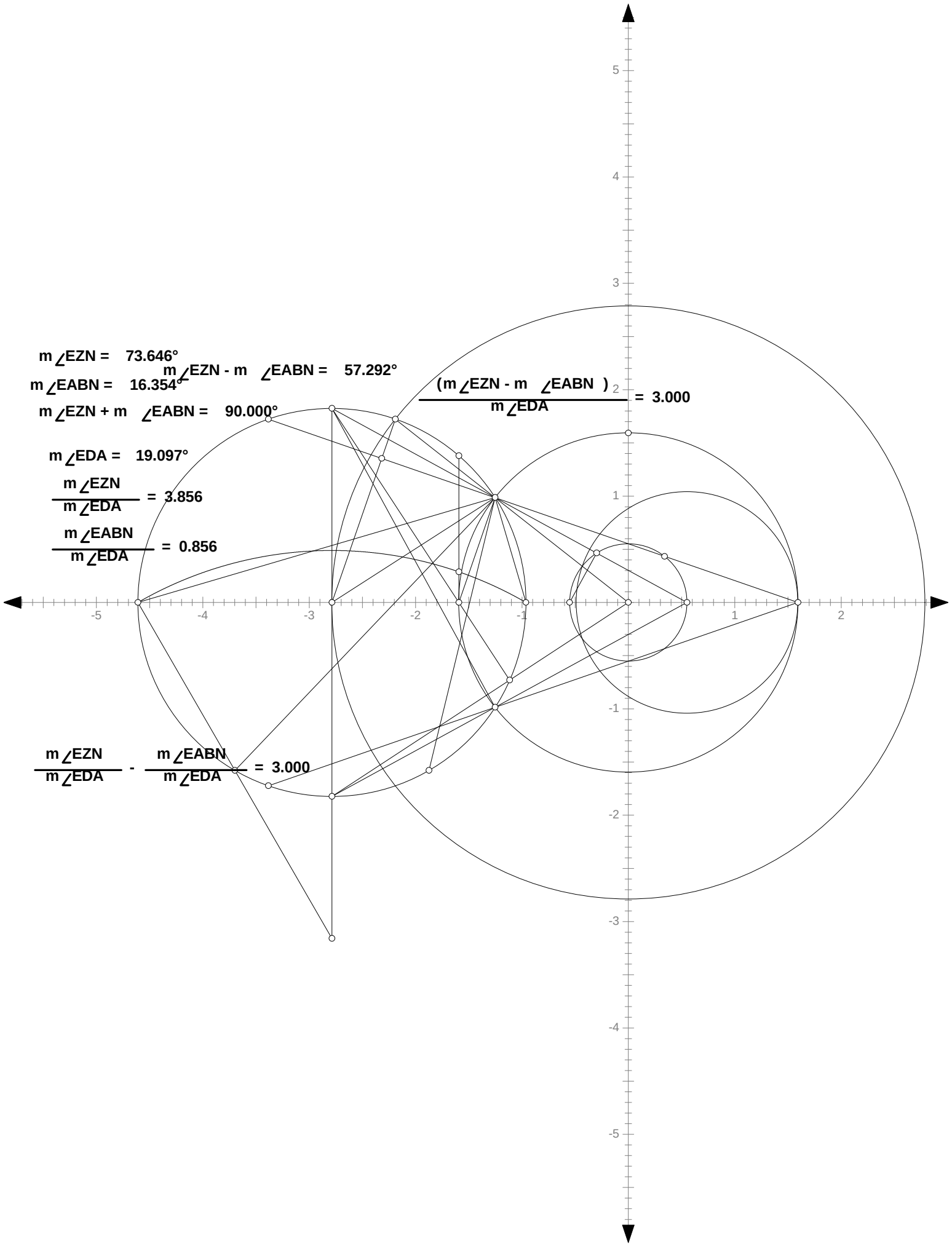
$m\angle BTAC = 16.826°$

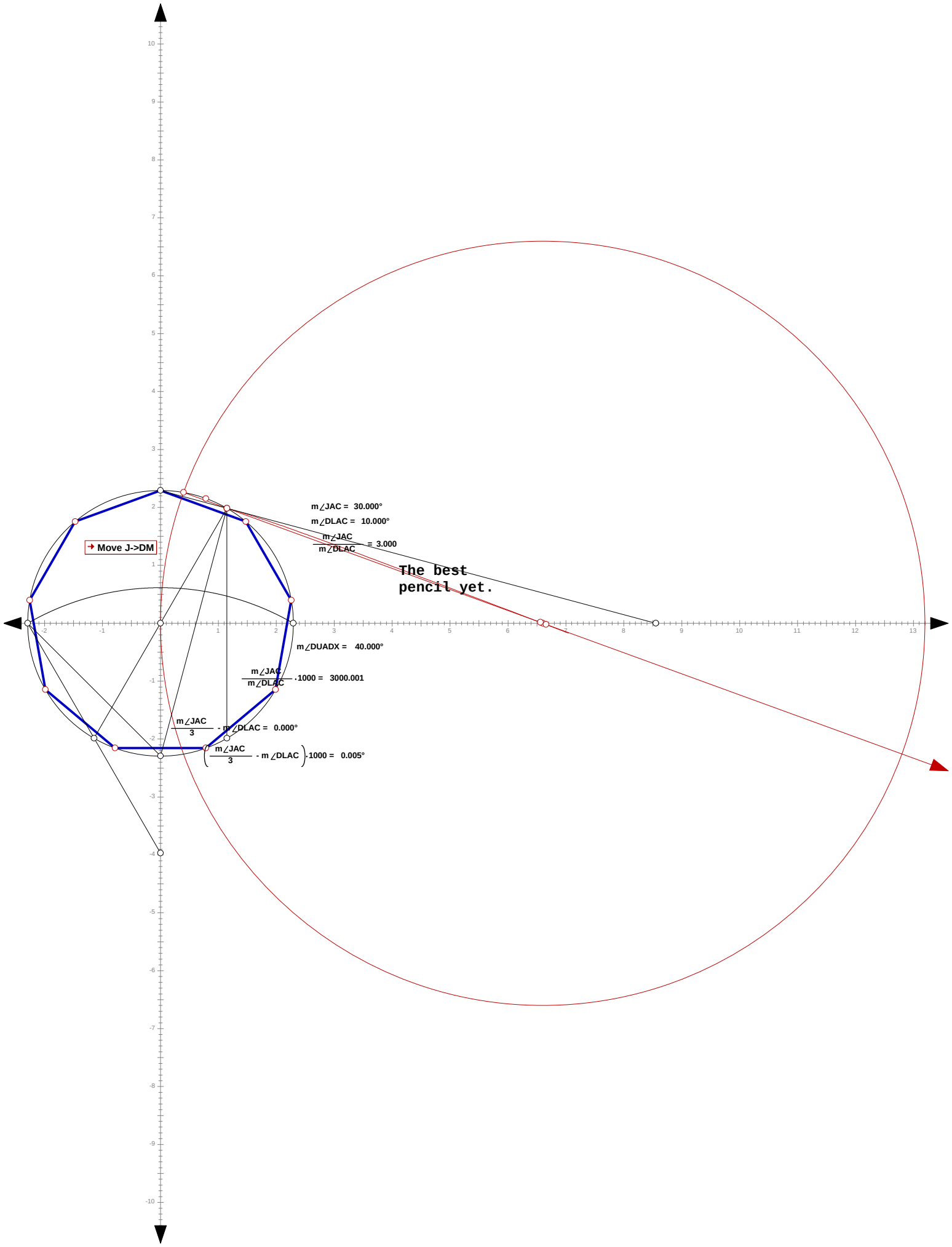
$m\angle CABXCB = 60.000°$

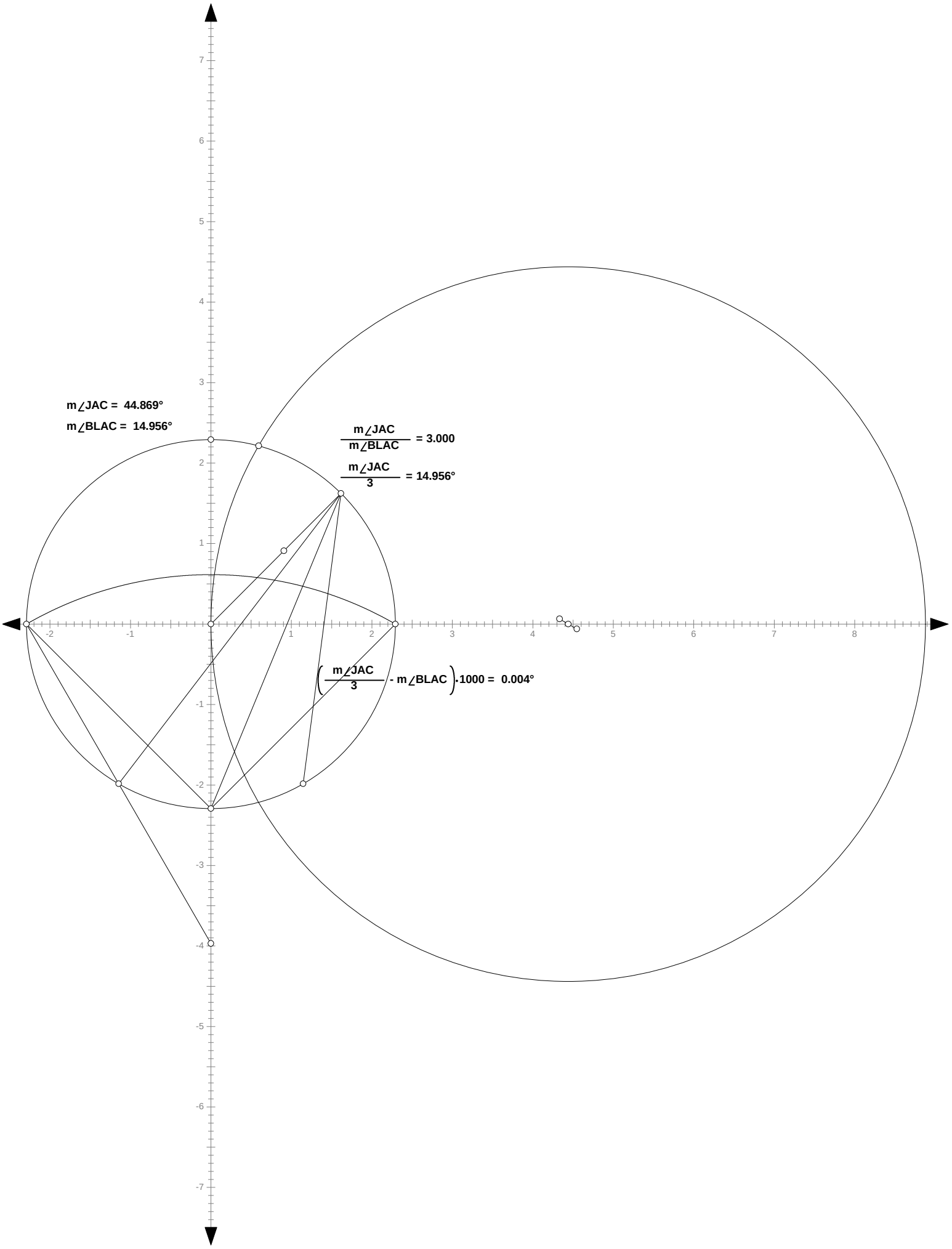

$\frac{m\angle JEA}{m\angle DREA}\cdot 1000 = 3000.007$
$m\angle JAC = 42.723°$
$m\angle DKAC = 14.241°$
$\frac{m\angle JAC}{m\angle DKAC}\cdot 1000 = 3000.022$
$m\angle DGDRDF = 59.997°$
$m\angle JEA = 66.362°$
$m\angle DREA = 22.120°$
$\frac{m\angle JEA}{m\angle DREA} = 3.000$

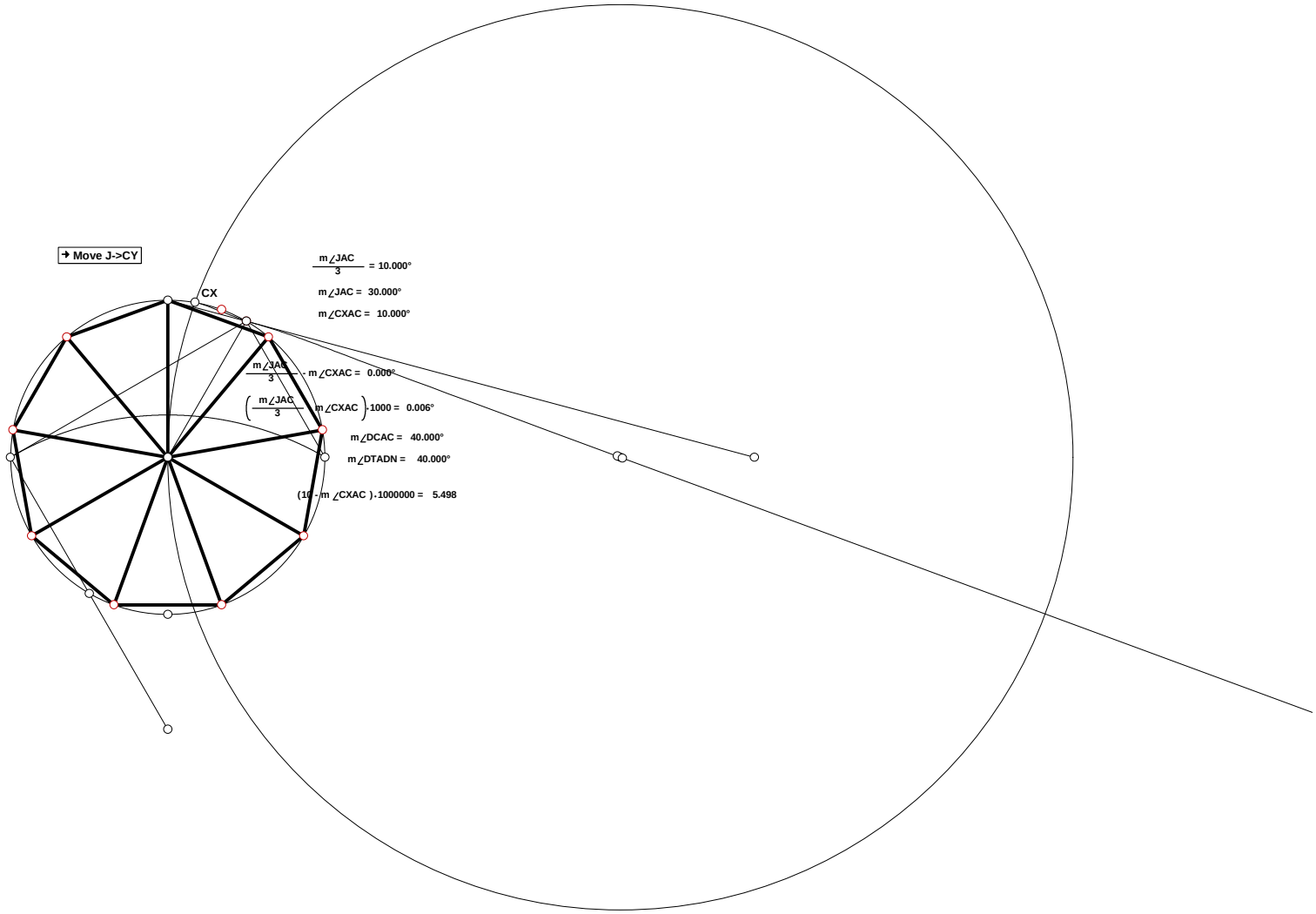

m∠JEA = 71.856°
m∠FNAC = 17.759°
m∠JAC = 53.712°
m∠JAC / m∠FNAC = 3.025
m∠JEA = 71.856°
m∠FHEA = 23.959°
m∠JEA / m∠FHEA = 2.999
m∠JEA / m∠FHEA ·1000 = 2999.163

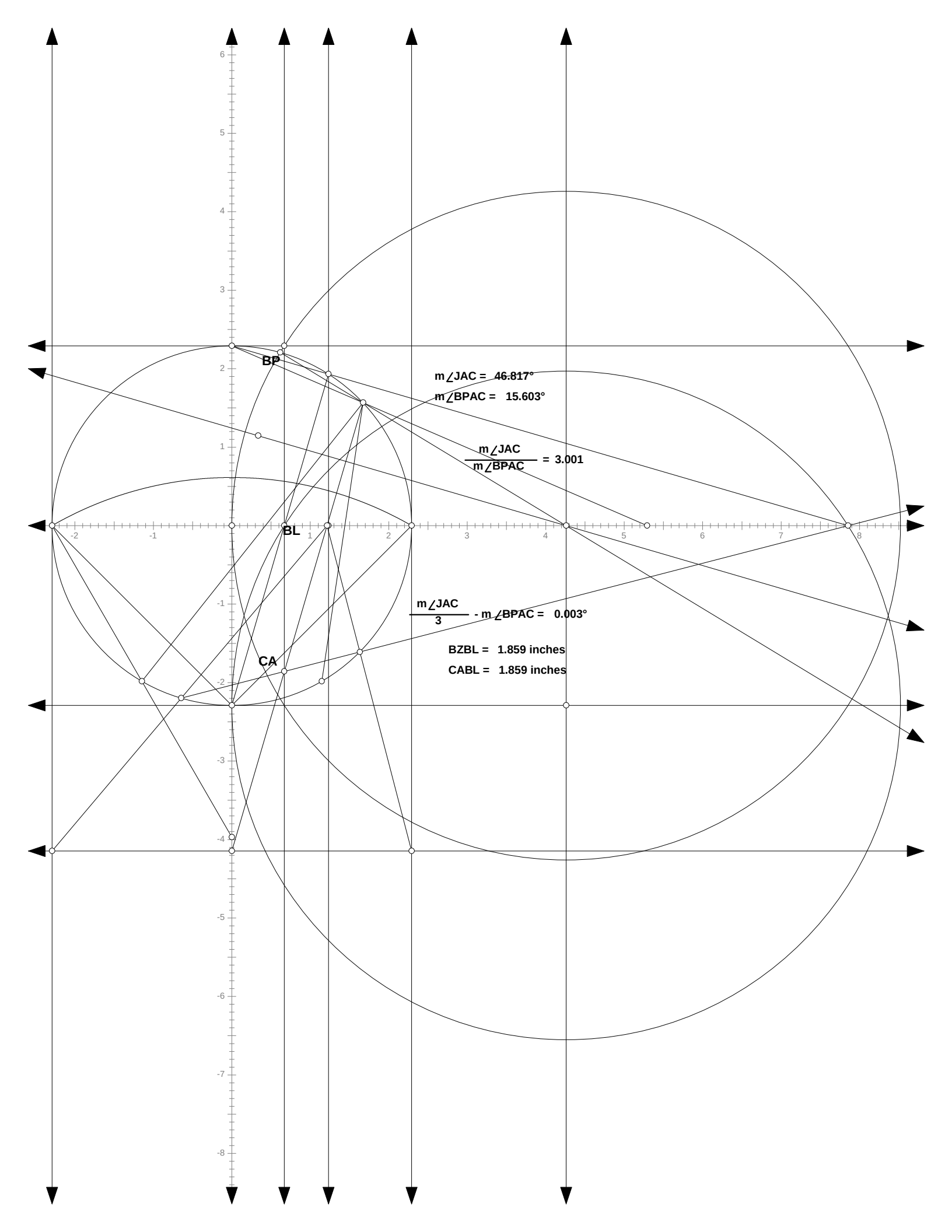

m∠JAC = 53.662°
m∠JAC = 53.662°
m∠FHAC = 17.887°
m∠JAC / m∠FHAC = 3.000

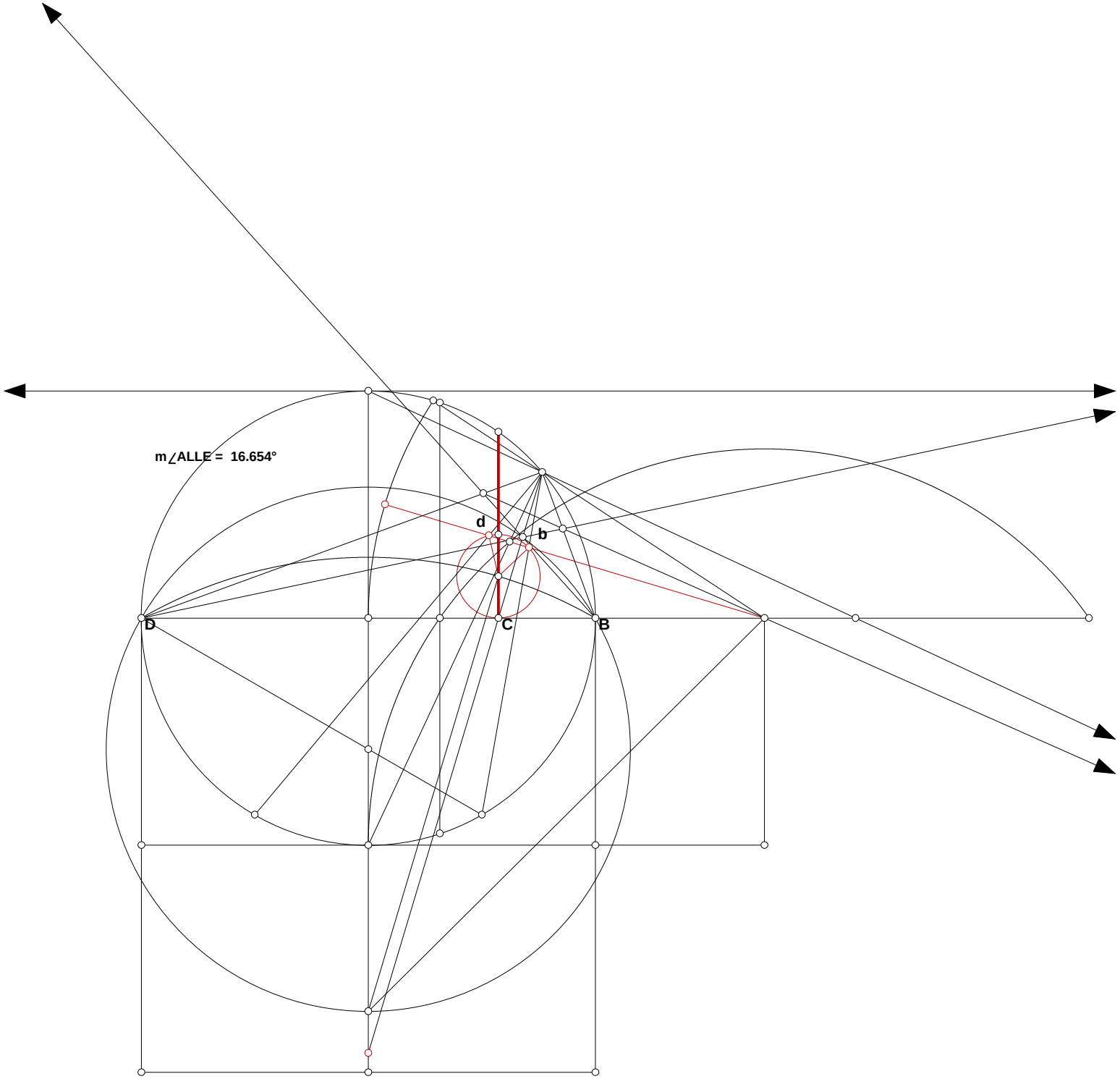

m∠EZN = 73.646°
m∠EABN = 16.354°
m∠EZN + m ∠EABN = 90.000°
m∠EZN - m ∠EABN = 57.292°
(m∠EZN - m ∠EABN) / m∠EDA = 3.000
m∠EDA = 19.097°
m∠EZN / m∠EDA = 3.856
m∠EABN / m∠EDA = 0.856
m∠EZN / m∠EDA - m∠EABN / m∠EDA = 3.000

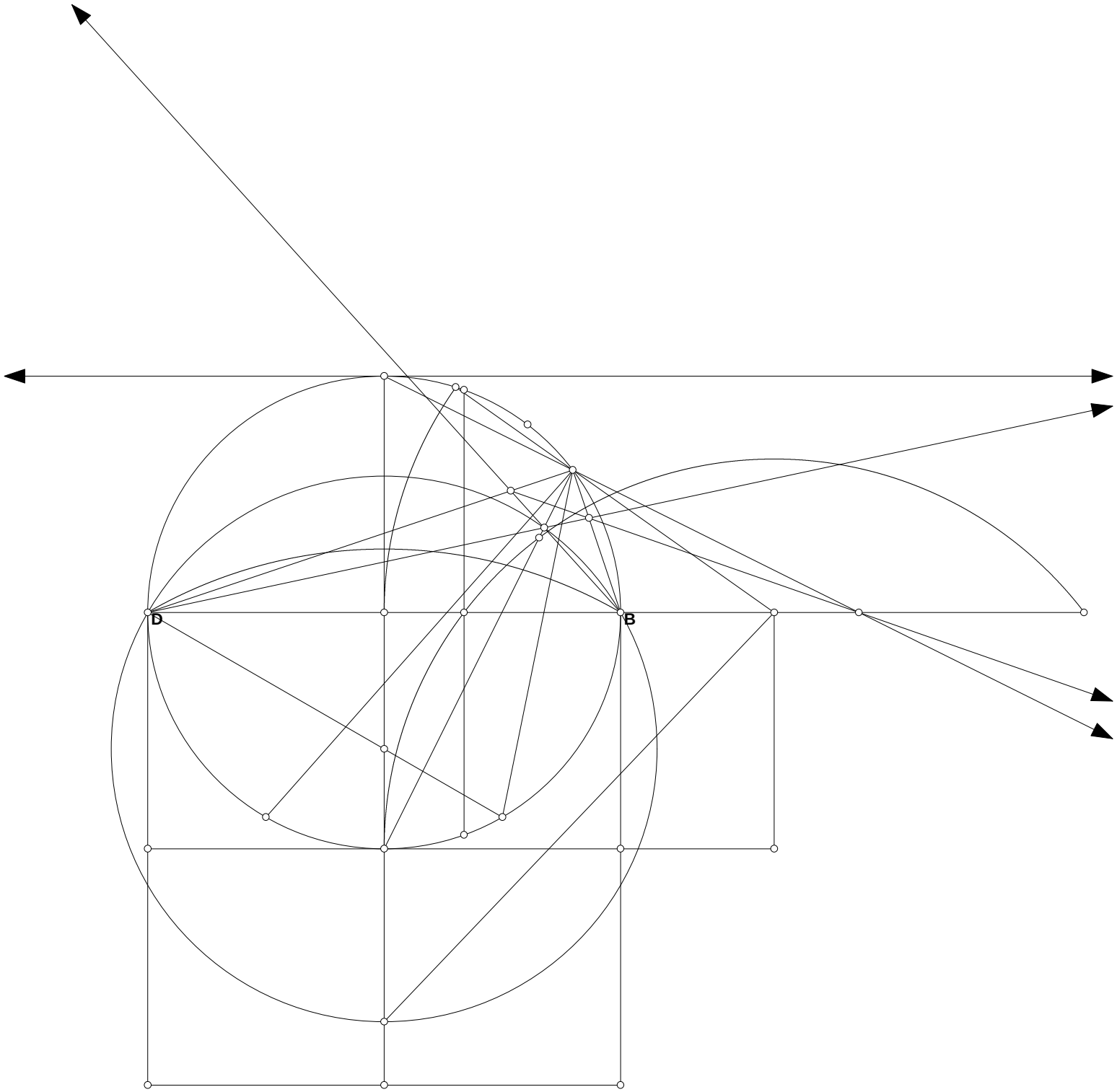

Move J->DM
m∠JAC = 30.000°
m∠DLAC = 10.000°
m∠JAC / m∠DLAC = 3.000
The best pencil yet.
m∠DUADX = 40.000°
m∠JAC / m∠DLAC ·1000 = 3000.001
m∠JAC / 3 - m∠DLAC = 0.000°
(m∠JAC / 3 - m∠DLAC) ·1000 = 0.005°

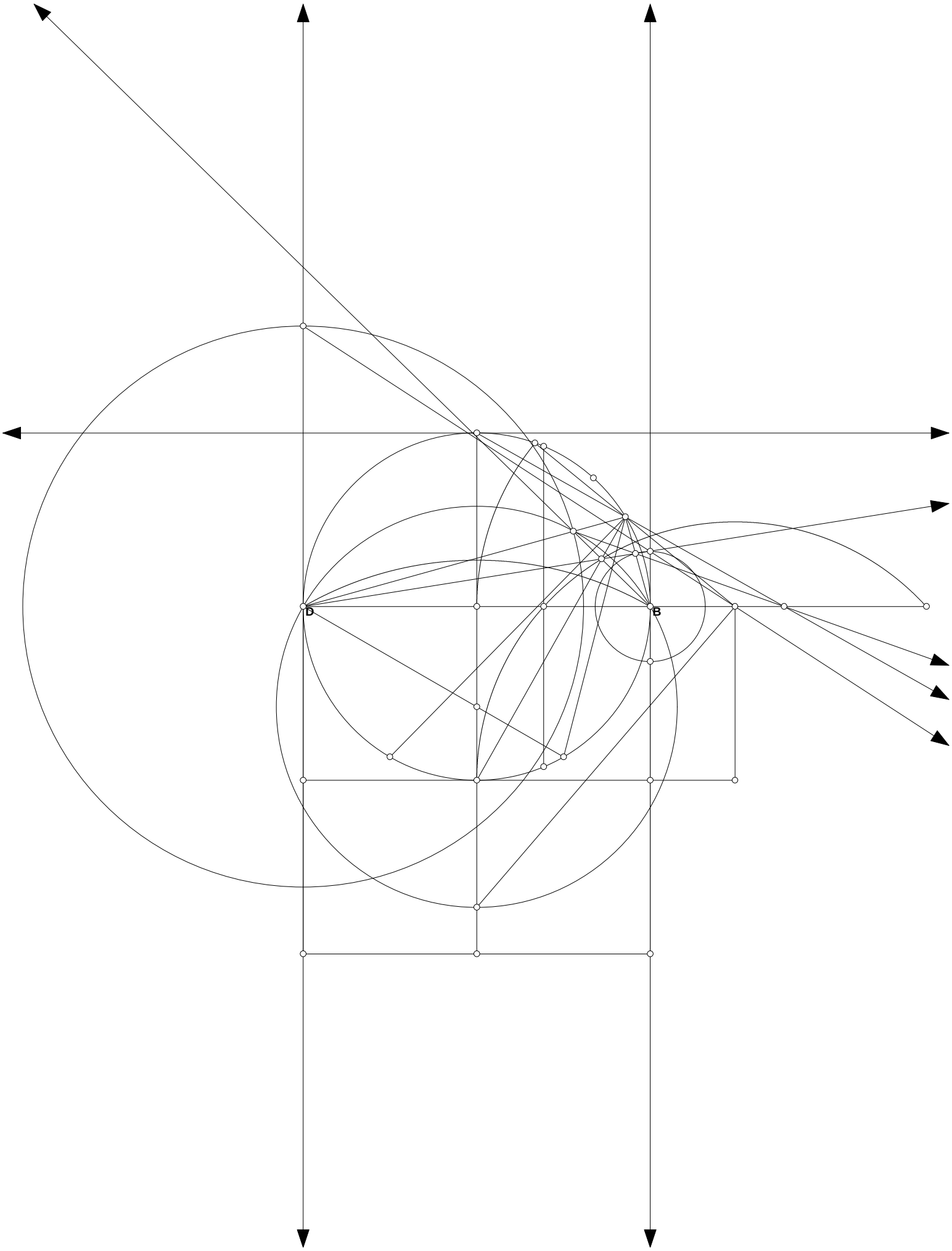

m∠JAC = 44.869°
m∠BLAC = 14.956°
m∠JAC / m∠BLAC = 3.000
m∠JAC / 3 = 14.956°
(m∠JAC / 3 - m∠BLAC)·1000 = 0.004°

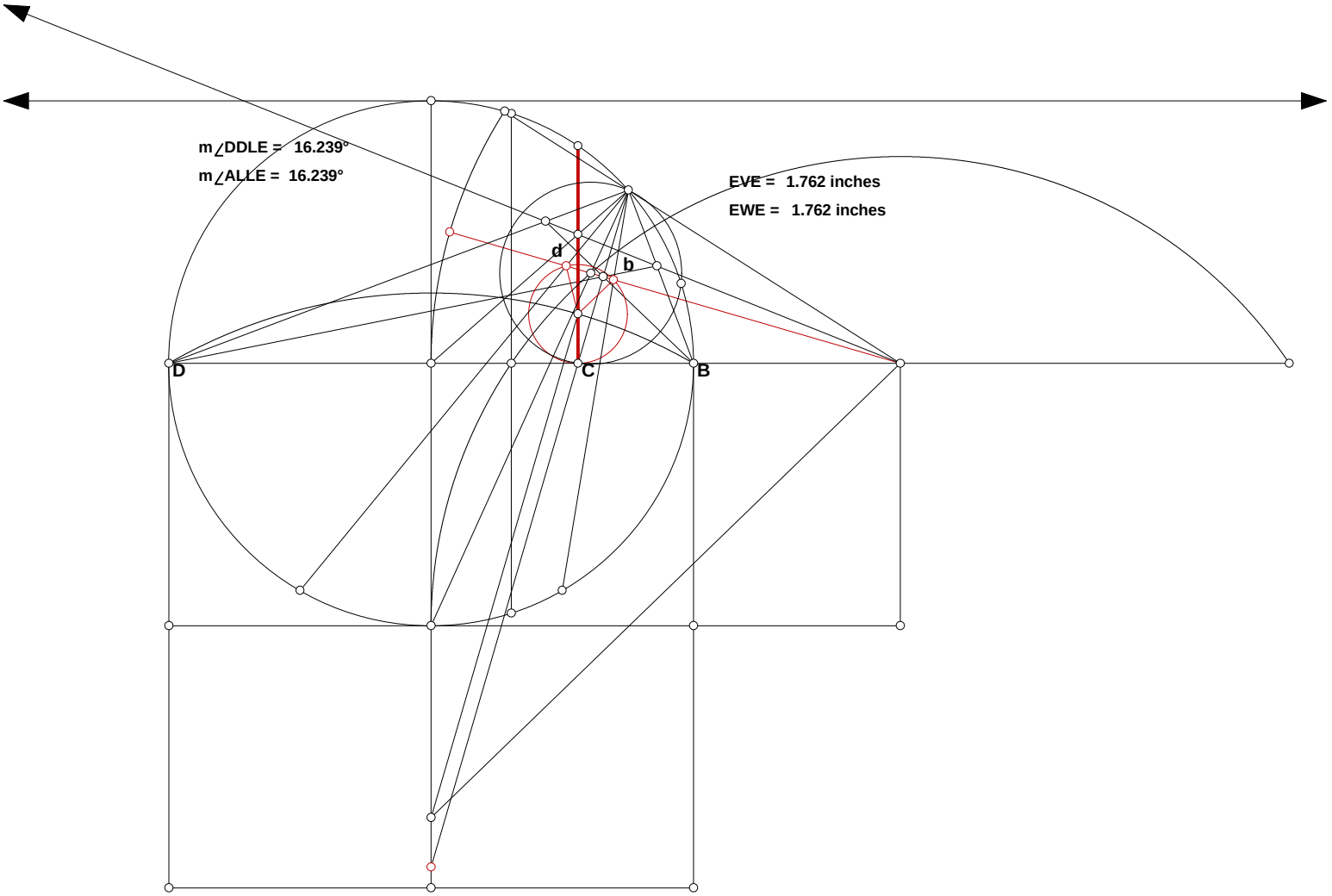

Move J->CY
CX
$\frac{m\angle JAC}{3}$ = 10.000°
m∠JAC = 30.000°
m∠CXAC = 10.000°
$\frac{m\angle JAC}{3}$ - m∠CXAC = 0.000°
$\left(\frac{m\angle JAC}{3} - m\angle CXAC\right)\cdot 1000$ = 0.006°
m∠DCAC = 40.000°
m∠DTADN = 40.000°
$(10 - m\angle CXAC)\cdot 1000000$ = 5.498

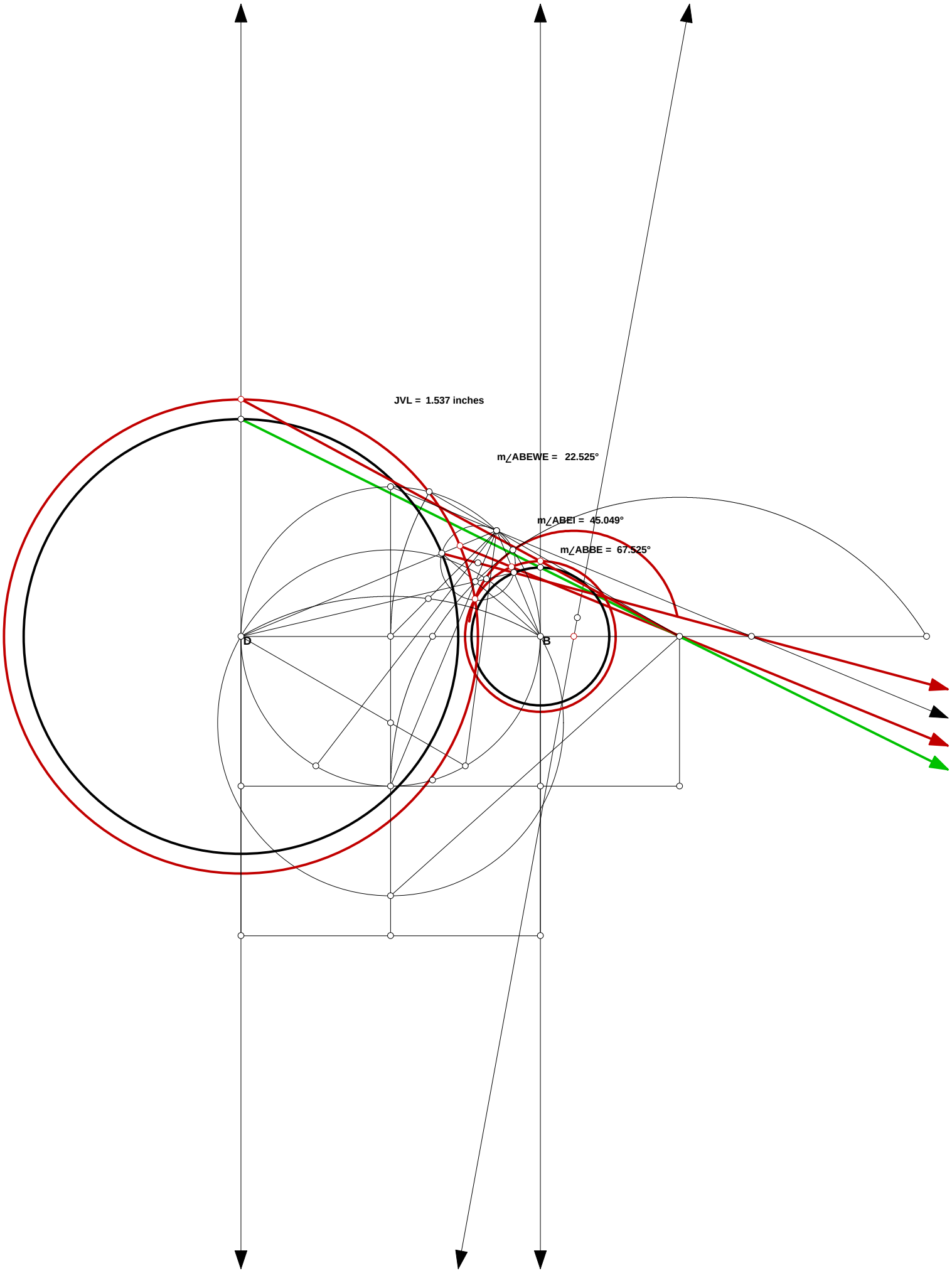

BP
BL
CA
m∠JAC = 46.817°
m∠BPAC = 15.603°
m∠JAC / m∠BPAC = 3.001
m∠JAC / 3 - m∠BPAC = 0.003°
BZBL = 1.859 inches
CABL = 1.859 inches

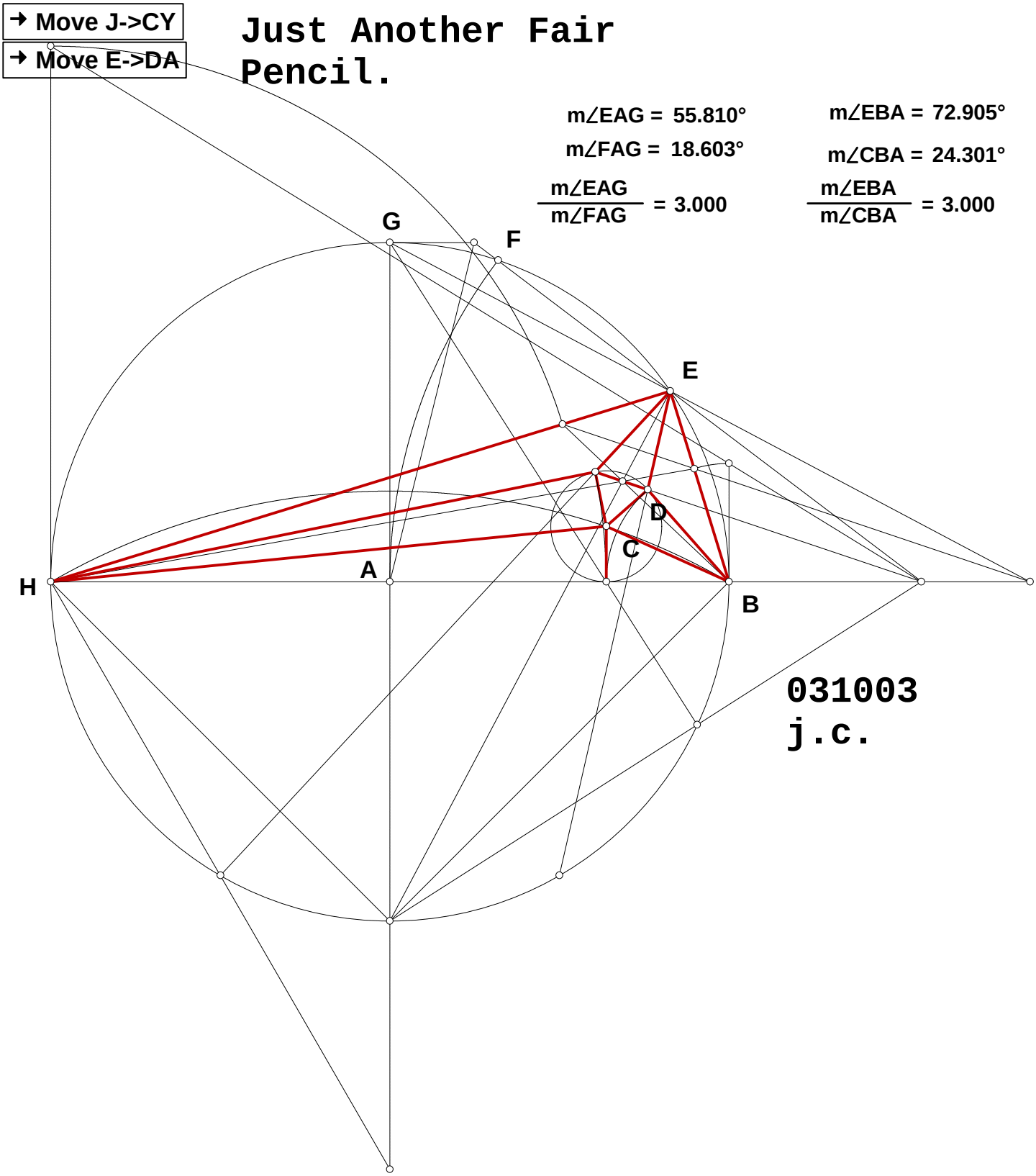

m∠ALLE = 16.654°
d
b
D
C
B

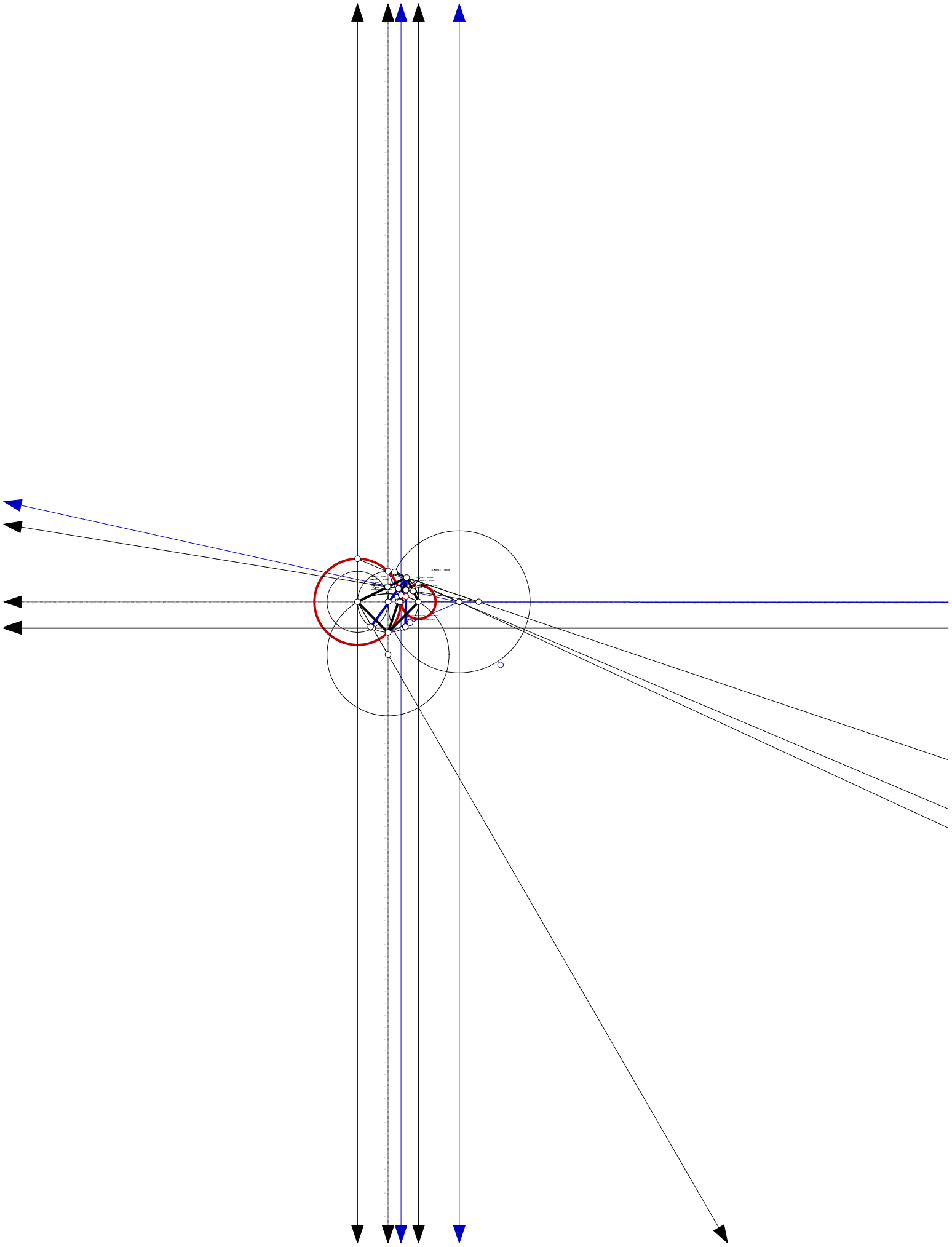

D
B

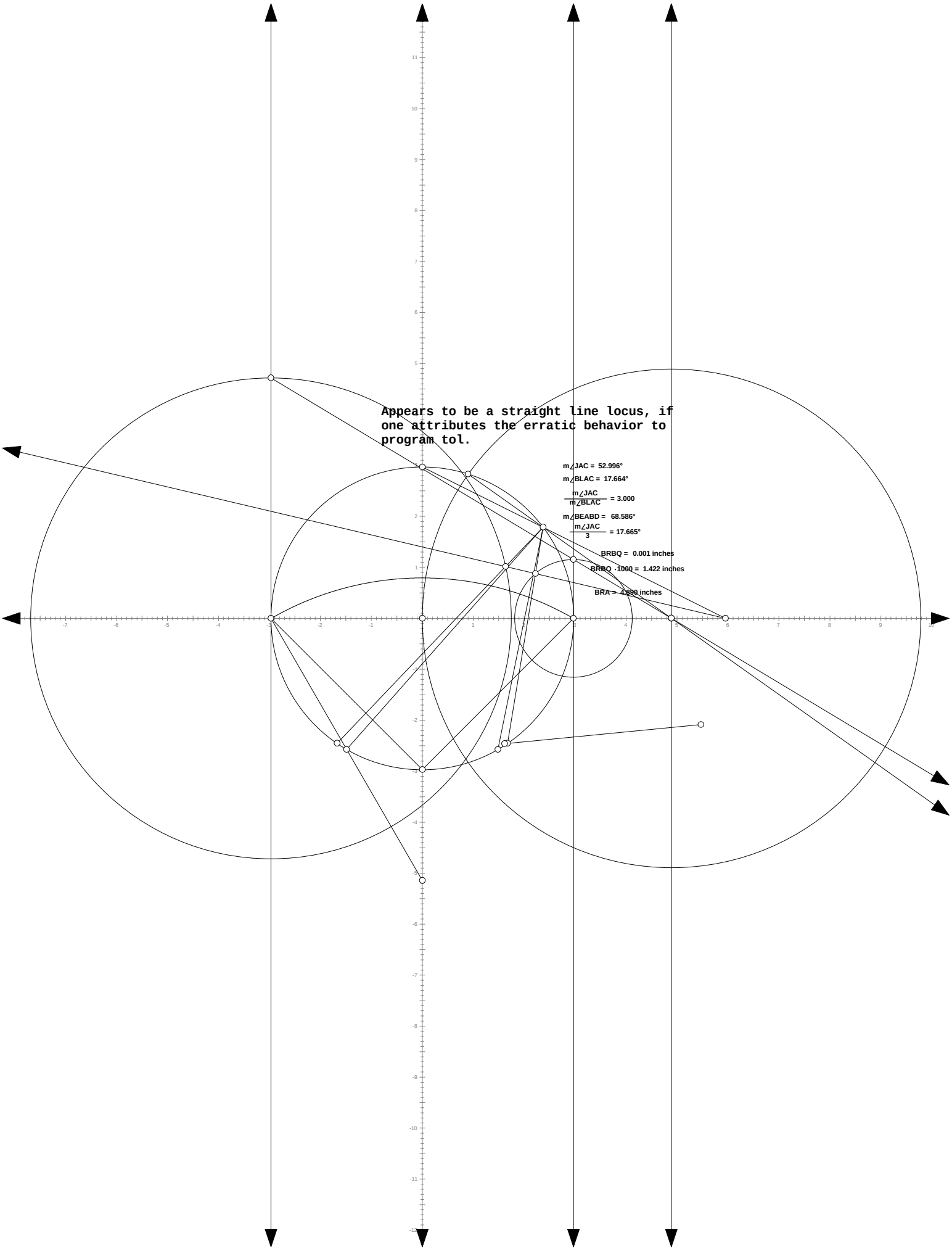

D
B

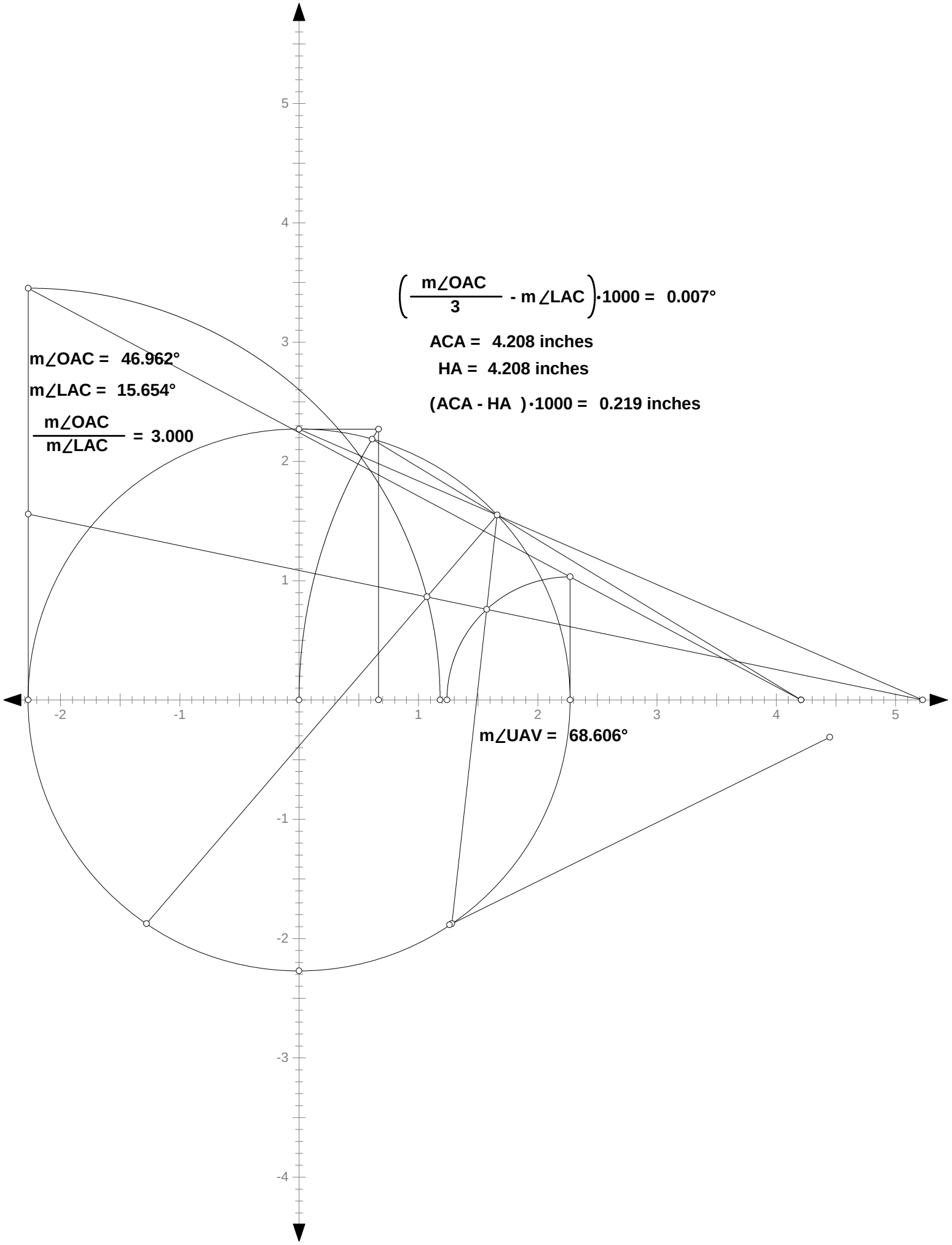

m∠DDLE = 16.239°
m∠ALLE = 16.239°
EVE = 1.762 inches
EWE = 1.762 inches
d
b
D
C
B

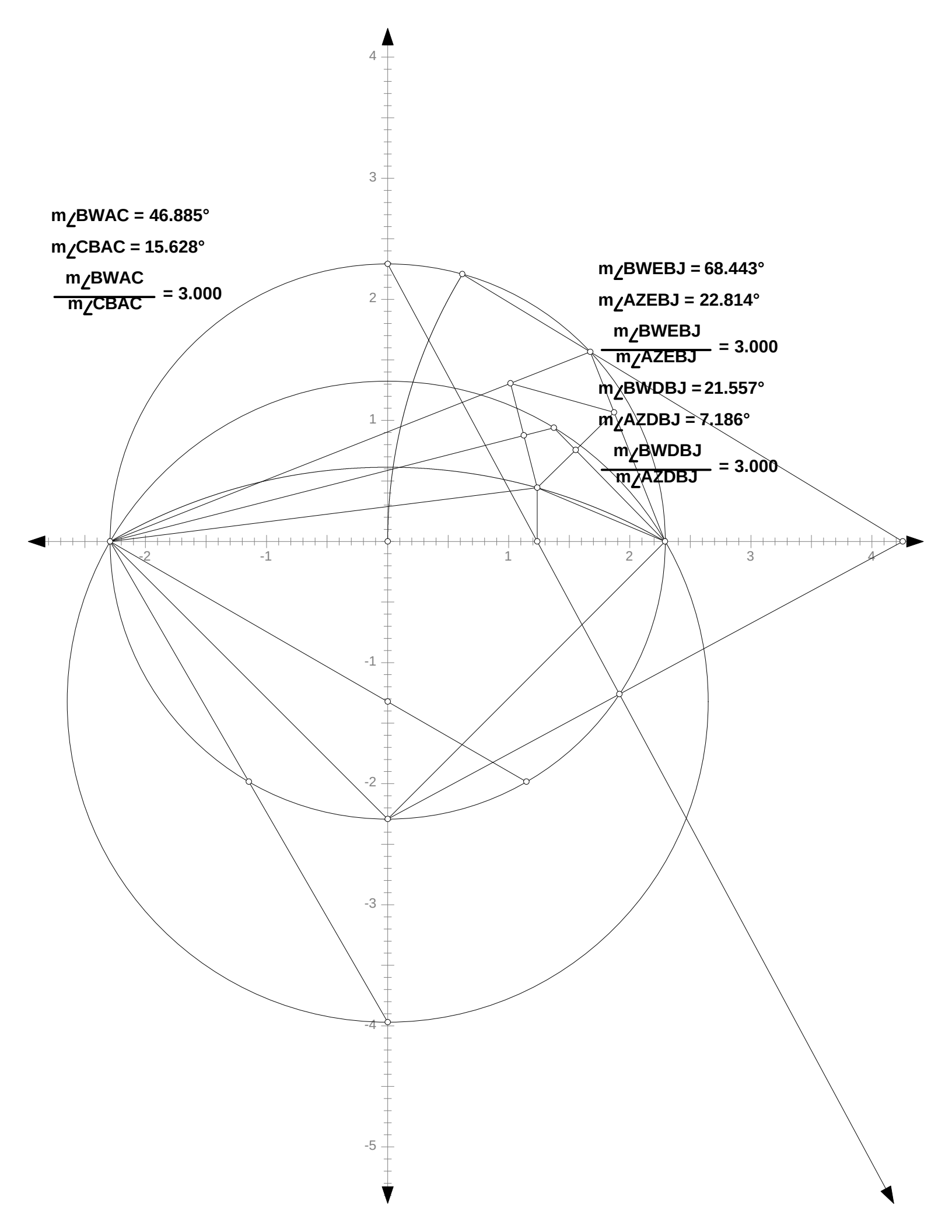

JVL = 1.537 inches
m∠ABEWE = 22.525°
m∠ABEI = 45.049°
m∠ABBE = 67.525°
D
B

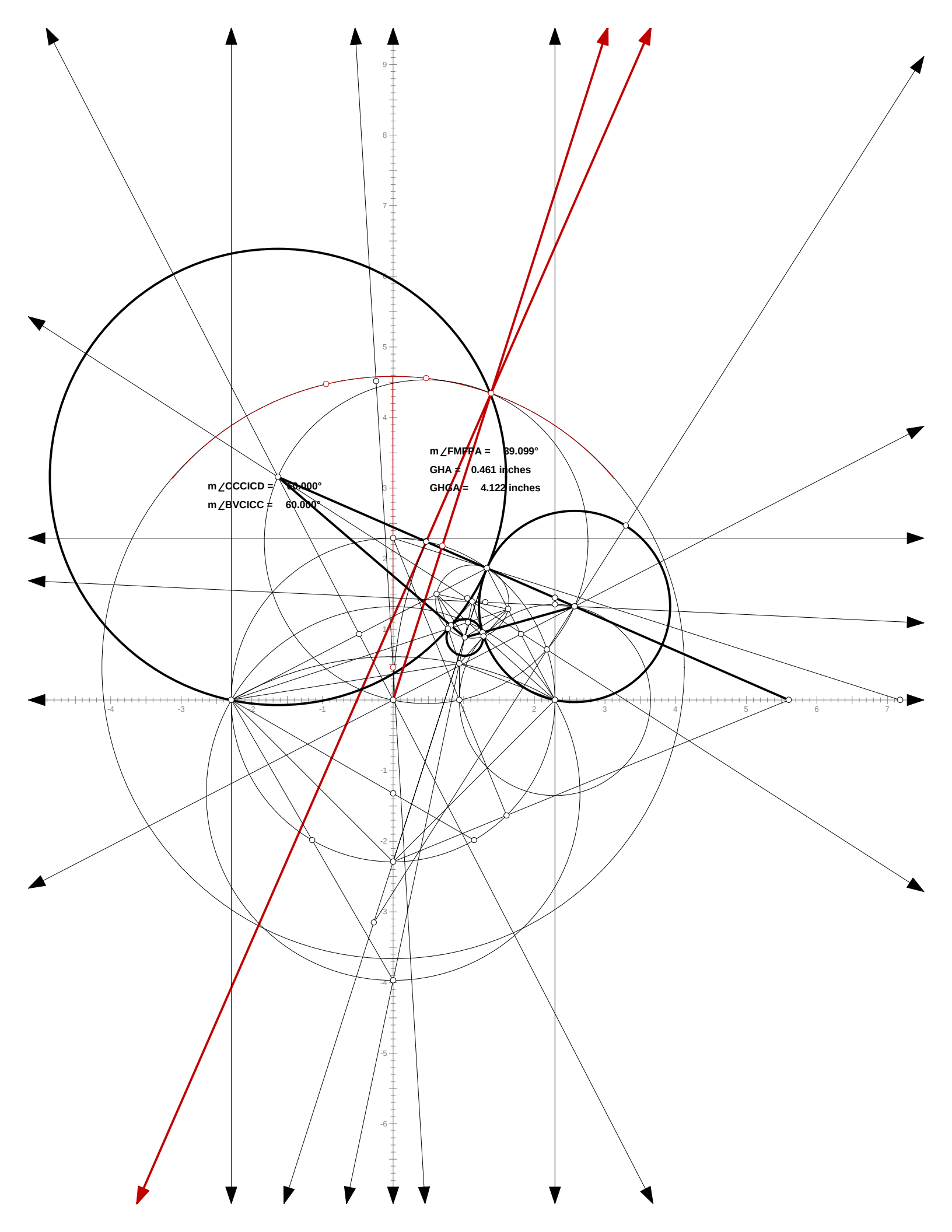

Move J->CY
Move E->DA
Just Another Fair Pencil.
m∠EAG = 55.810°
m∠FAG = 18.603°
m∠EAG / m∠FAG = 3.000
m∠EBA = 72.905°
m∠CBA = 24.301°
m∠EBA / m∠CBA = 3.000
G
F
E
D
C
A
H
B
031003
j.c.

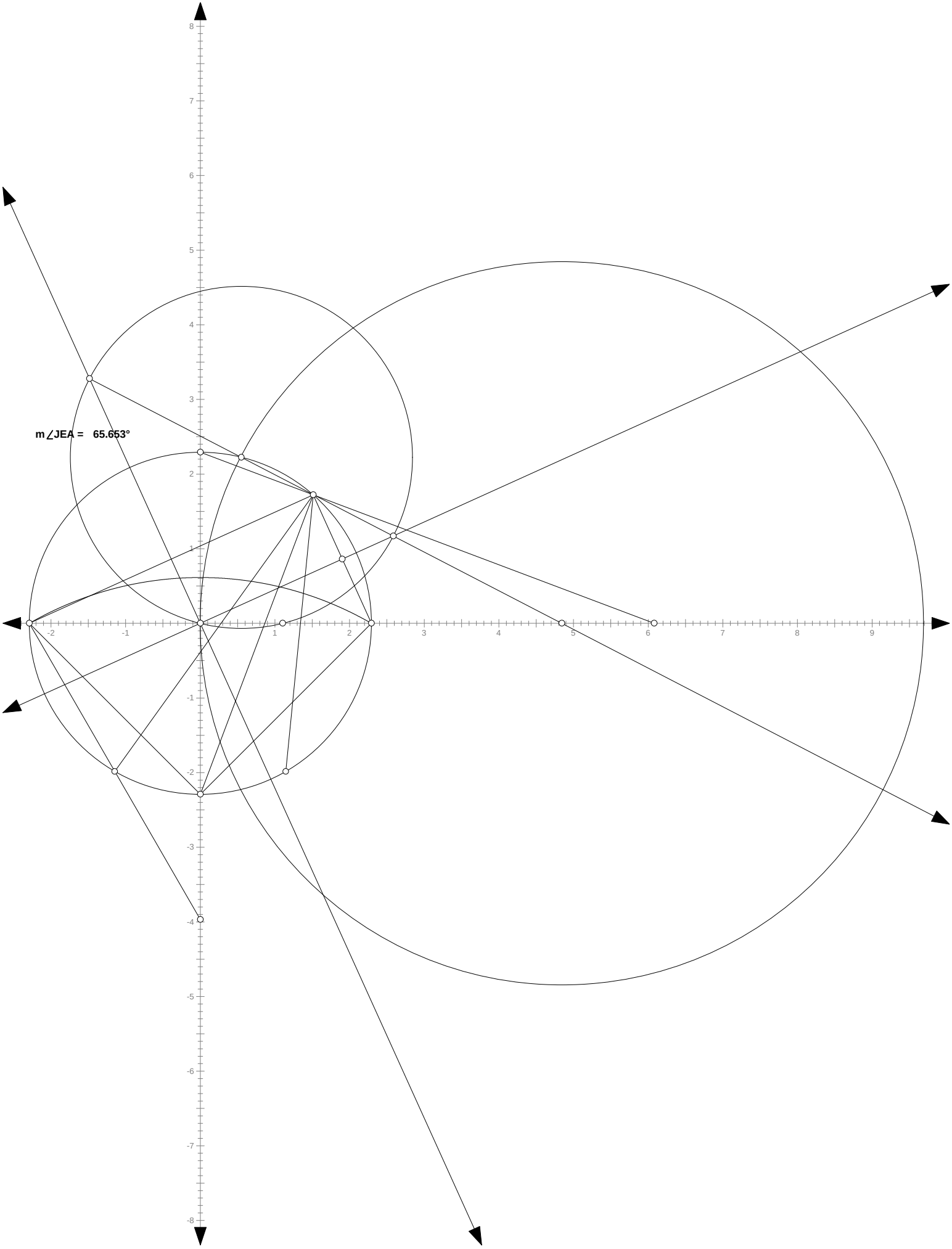


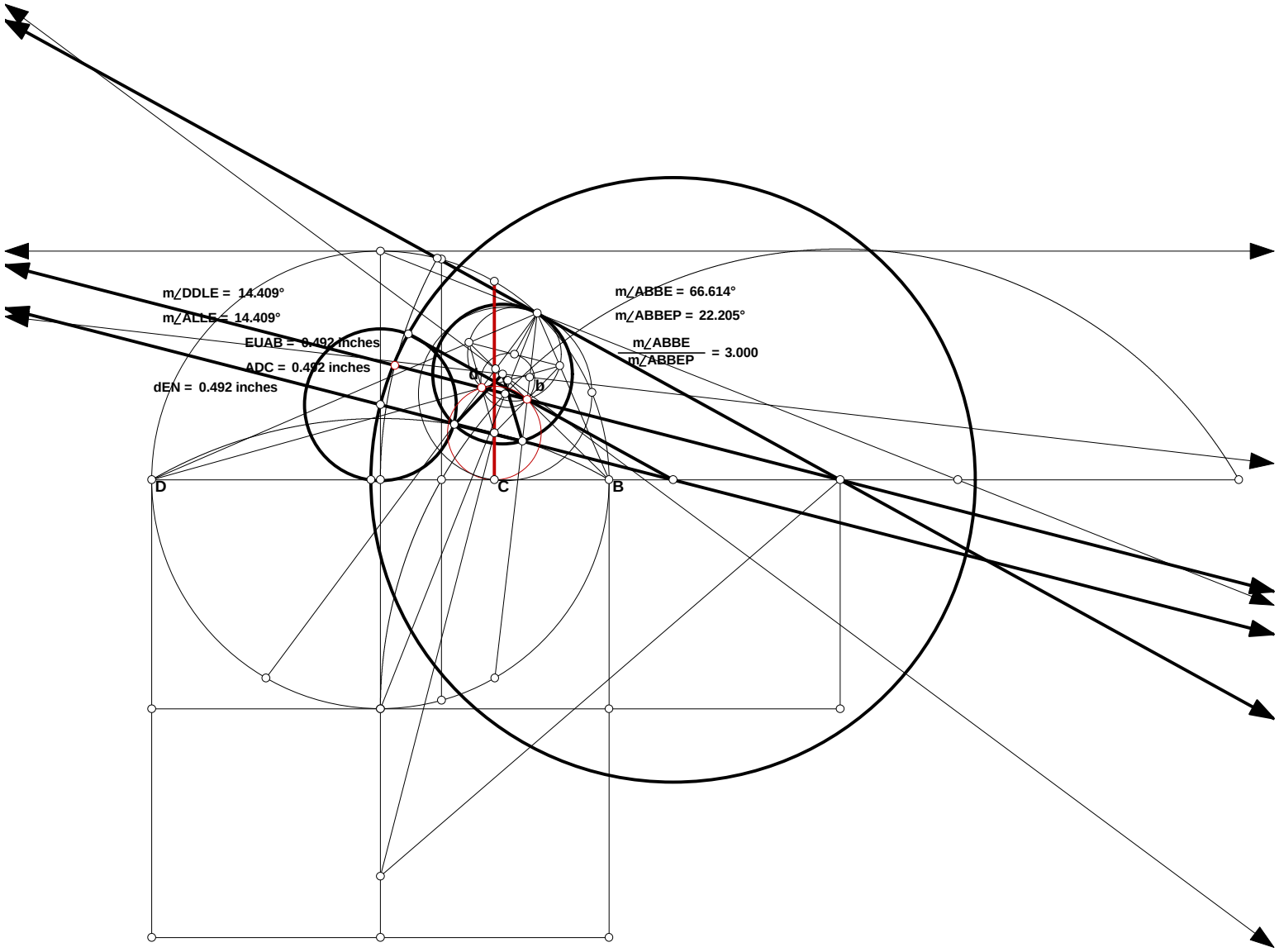

Appears to be a straight line locus, if one attributes the erratic behavior to program tol.
m∠JAC = 52.996°
m∠BLAC = 17.664°
m∠JAC / m∠BLAC = 3.000
m∠BEABD = 68.586°
m∠JAC / 3 = 17.665°
BRBQ = 0.001 inches
BRBQ ·1000 = 1.422 inches
BRA = 4.890 inches

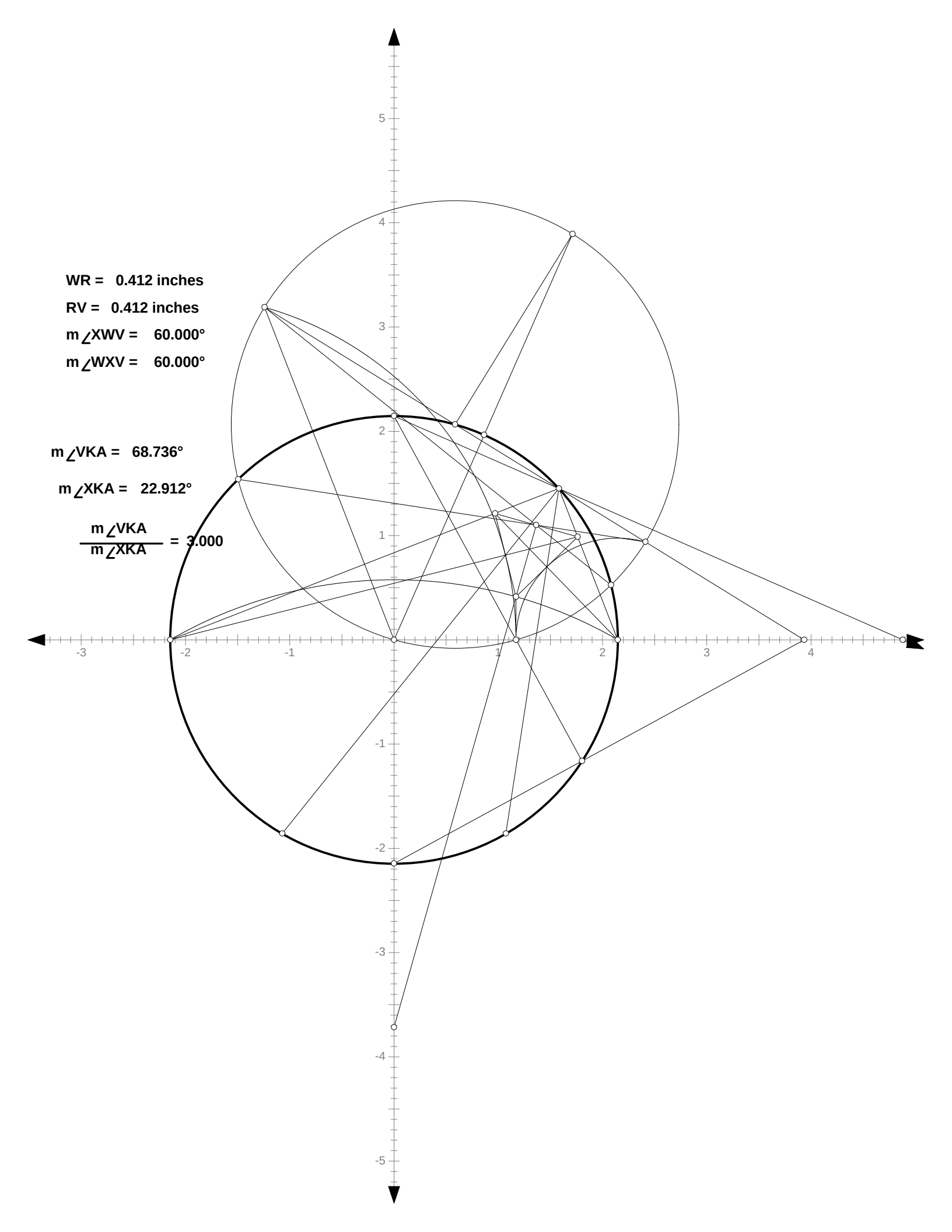

$\left(\frac{m\angle OAC}{3} - m\angle LAC\right)\cdot 1000 = 0.007°$
ACA = 4.208 inches
HA = 4.208 inches
(ACA - HA)·1000 = 0.219 inches
m∠OAC = 46.962°
m∠LAC = 15.654°
$\frac{m\angle OAC}{m\angle LAC} = 3.000$
m∠UAV = 68.606°

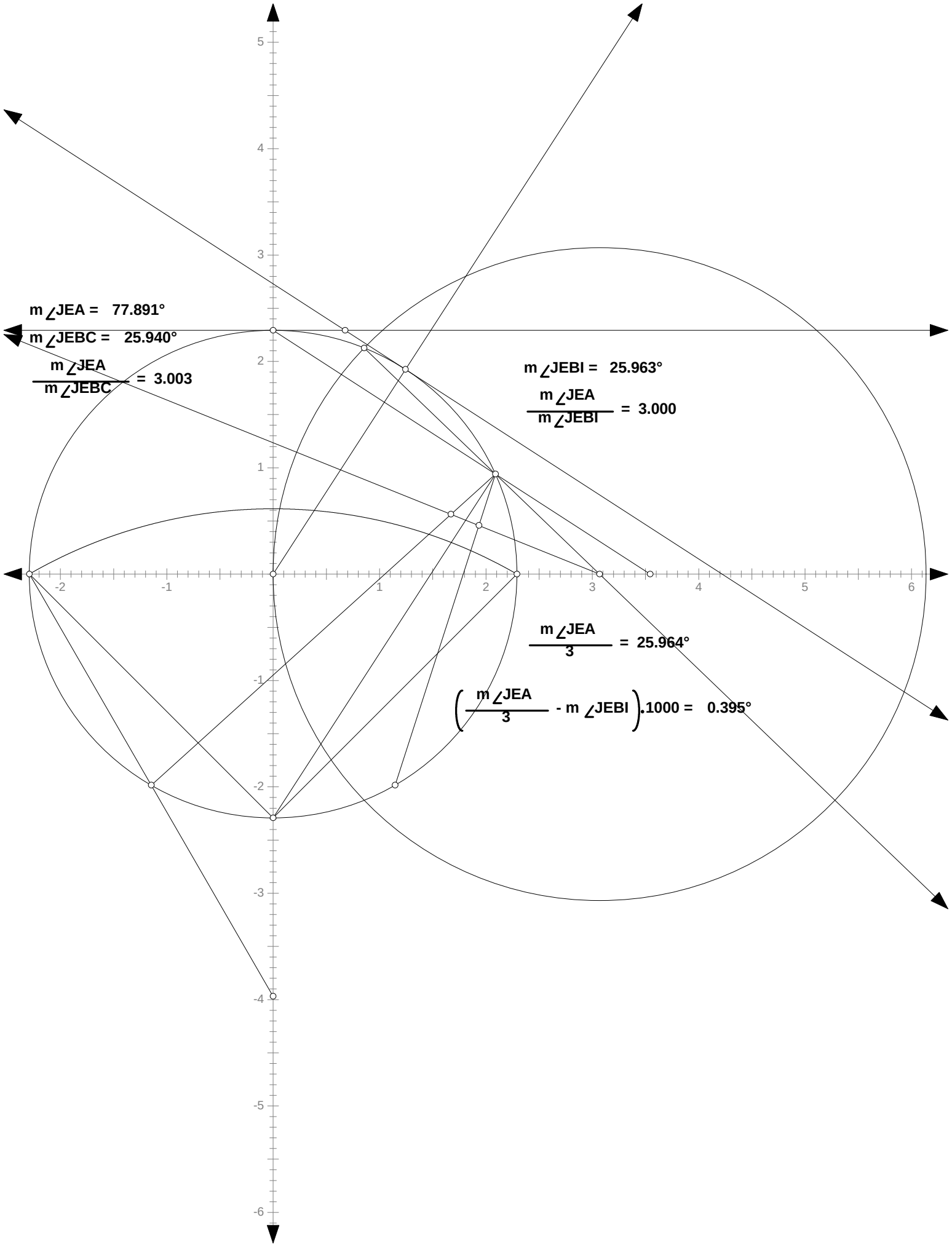

m∠BWAC = 46.885°
m∠CBAC = 15.628°
m∠BWAC / m∠CBAC = 3.000
m∠BWEBJ = 68.443°
m∠AZEBJ = 22.814°
m∠BWEBJ / m∠AZEBJ = 3.000
m∠BWDBJ = 21.557°
m∠AZDBJ = 7.186°
m∠BWDBJ / m∠AZDBJ = 3.000
4
3
2
1
-1
-2
-3
-4
-5
-2
-1
1
2
3
4

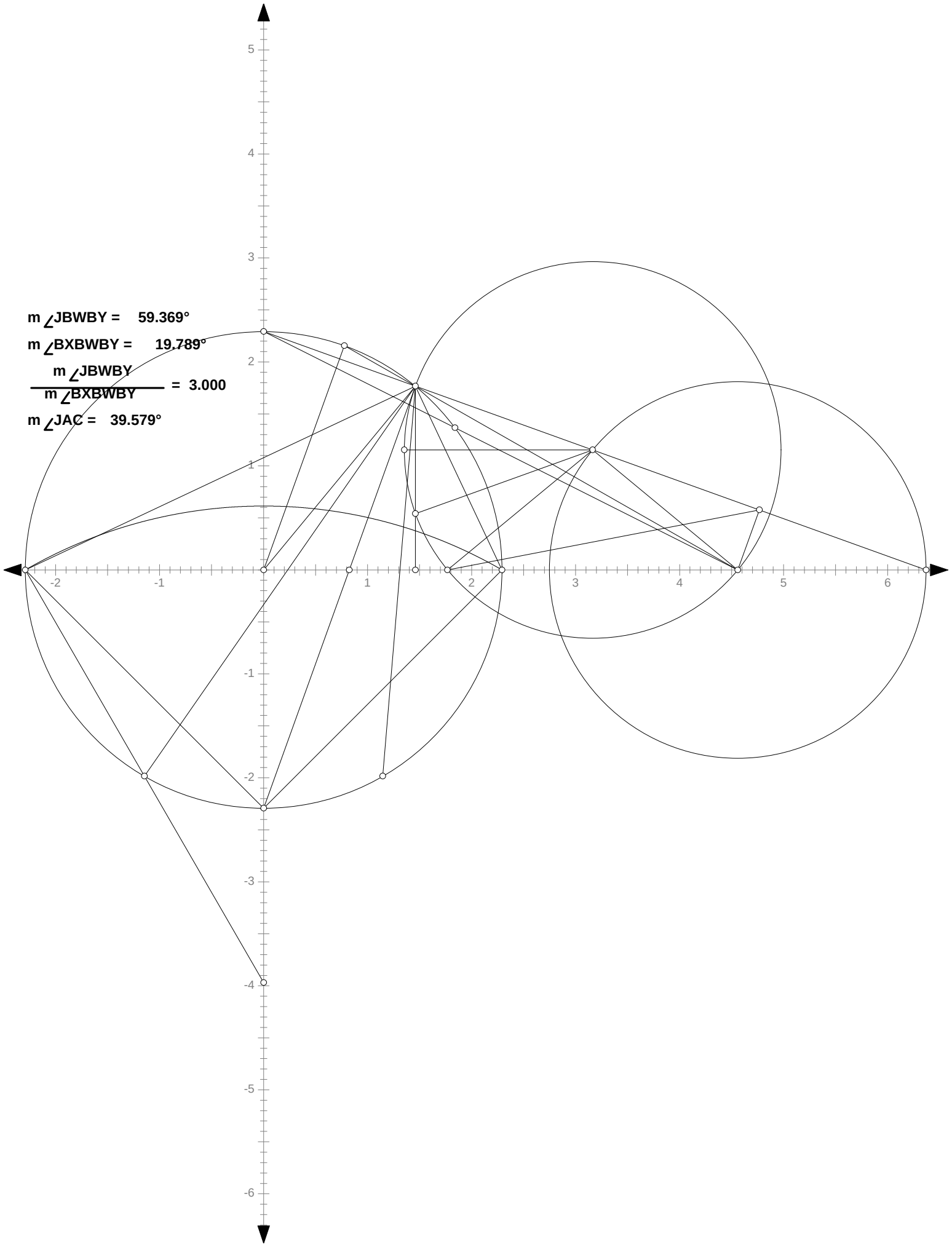

m∠FMPPA = 39.099°
GHA = 0.461 inches
GHGA = 4.122 inches
m∠CCCICD = 60.000°
m∠BVCICC = 60.000°

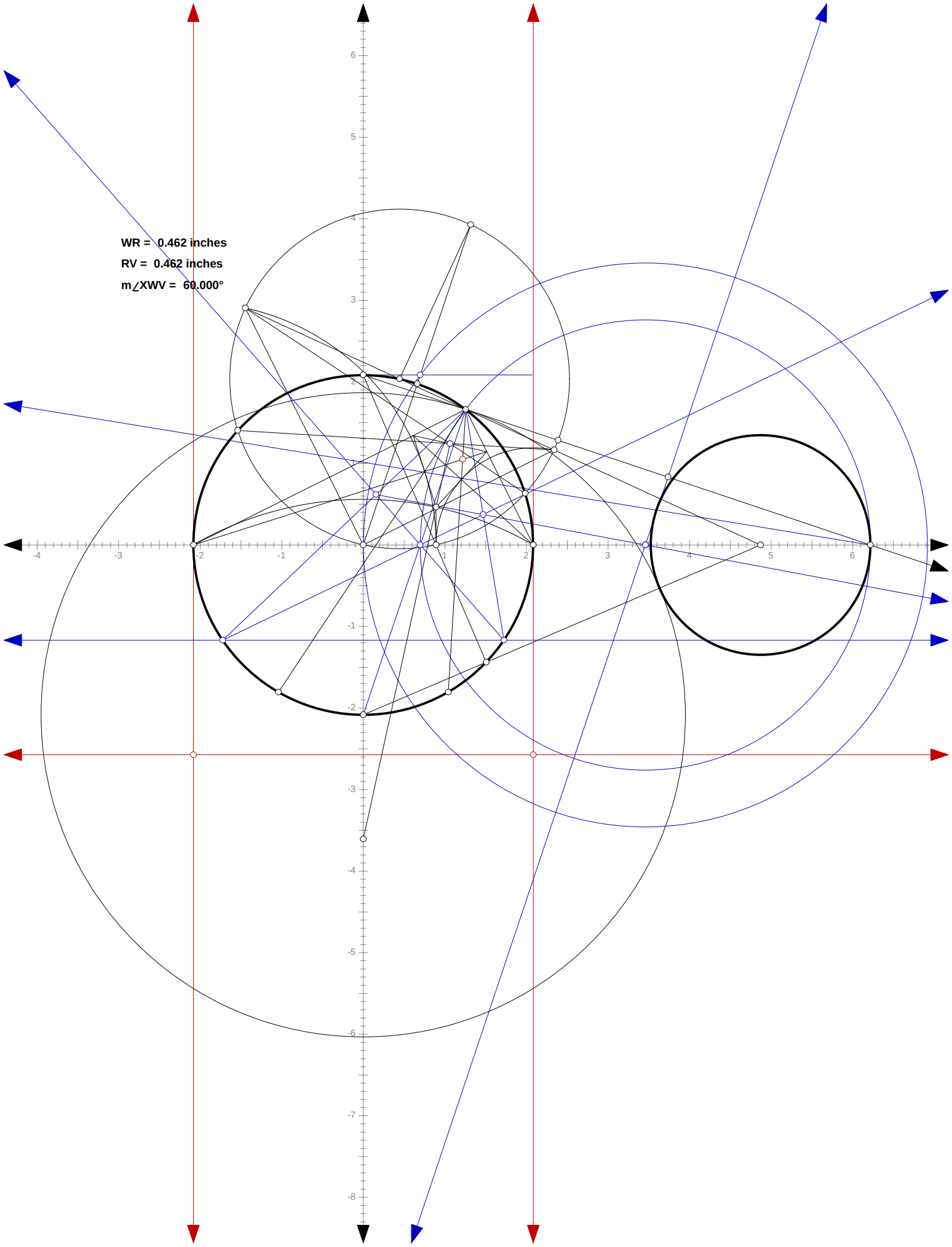

m∠JEA = 65.653°

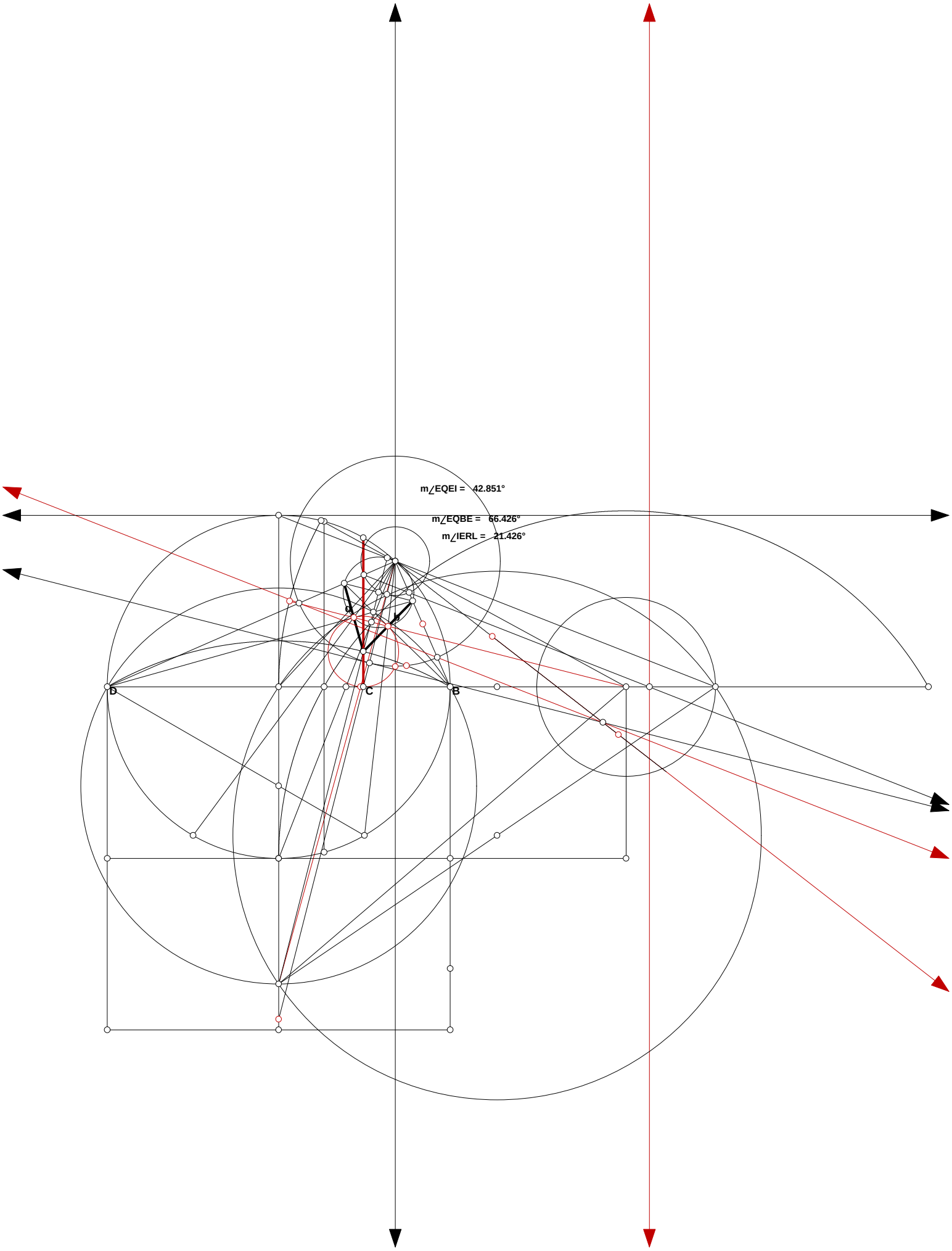

m∠DDLE = 14.409°
m∠ALLE = 14.409°
EUAB = 0.492 inches
ADC = 0.492 inches
dEN = 0.492 inches
m∠ABBE = 66.614°
m∠ABBEP = 22.205°
m∠ABBE / m∠ABBEP = 3.000
D
C
B
a
b

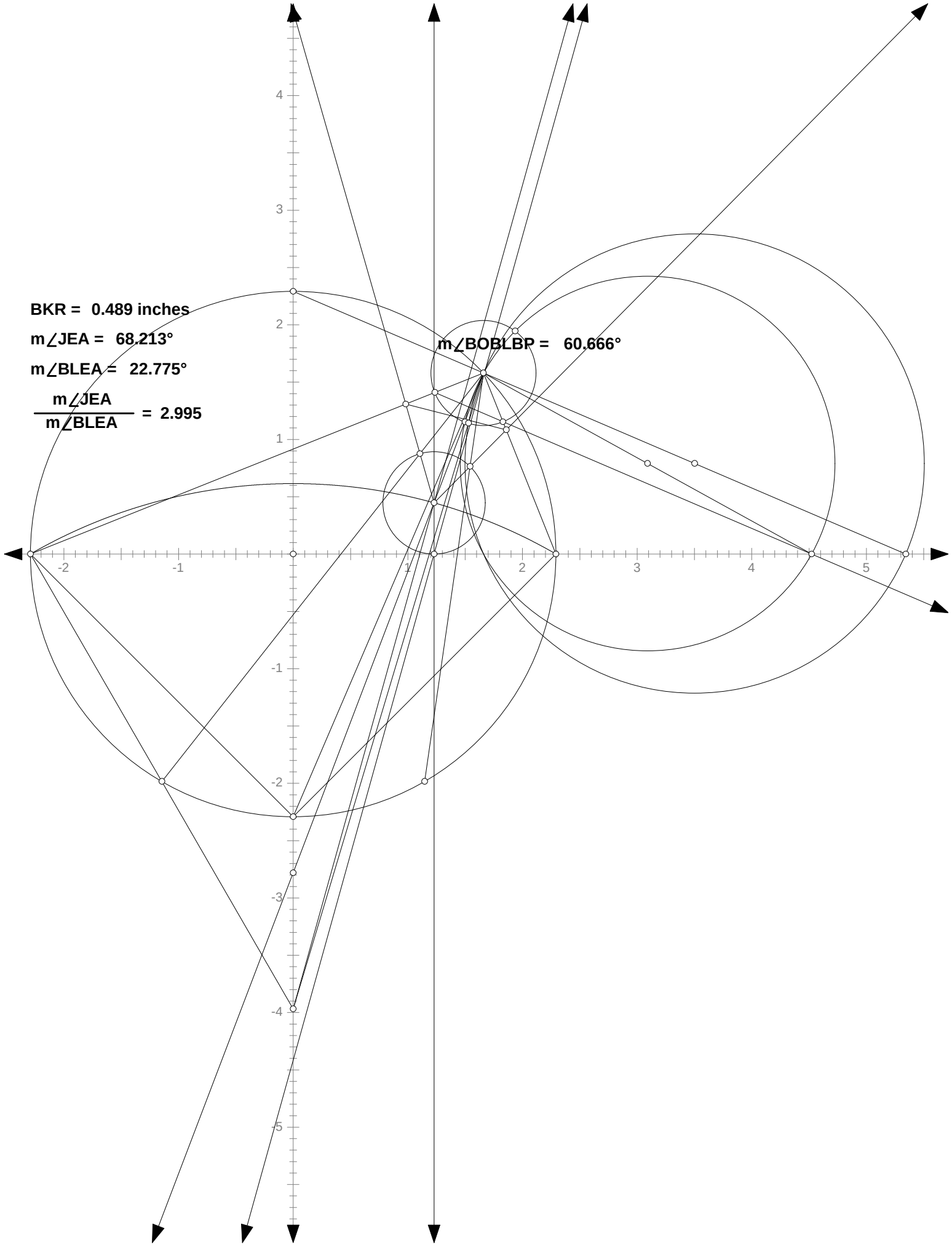

WR = 0.412 inches
RV = 0.412 inches
m∠XWV = 60.000°
m∠WXV = 60.000°
m∠VKA = 68.736°
m∠XKA = 22.912°
m∠VKA / m∠XKA = 3.000
5
4
3
2
1
-1
-2
-3
-4
-5
-3
-2
-1
1
2
3
4

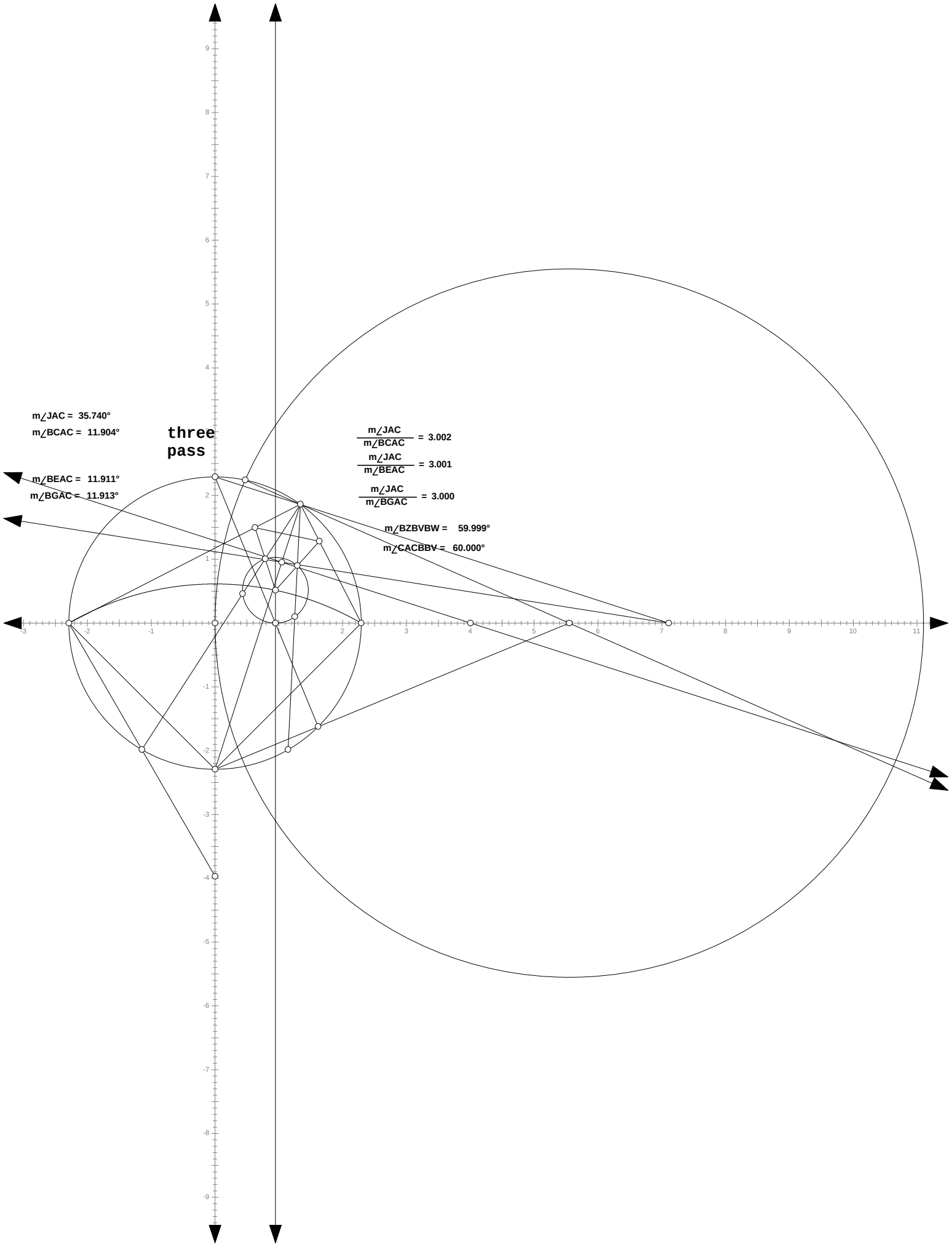

m∠JEA = 77.891°
m∠JEBC = 25.940°
m∠JEA / m∠JEBC = 3.003
m∠JEBI = 25.963°
m∠JEA / m∠JEBI = 3.000
m∠JEA / 3 = 25.964°
(m∠JEA / 3 - m∠JEBI).1000 = 0.395°

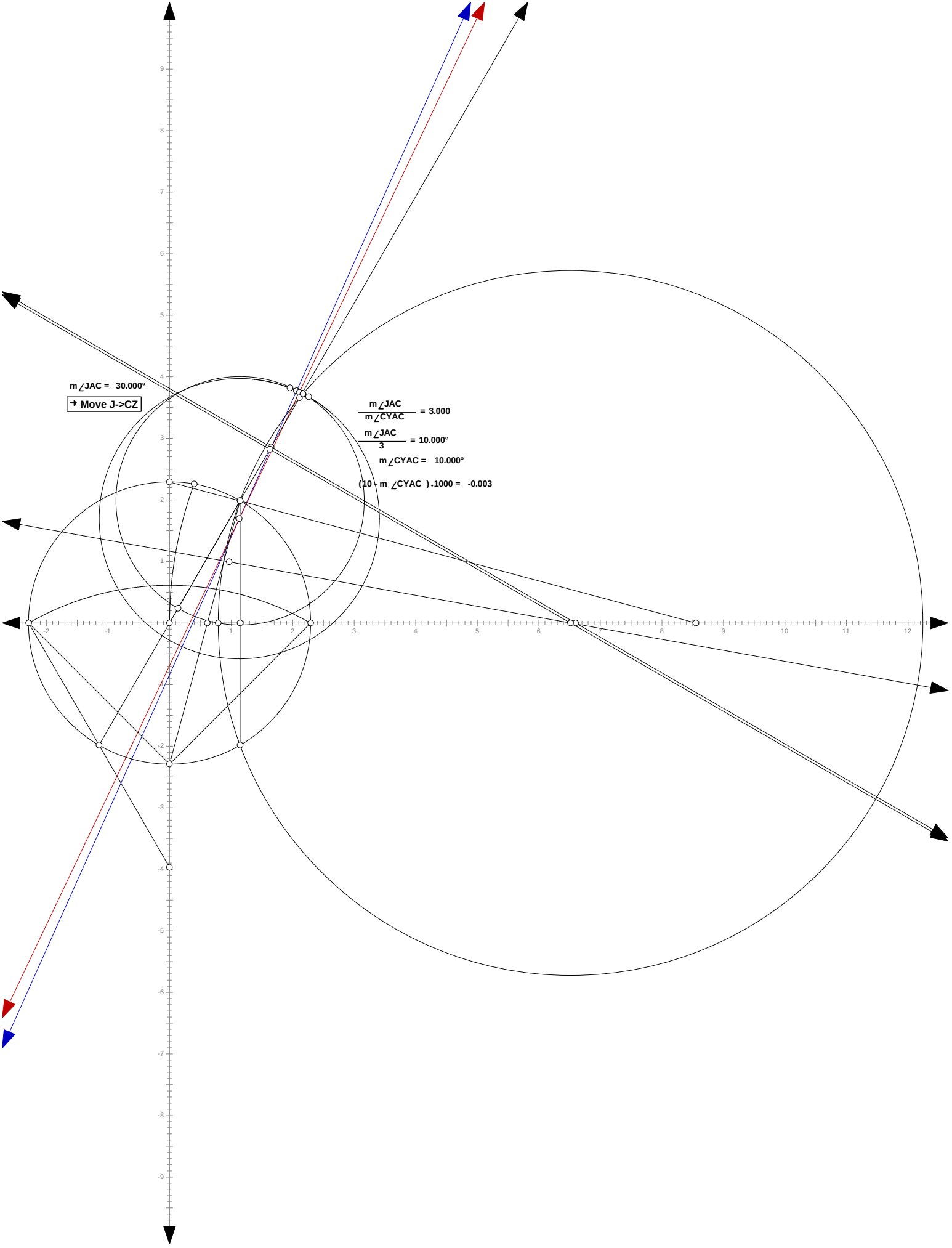

m ∠JBWBY = 59.369°
m ∠BXBWBY = 19.789°
m ∠JBWBY / m ∠BXBWBY = 3.000
m ∠JAC = 39.579°
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-2
-1
1
2
3
4
5
6

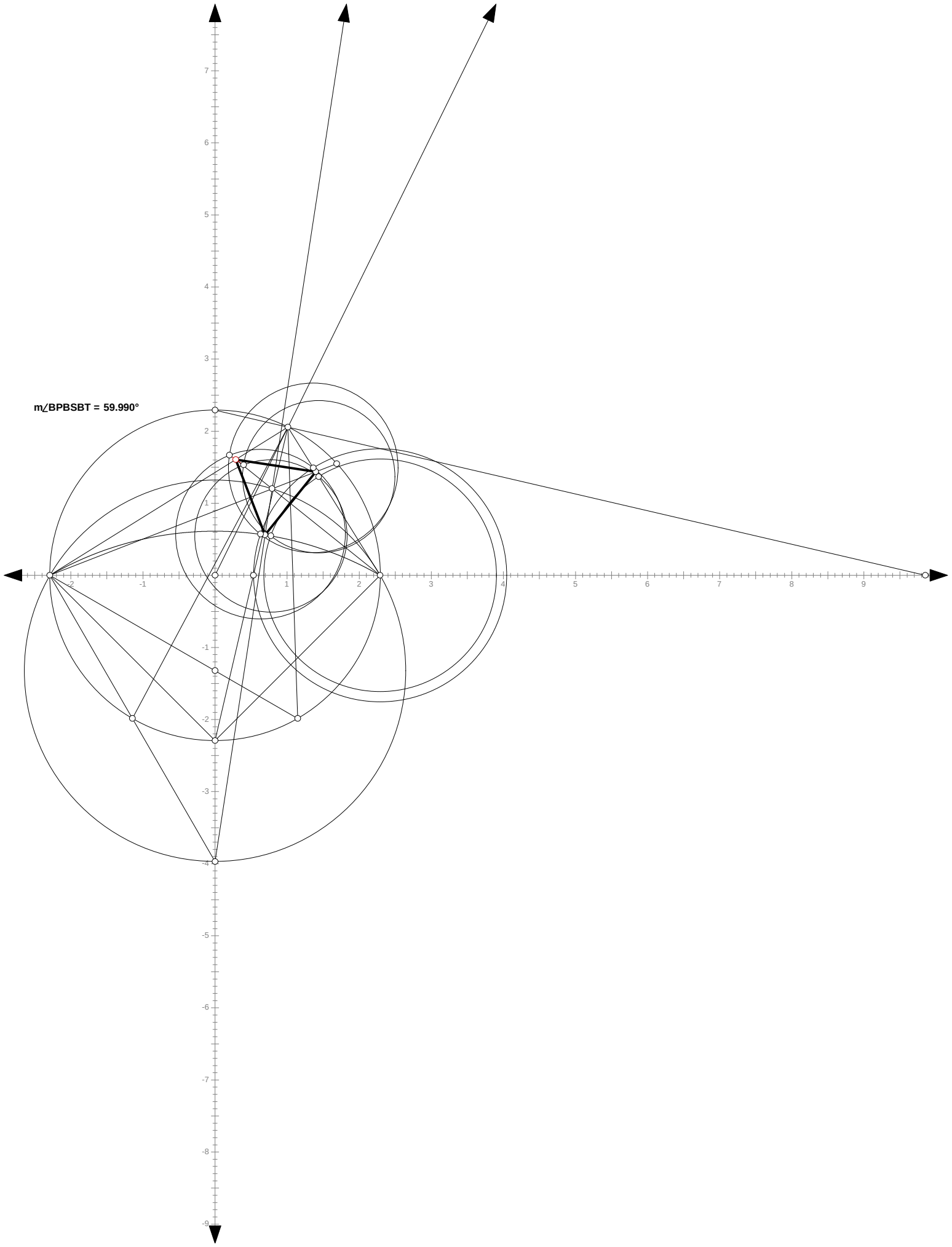

WR = 0.462 inches
RV = 0.462 inches
m∠XWV = 60.000°

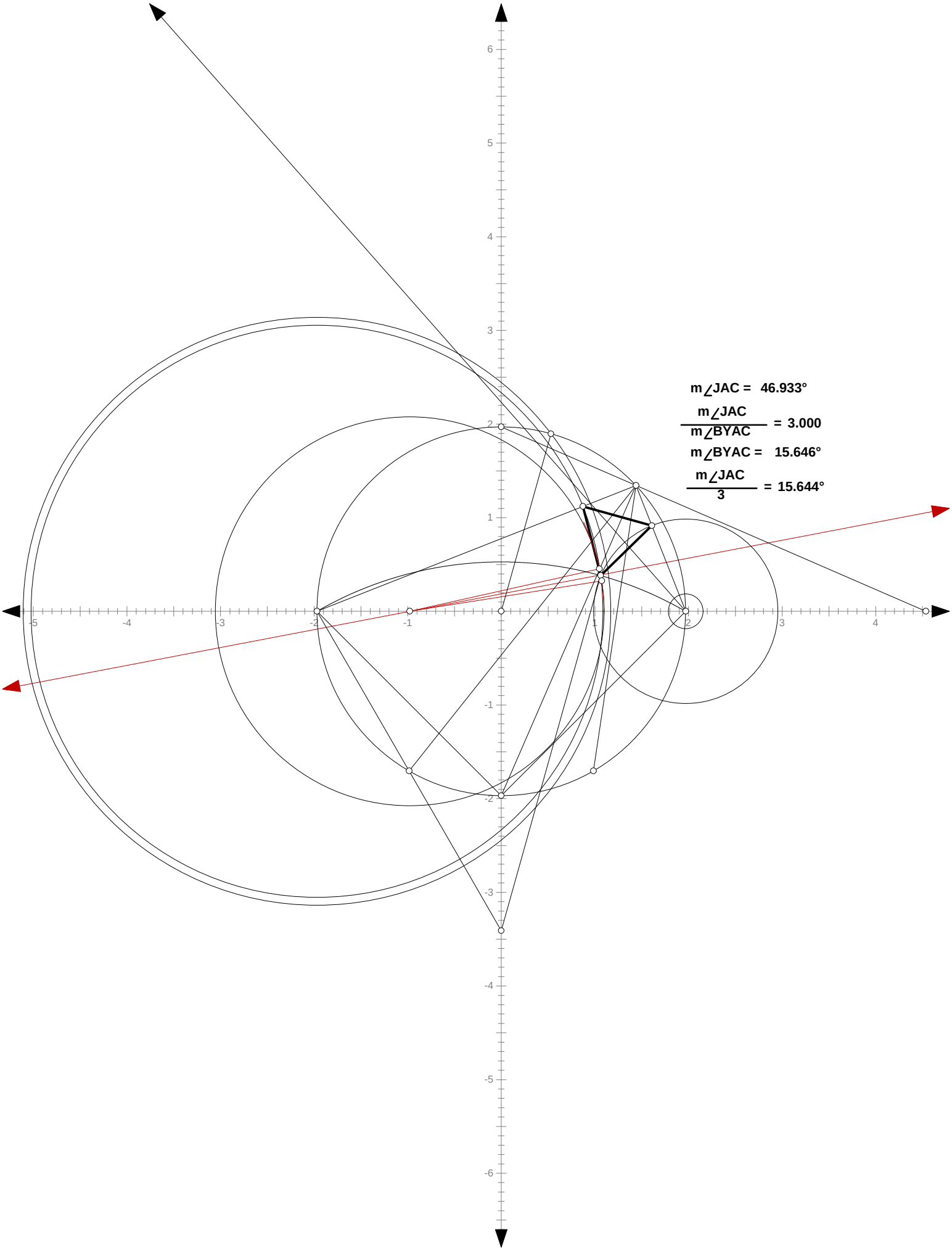

m∠EQEI = 42.851°
m∠EQBE = 66.426°
m∠IERL = 21.426°
D
C
B

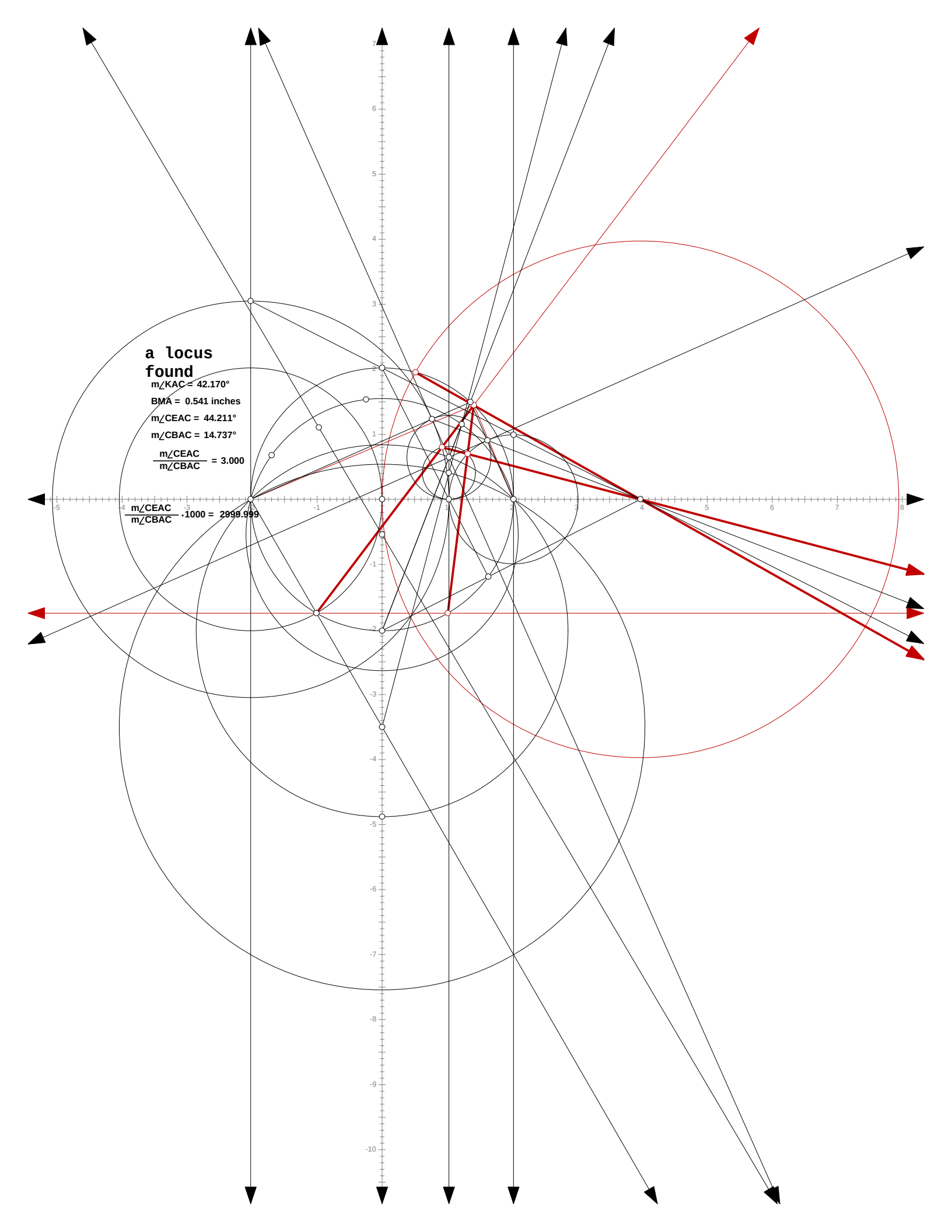

BKR = 0.489 inches
m∠JEA = 68.213°
m∠BLEA = 22.775°
m∠JEA / m∠BLEA = 2.995
m∠BOBLBP = 60.666°
-2
-1
1
2
3
4
5
4
3
2
1
-1
-2
-3
-4
-5

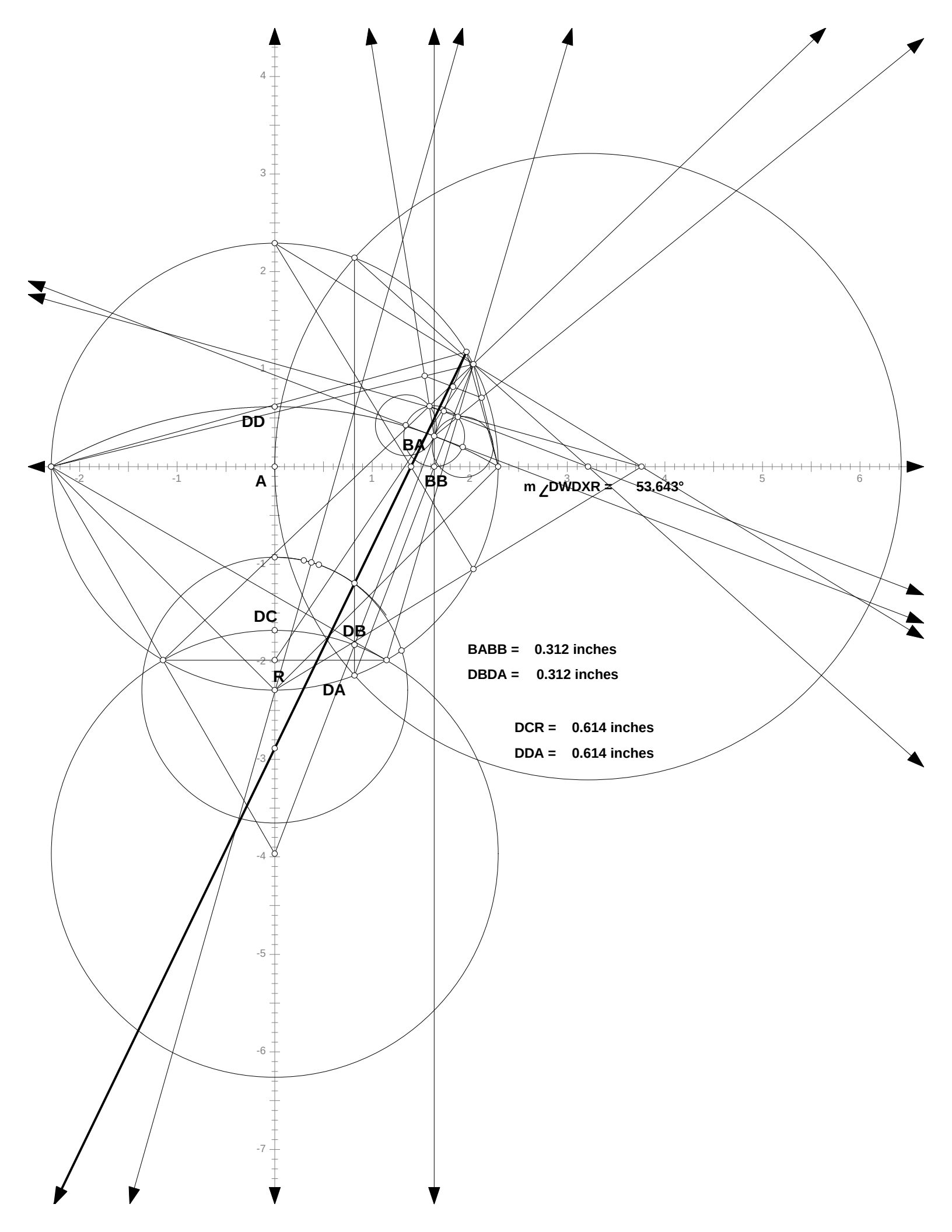

m∠JAC = 35.740°
m∠BCAC = 11.904°
three
pass
m∠BEAC = 11.911°
m∠BGAC = 11.913°
m∠JAC / m∠BCAC = 3.002
m∠JAC / m∠BEAC = 3.001
m∠JAC / m∠BGAC = 3.000
m∠BZBVBW = 59.999°
m∠CACBBV = 60.000°

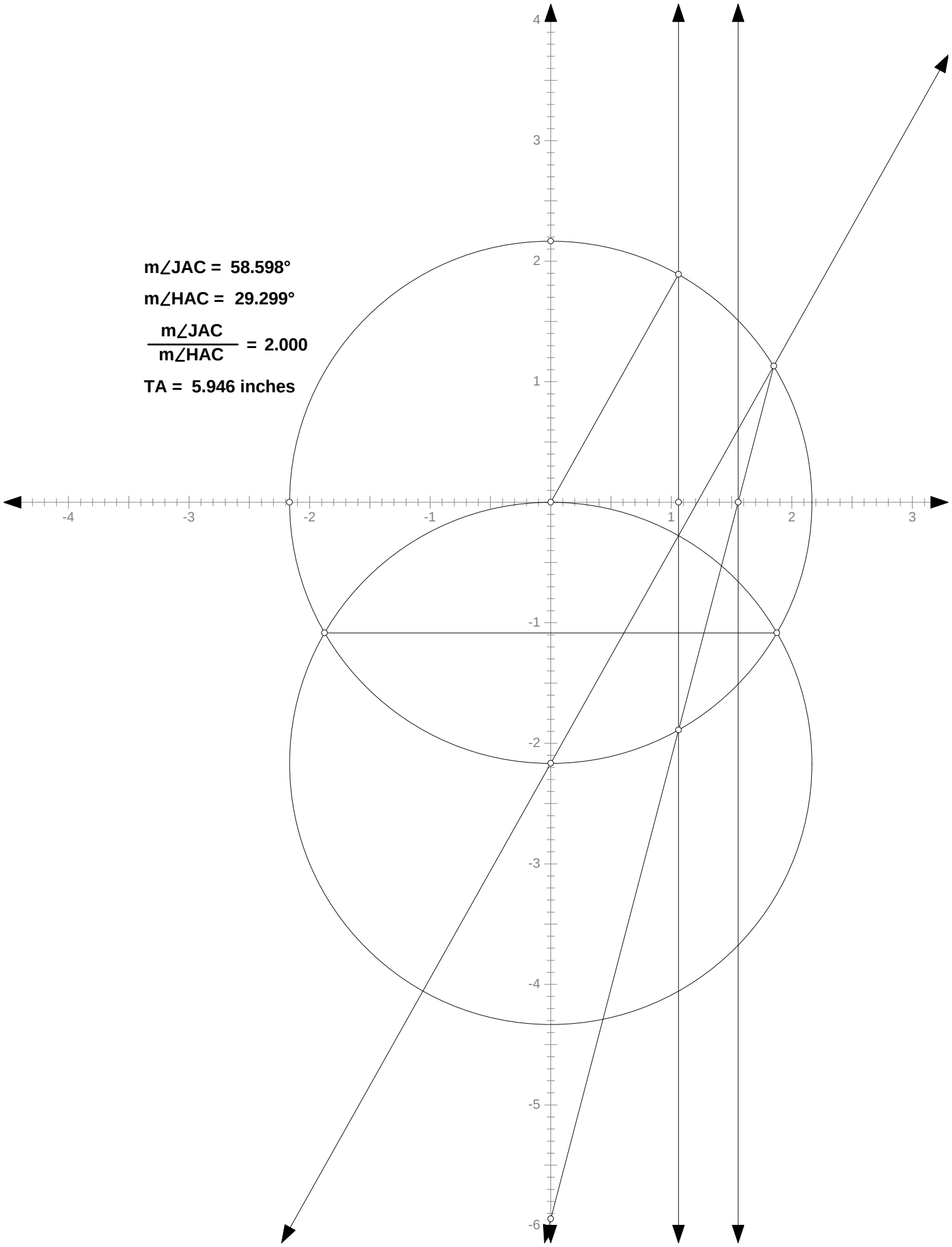

m∠JAC = 30.000°
Move J->CZ
m∠JAC / m∠CYAC = 3.000
m∠JAC / 3 = 10.000°
m∠CYAC = 10.000°
(10 - m∠CYAC).1000 = -0.003

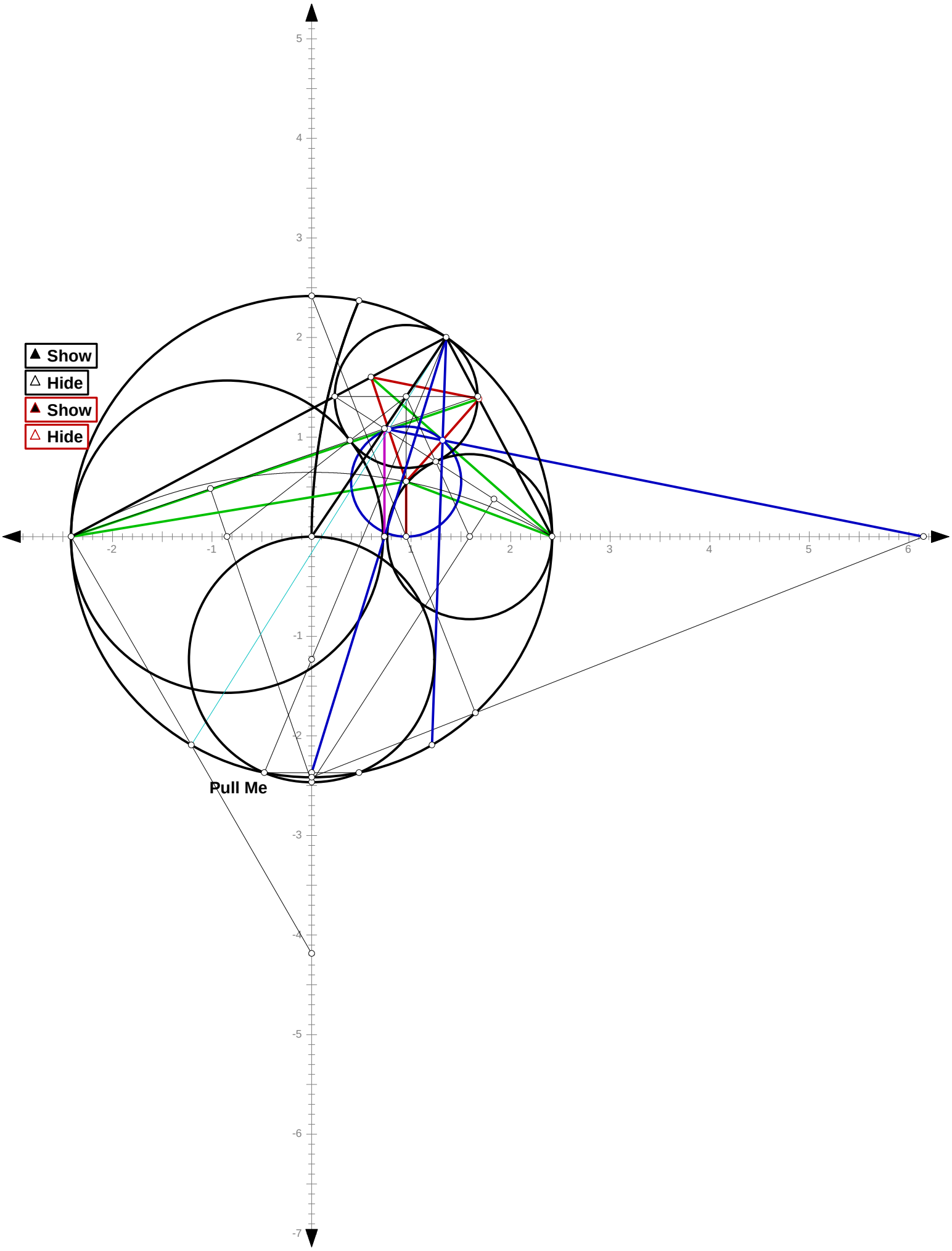

m∠BPBSBT = 59.990°

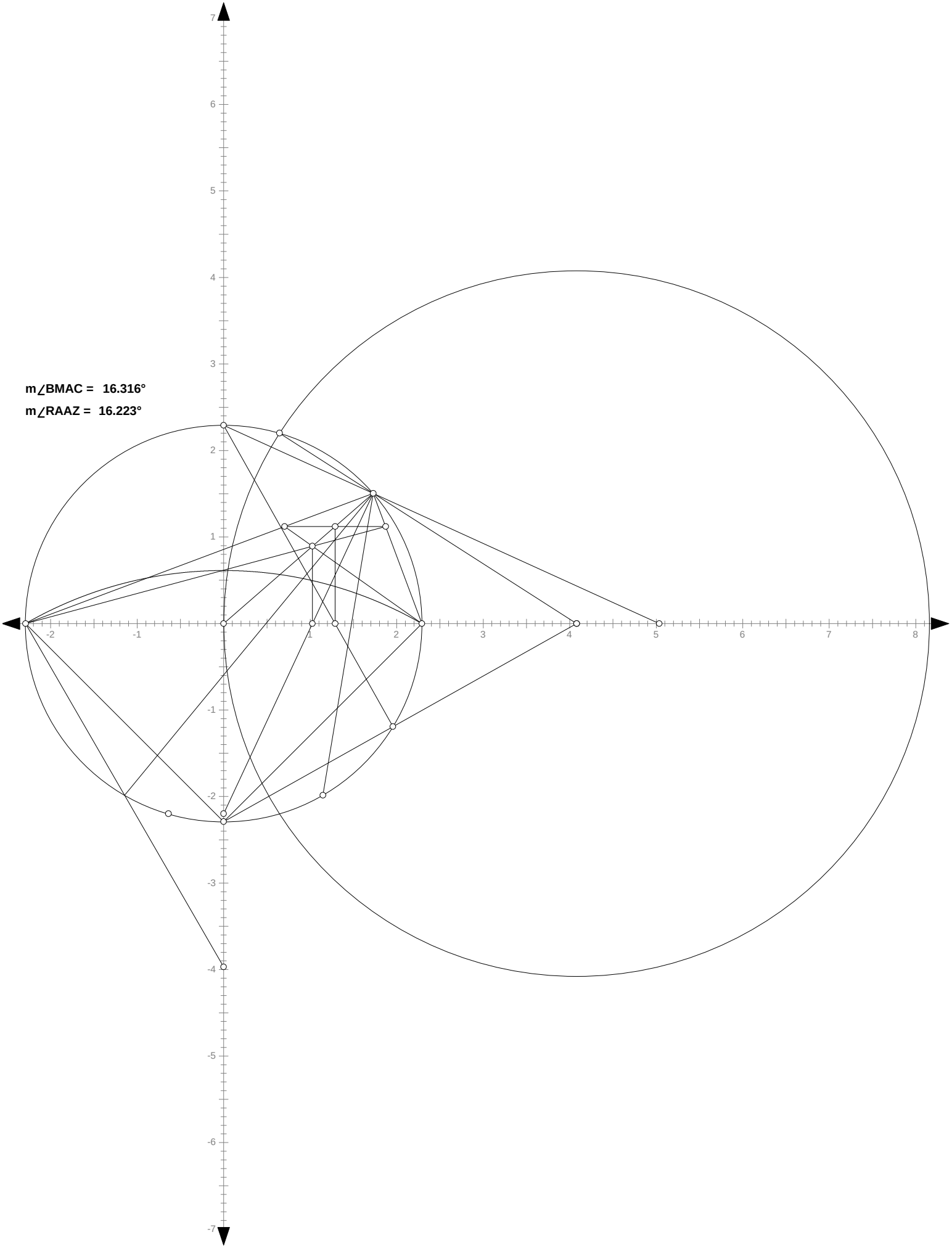

m∠JAC = 46.933°
m∠JAC / m∠BYAC = 3.000
m∠BYAC = 15.646°
m∠JAC / 3 = 15.644°

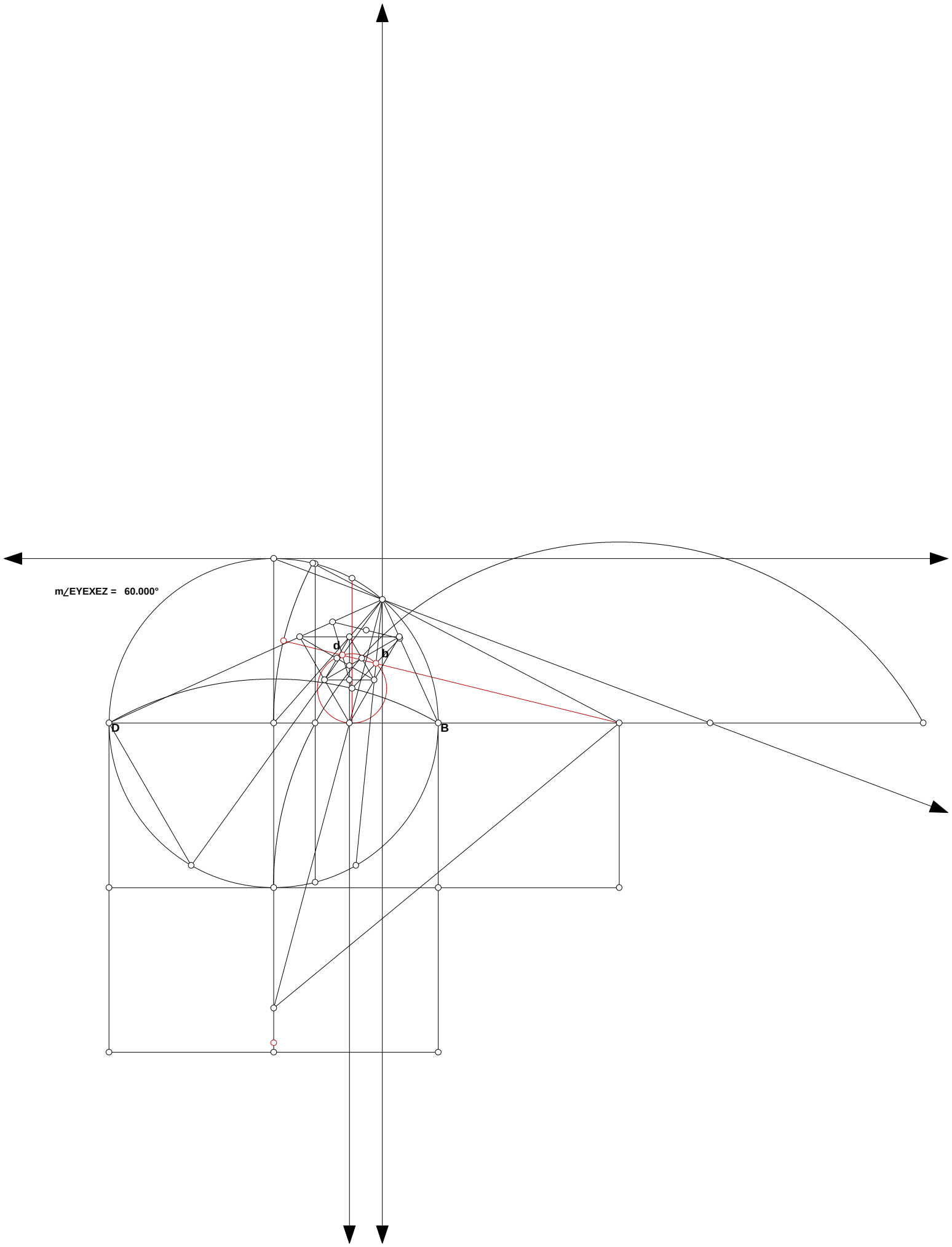

a locus found
m∠KAC = 42.170°
BMA = 0.541 inches
m∠CEAC = 44.211°
m∠CBAC = 14.737°
m∠CEAC / m∠CBAC = 3.000
m∠CEAC / m∠CBAC ·1000 = 2999.999

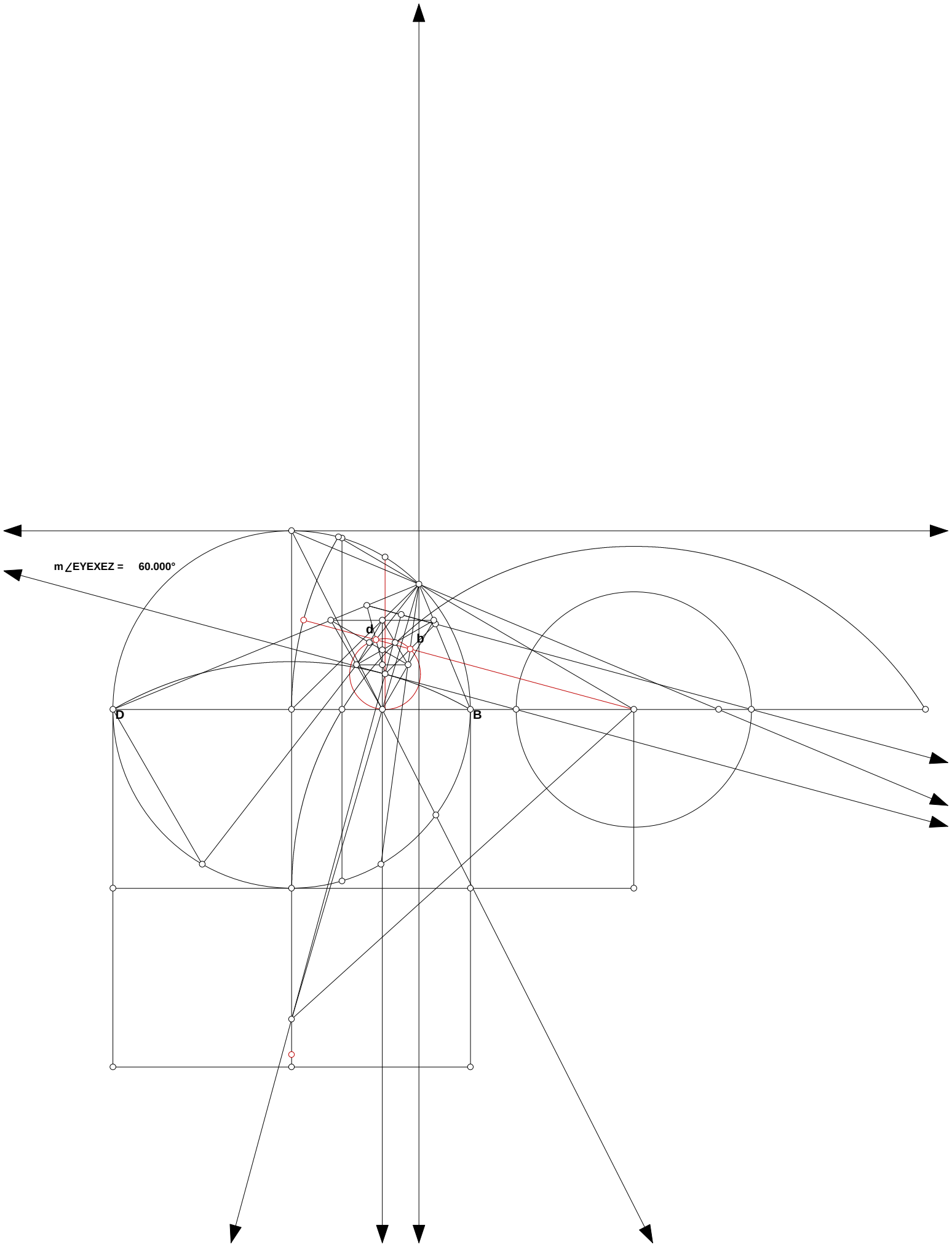

DD
BA
A
BB
m ∠DWDXR = 53.643°
DC
DB
R
DA
BABB = 0.312 inches
DBDA = 0.312 inches
DCR = 0.614 inches
DDA = 0.614 inches

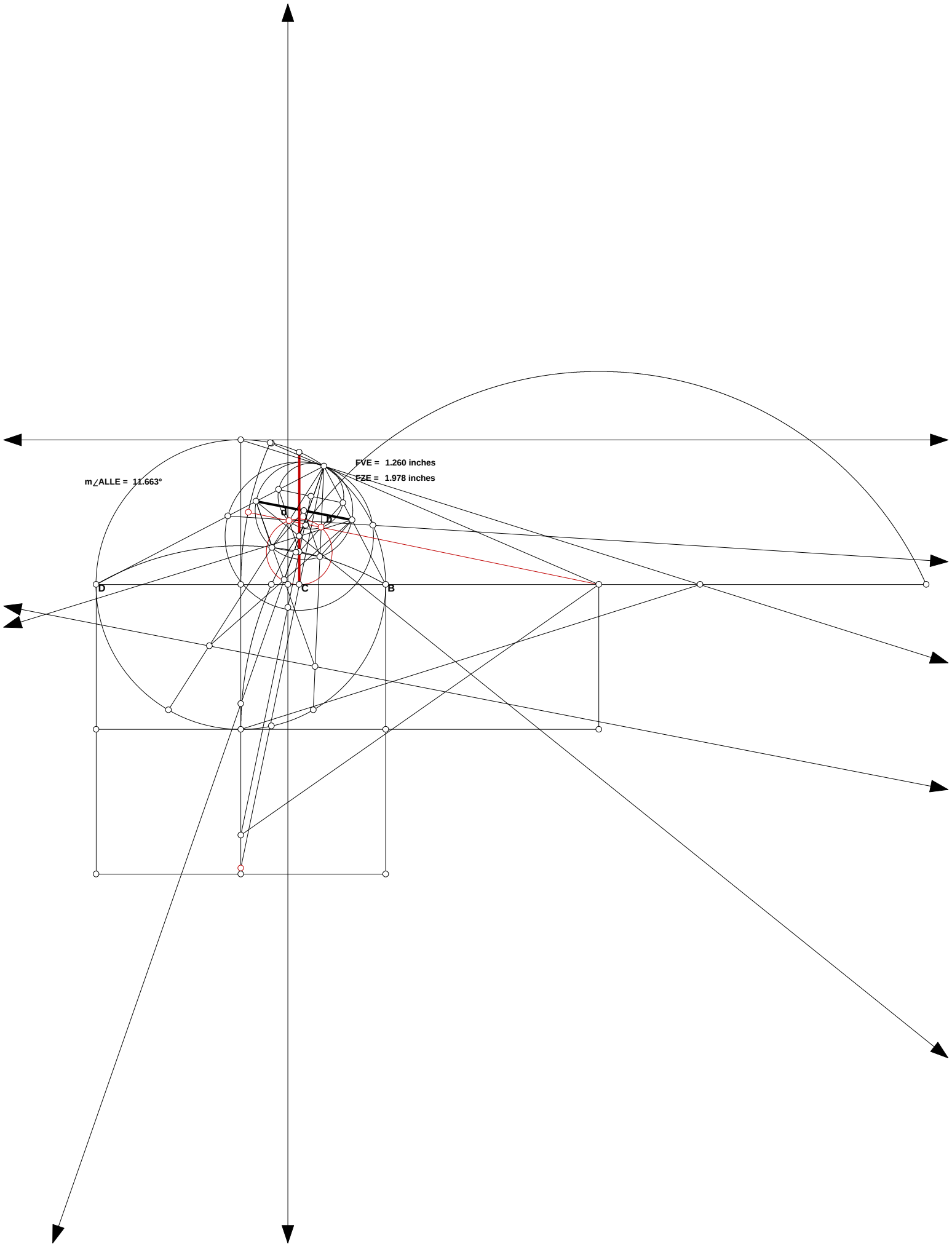

m∠JAC = 58.598°
m∠HAC = 29.299°
m∠JAC / m∠HAC = 2.000
TA = 5.946 inches
4
3
2
1
-1
-2
-3
-4
-5
-6
-4
-3
-2
-1
1
2
3

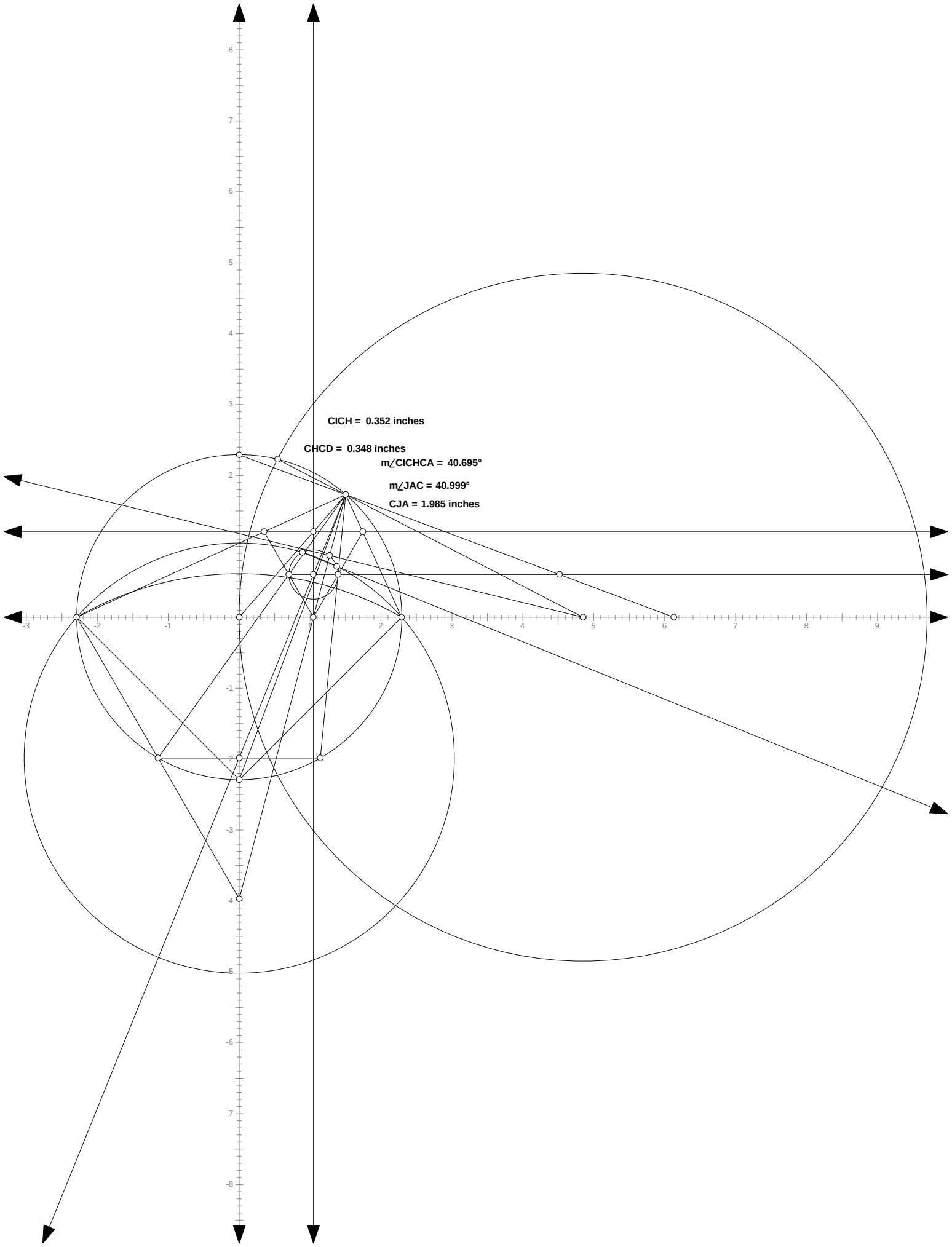

Show
Hide
Show
Hide
Pull Me

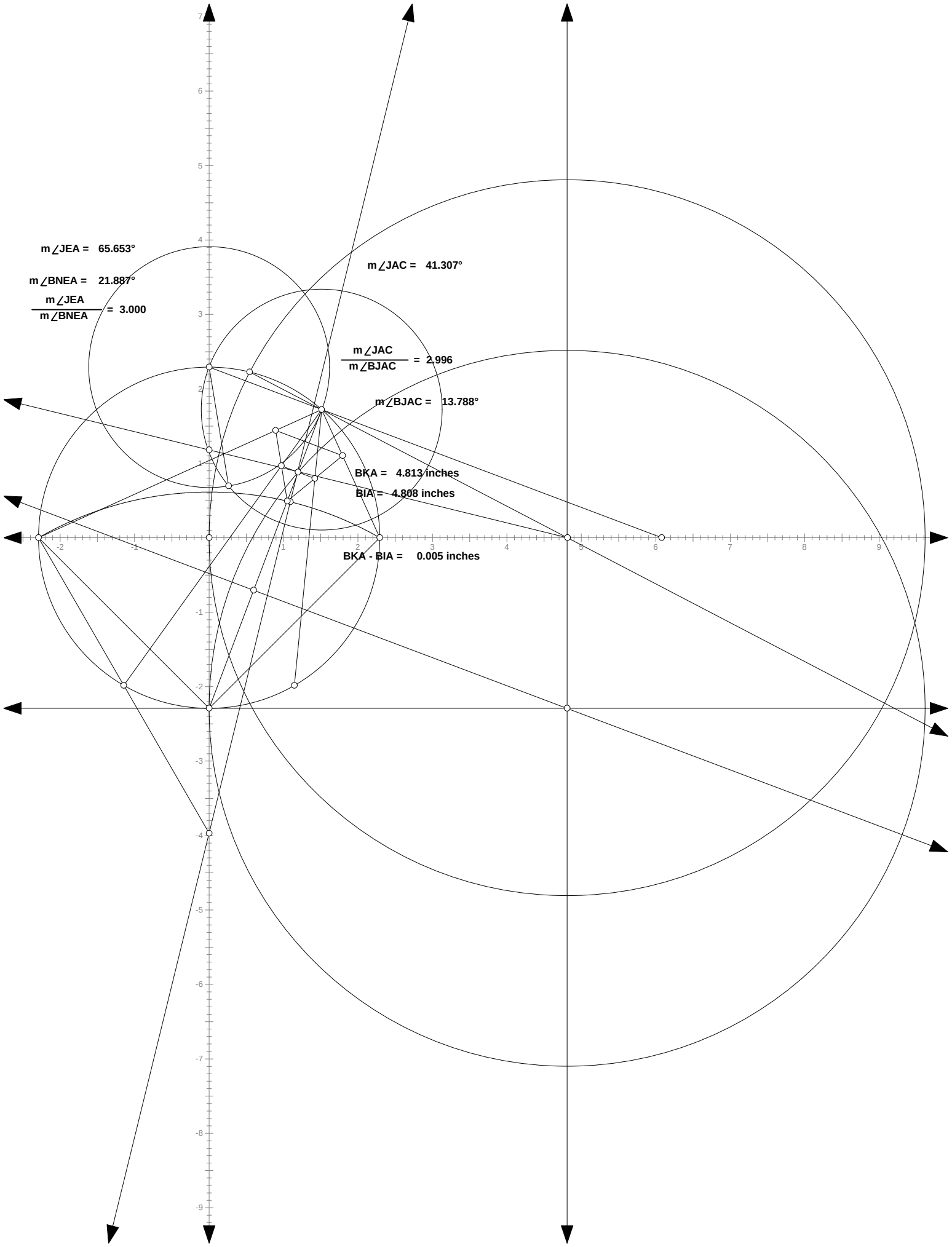

m∠BMAC = 16.316°
m∠RAAZ = 16.223°

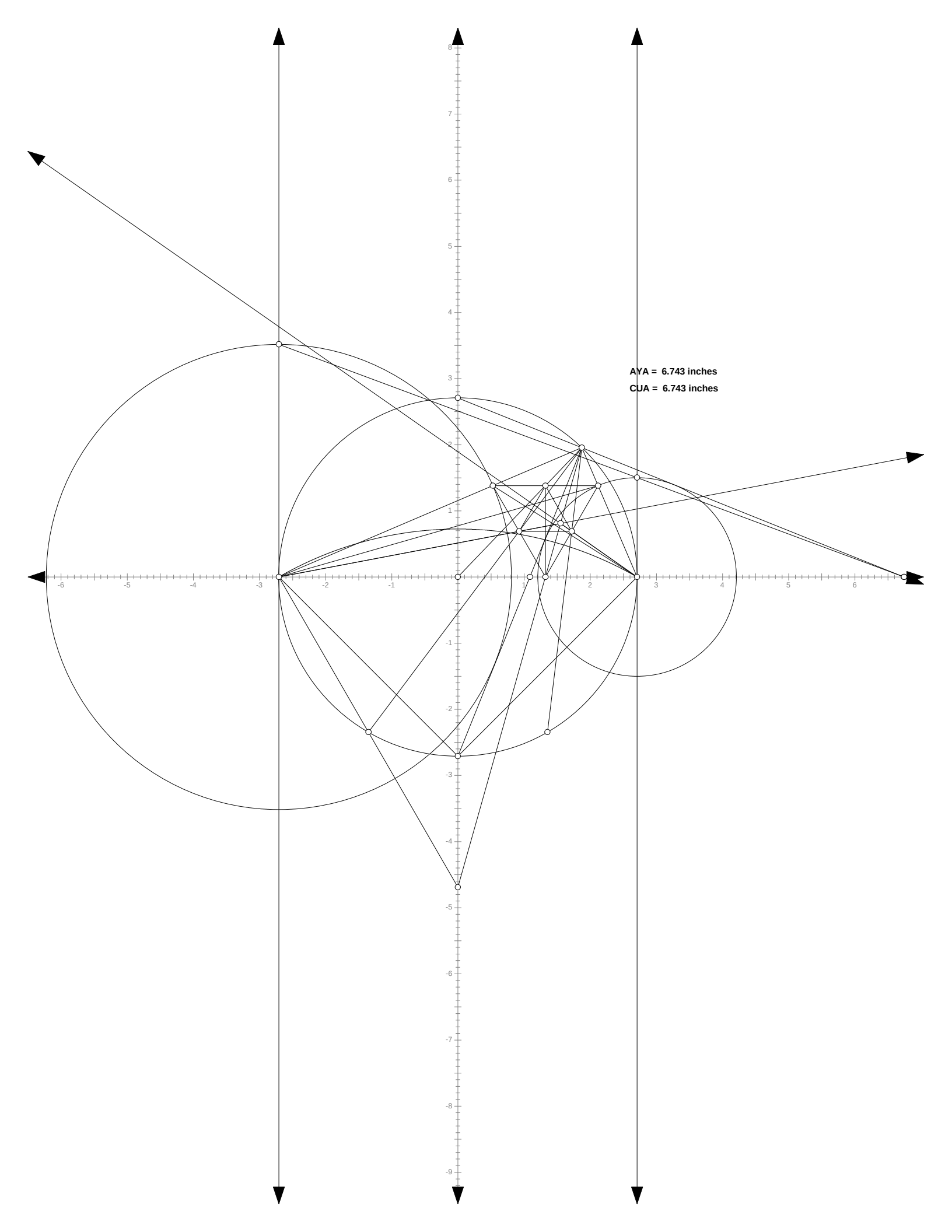

m∠EYEXEZ = 60.000°
d
b
D
B

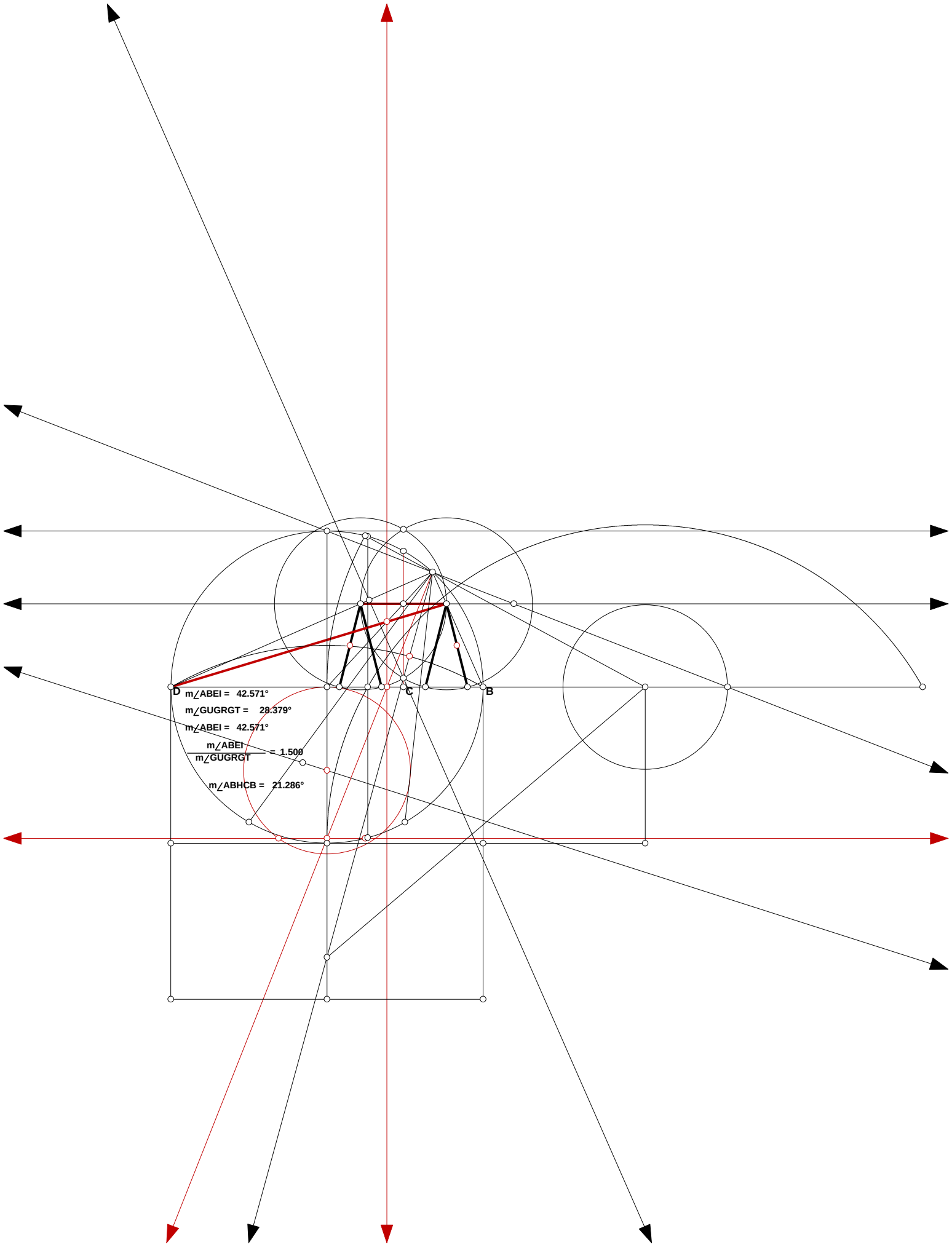

m ∠EYEXEZ = 60.000°
d
b
D
B

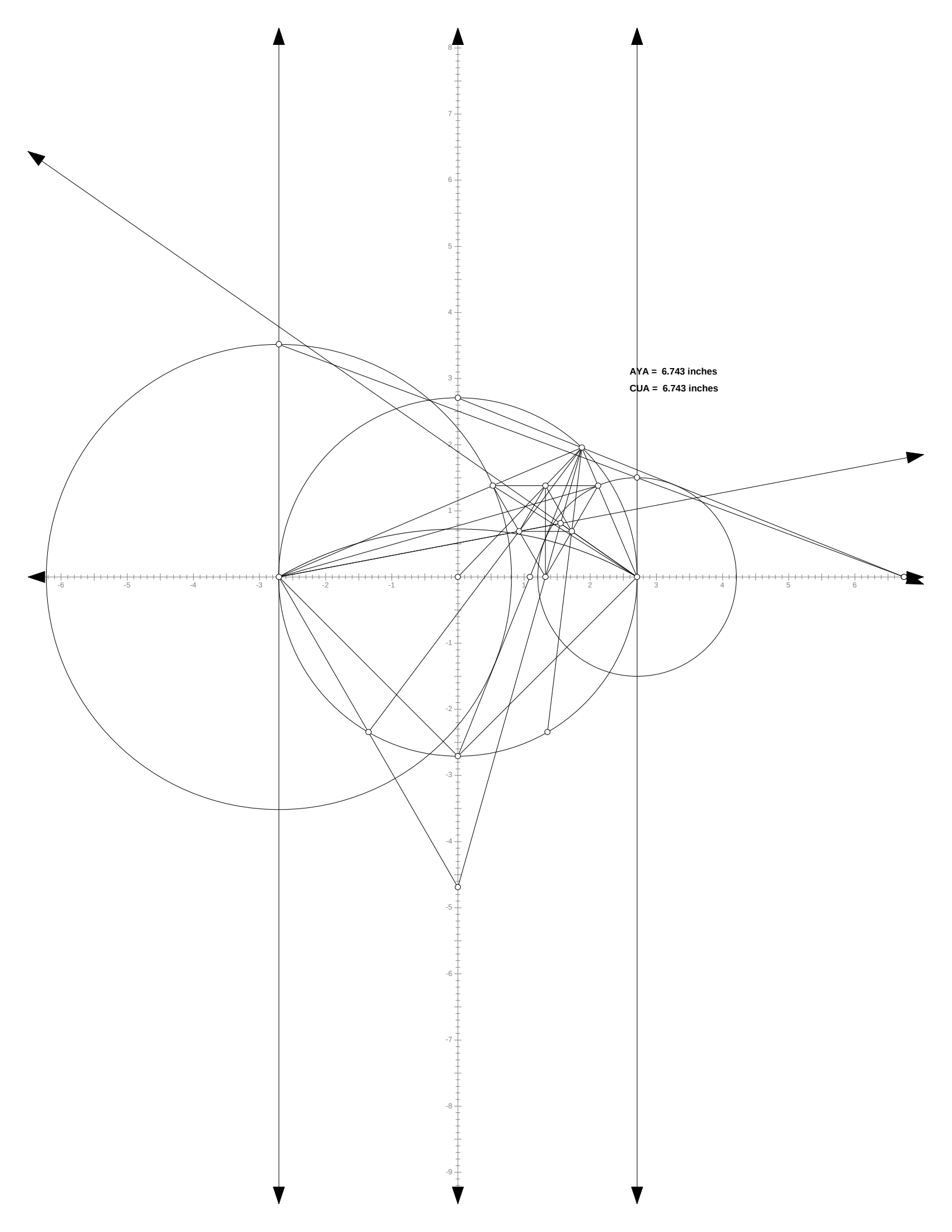

FVE = 1.260 inches
FZE = 1.978 inches
m∠ALLE = 11.663°
Q
O
D
D
C
B

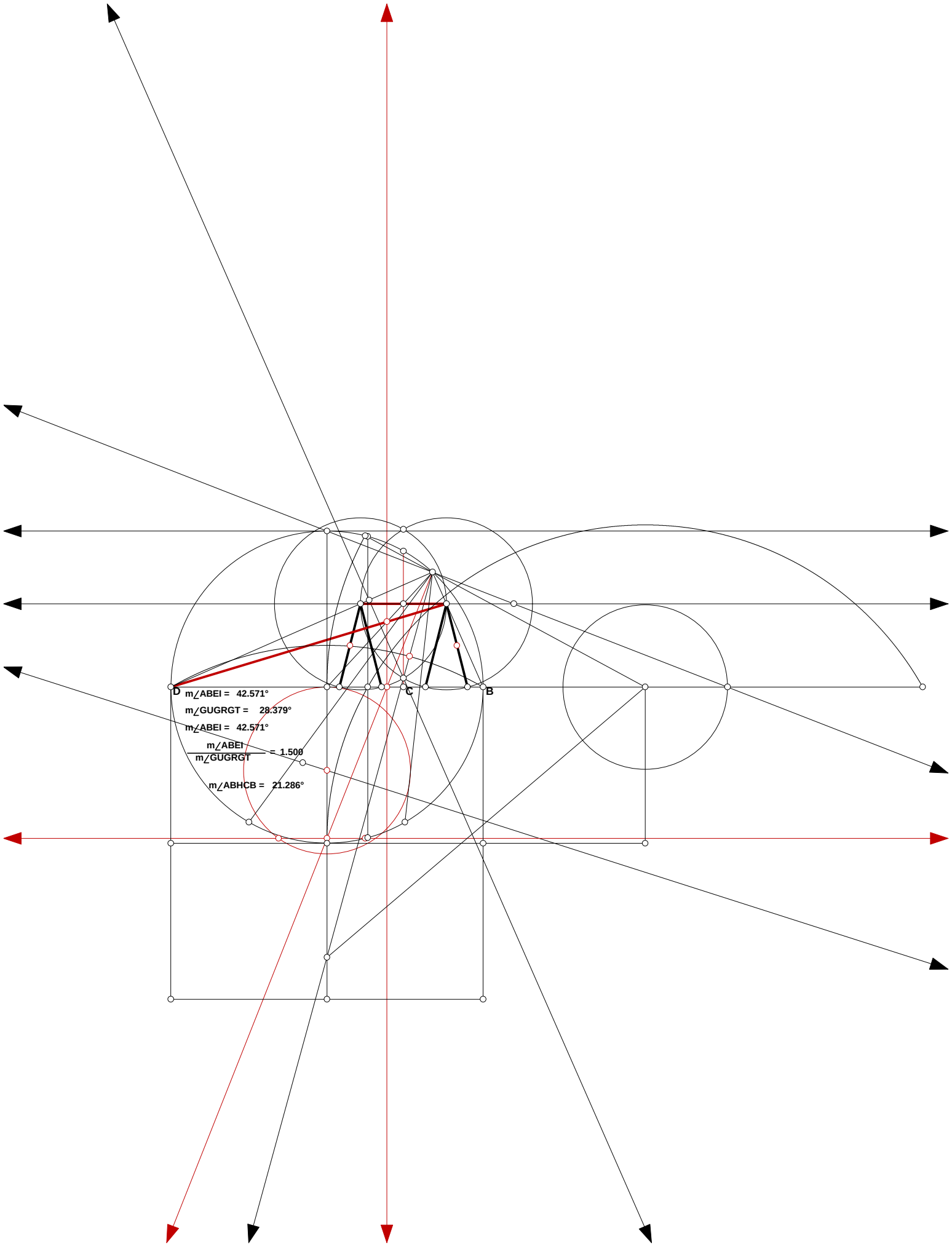

CICH = 0.352 inches
CHCD = 0.348 inches
m∠CICHCA = 40.695°
m∠JAC = 40.999°
CJA = 1.985 inches

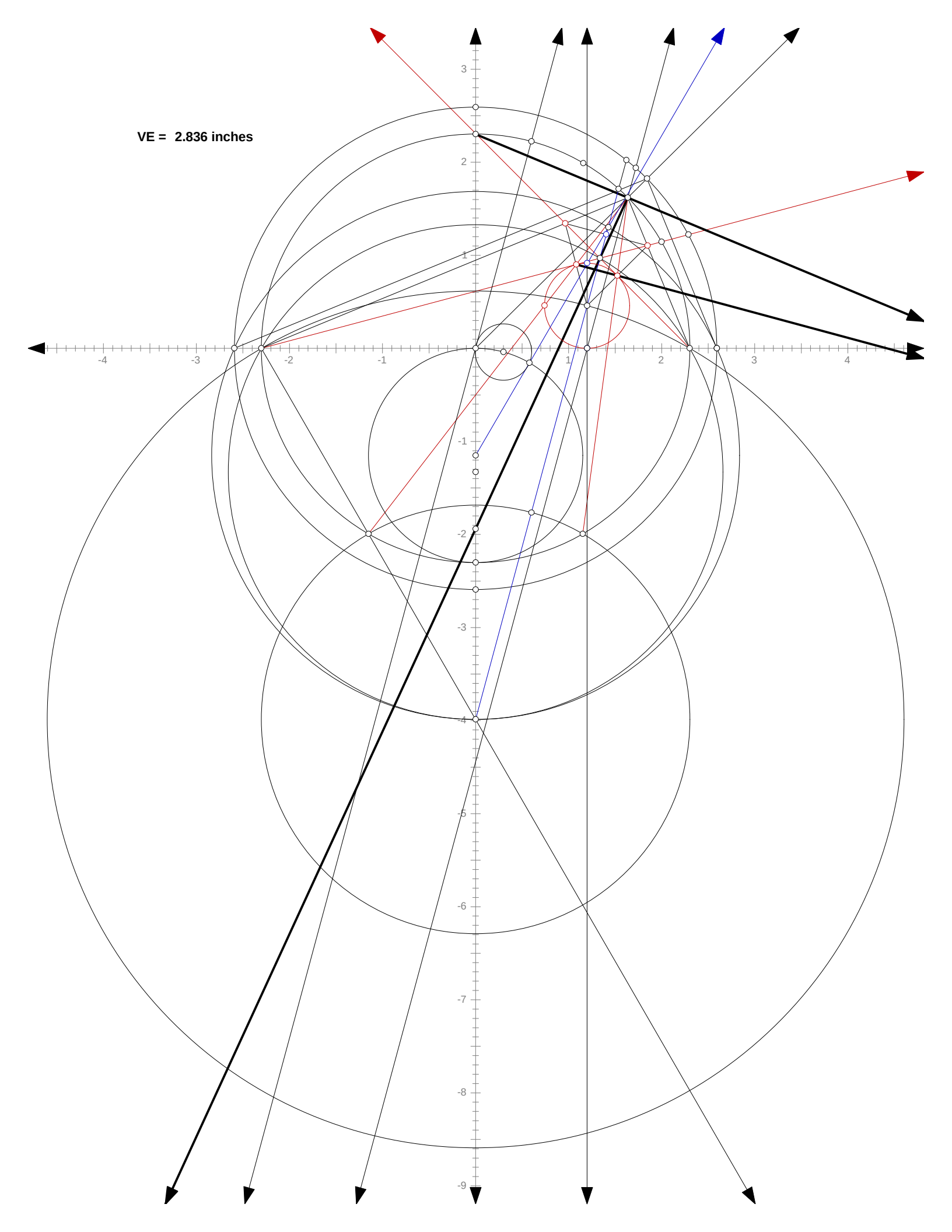

m∠JEA = 65.653°
m∠BNEA = 21.887°
m∠JEA / m∠BNEA = 3.000
m∠JAC = 41.307°
m∠JAC / m∠BJAC = 2.996
m∠BJAC = 13.788°
BKA = 4.813 inches
BIA = 4.808 inches
BKA - BIA = 0.005 inches

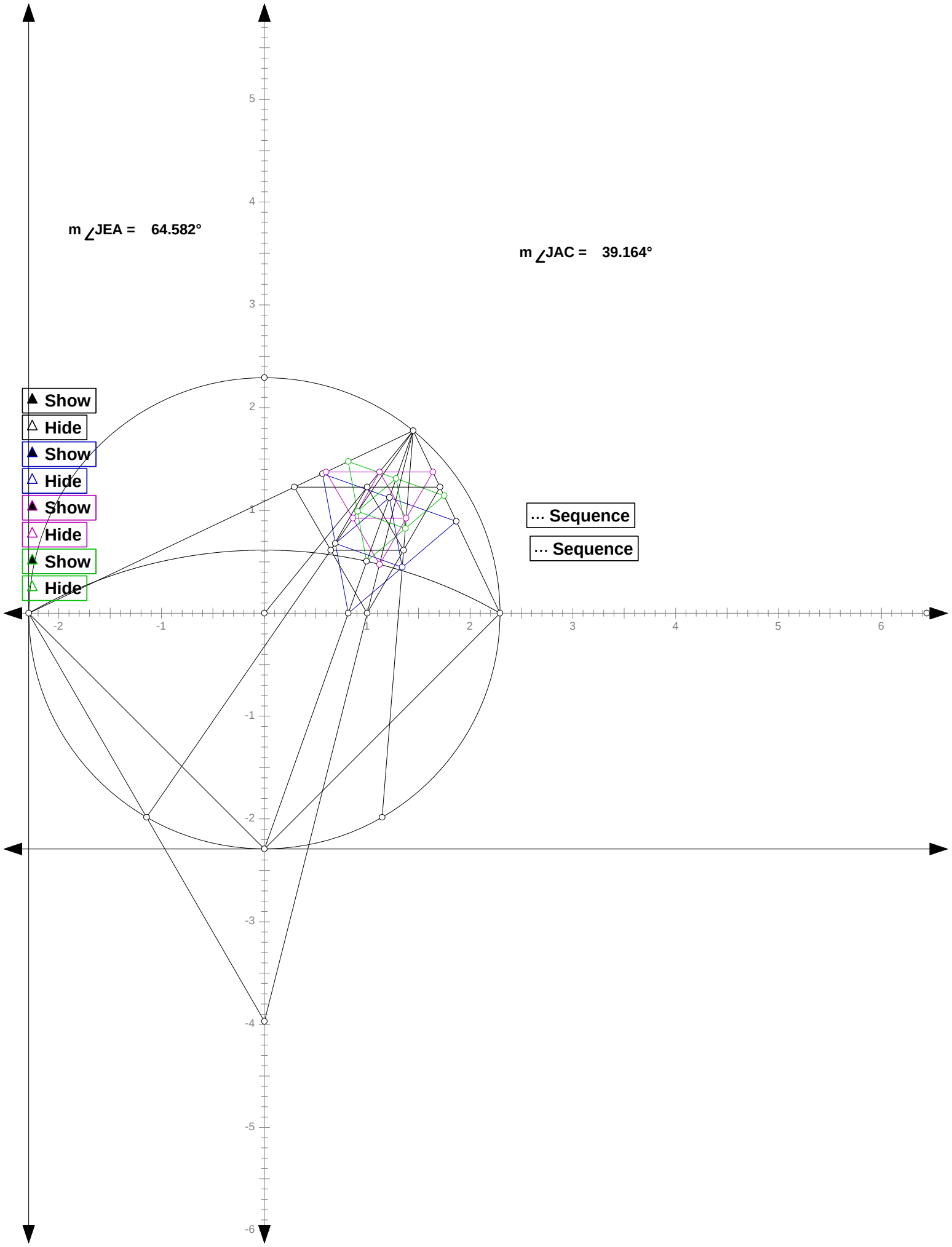

AYA = 6.743 inches
CUA = 6.743 inches

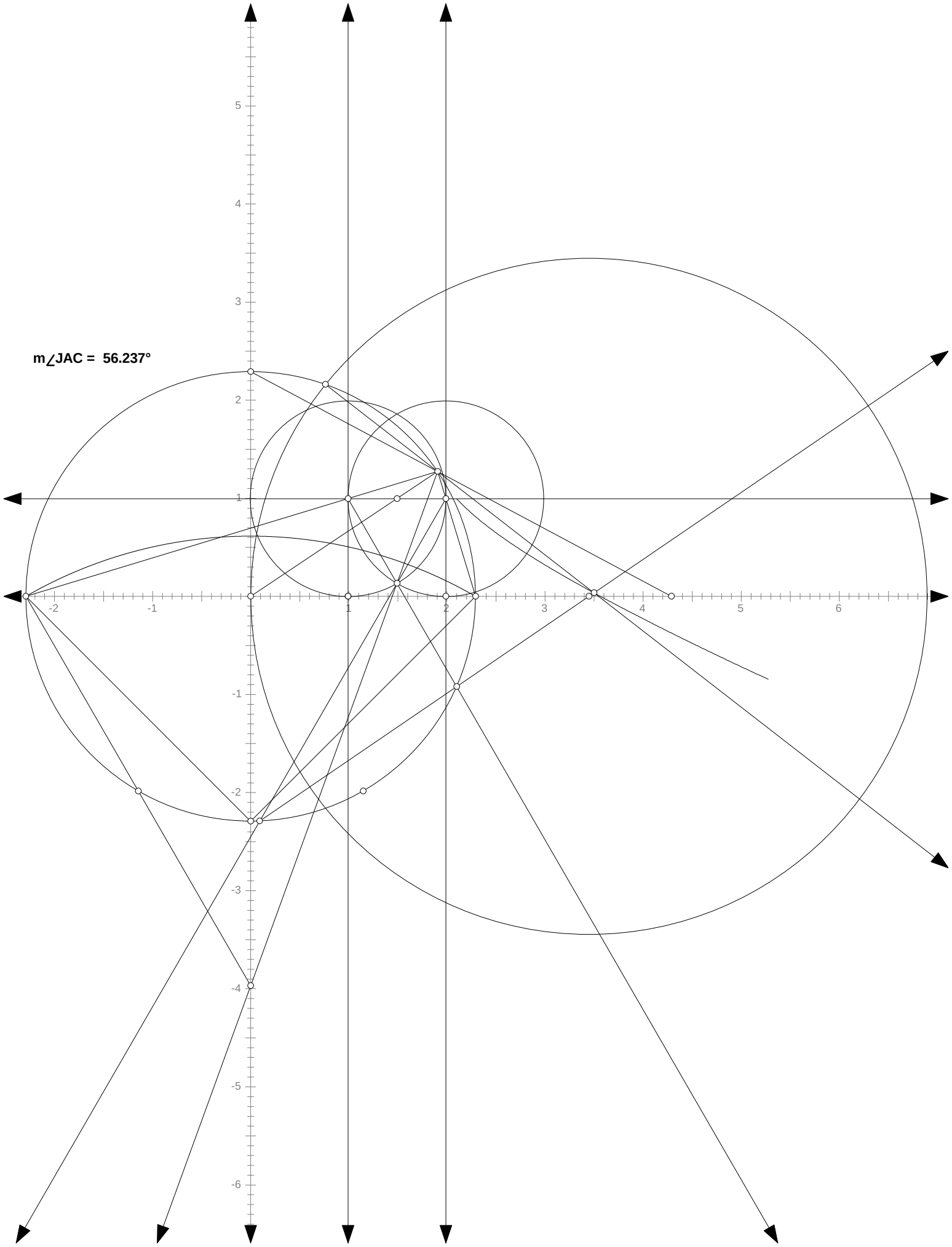

D
C
B
m∠ABEI = 42.571°
m∠GUGRGT = 28.379°
m∠ABEI = 42.571°
m∠ABEI / m∠GUGRGT = 1.500
m∠ABHCB = 21.286°

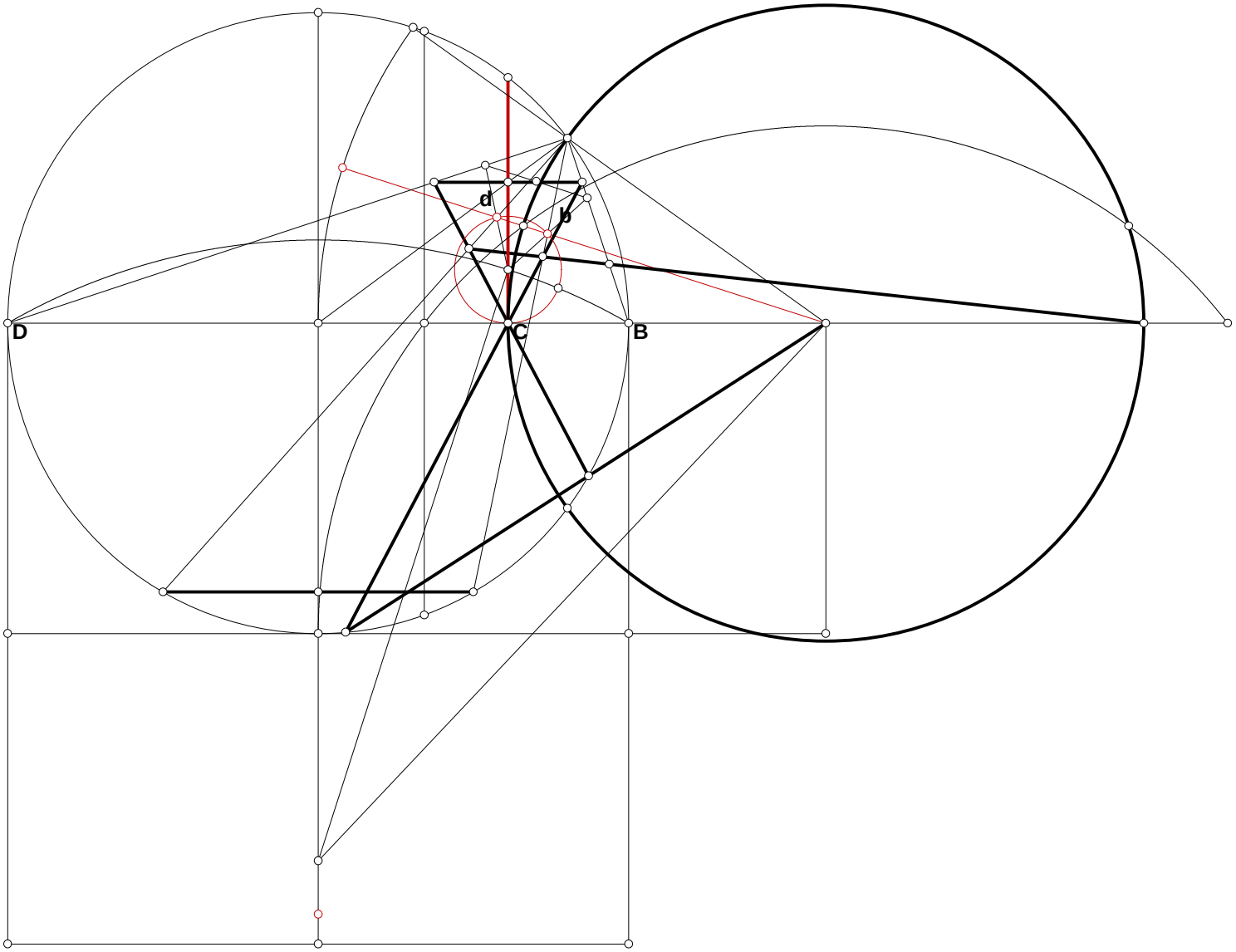

AYA = 6.743 inches
CUA = 6.743 inches

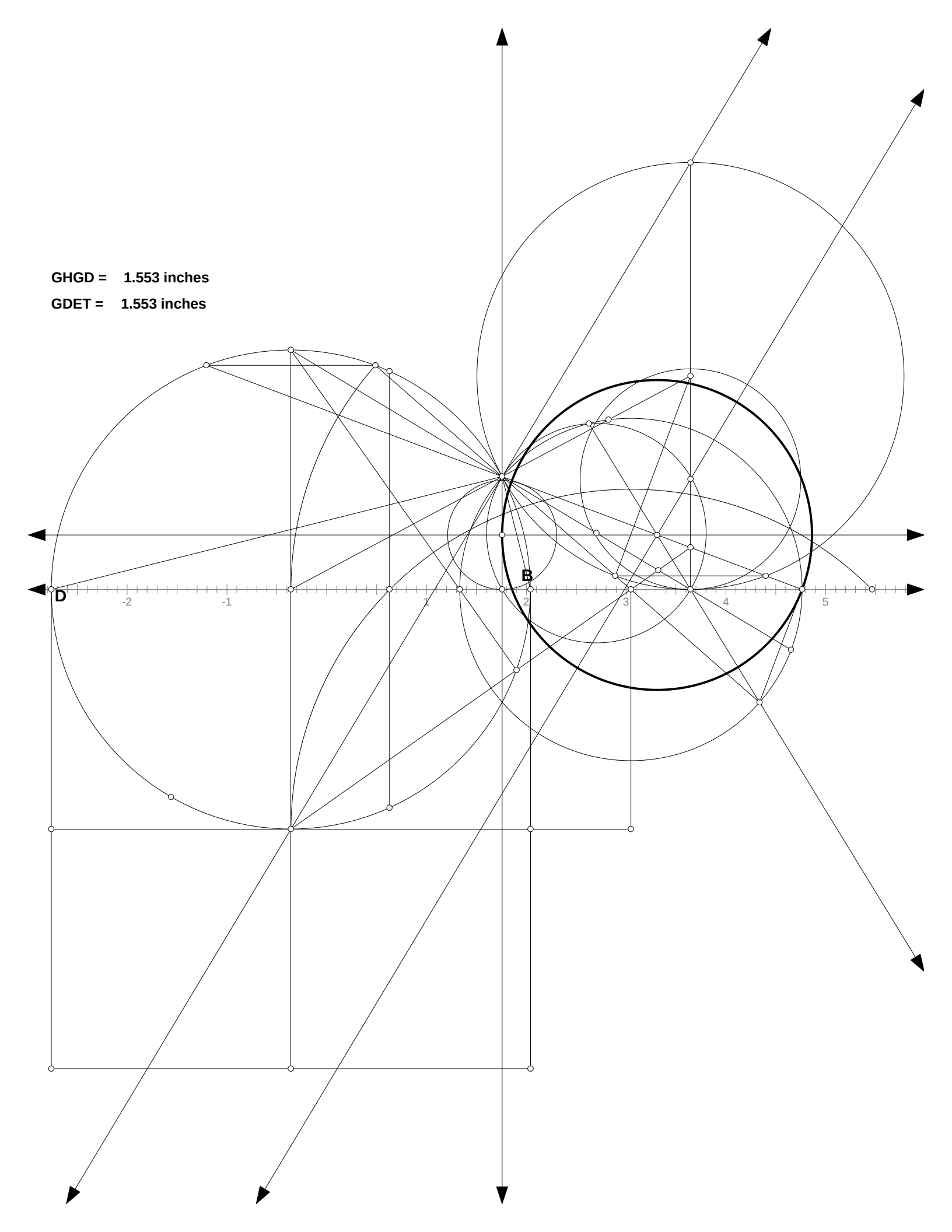

D
C
B
m∠ABEI = 42.571°
m∠GUGRGT = 28.379°
m∠ABEI = 42.571°
m∠ABEI / m∠GUGRGT = 1.500
m∠ABHCB = 21.286°

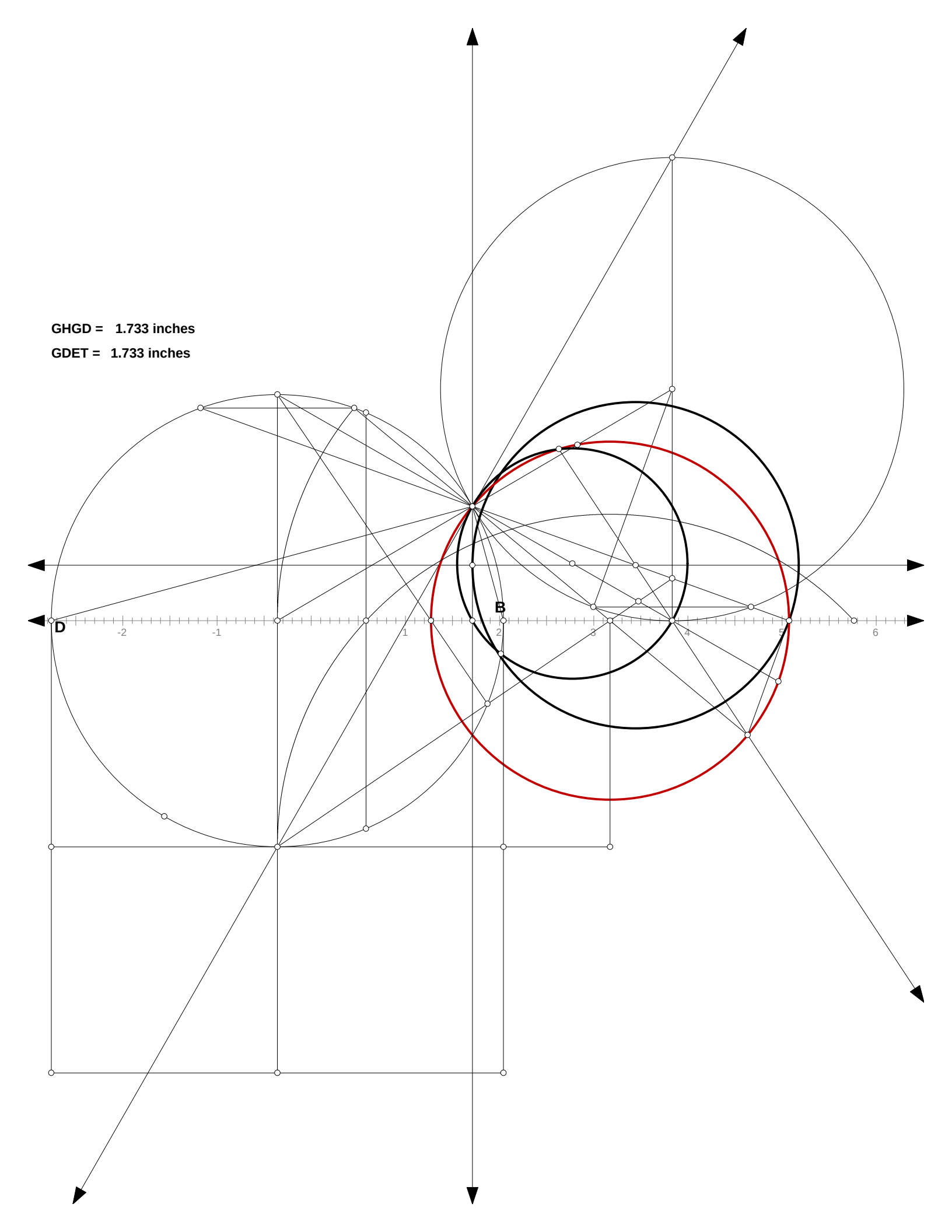

VE = 2.836 inches
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-4
-3
-2
-1
1
2
3
4

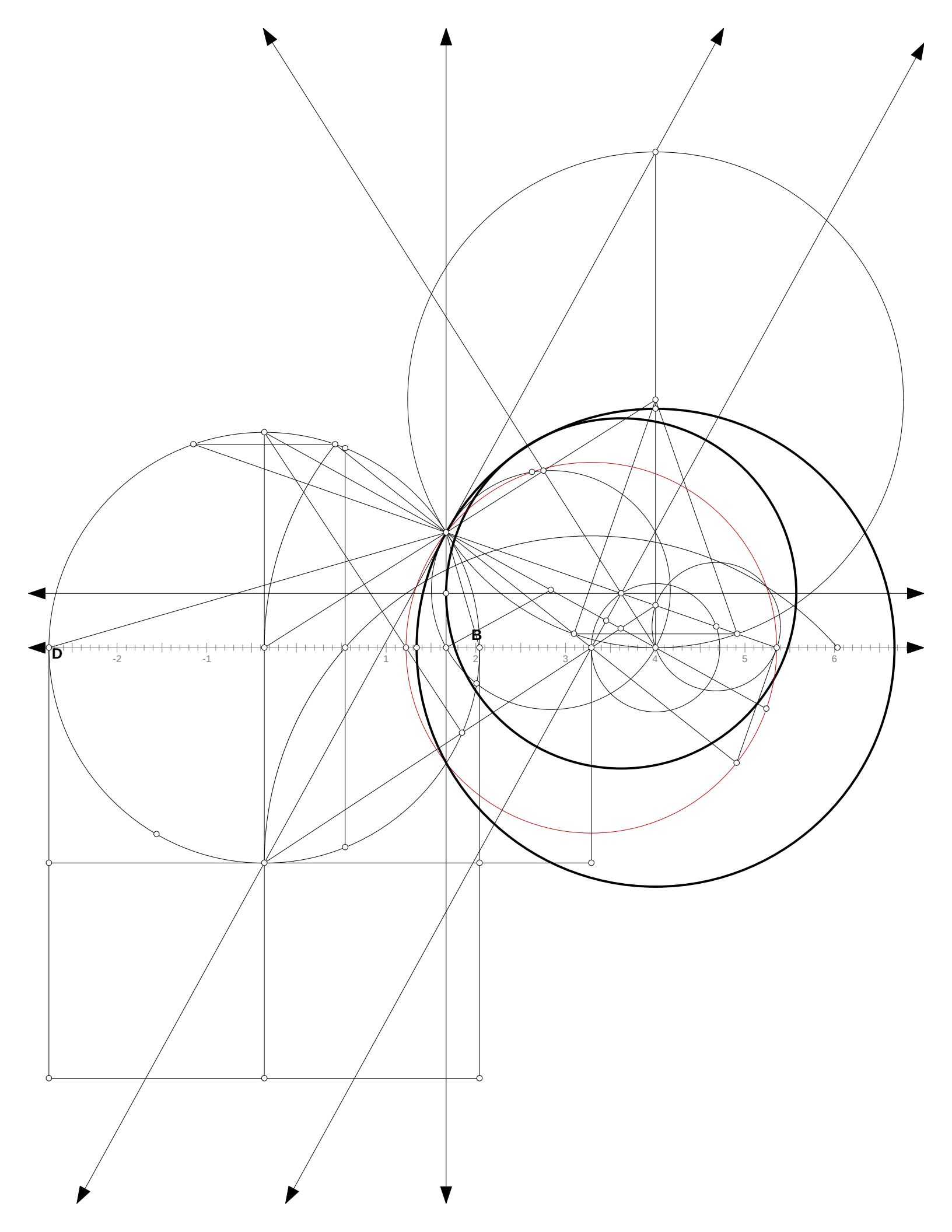

m ∠JEA = 64.582°
m ∠JAC = 39.164°
Show
Hide
Show
Hide
Show
Hide
Show
Hide
··· Sequence
··· Sequence

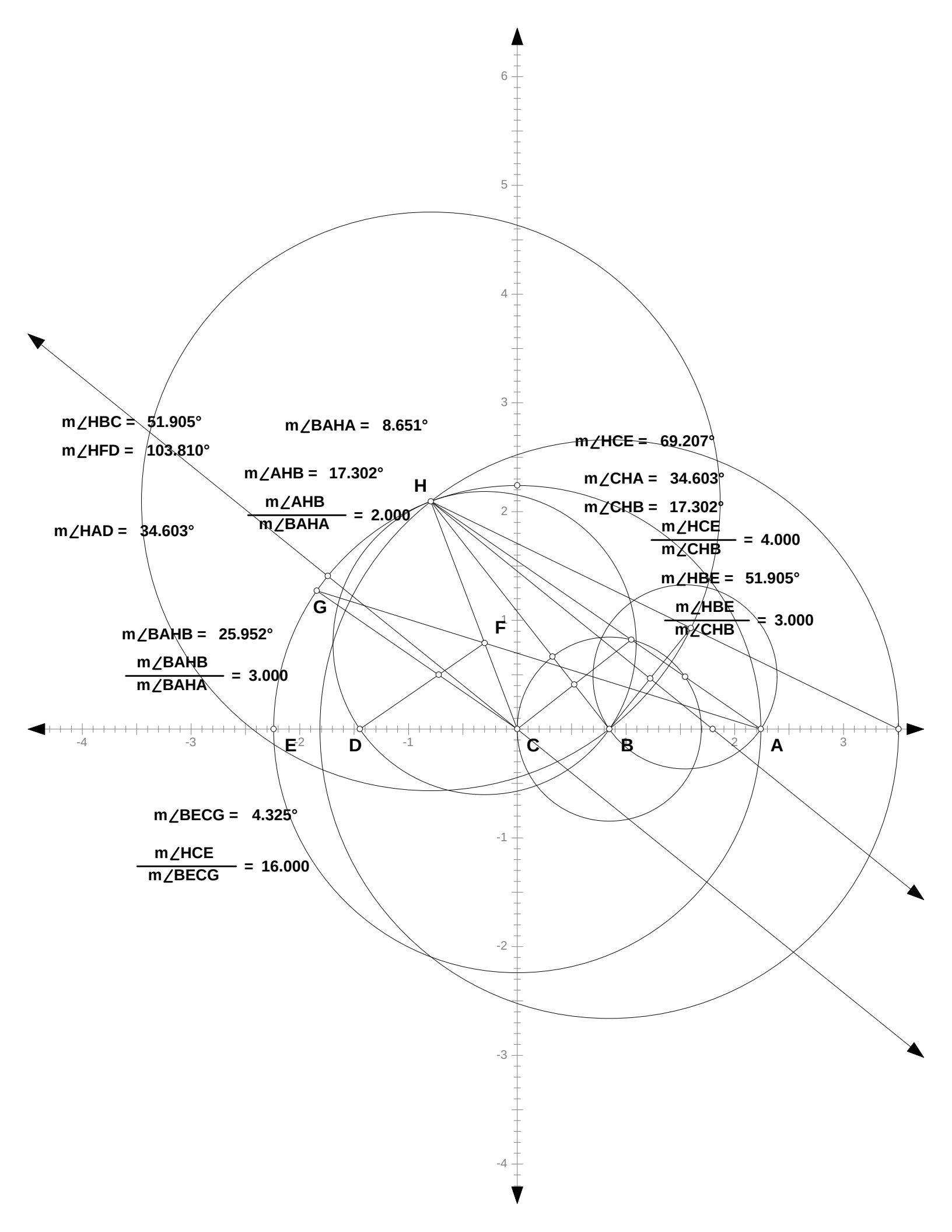

m∠JAC = 56.237°
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-2
-1
1
2
3
4
5
6

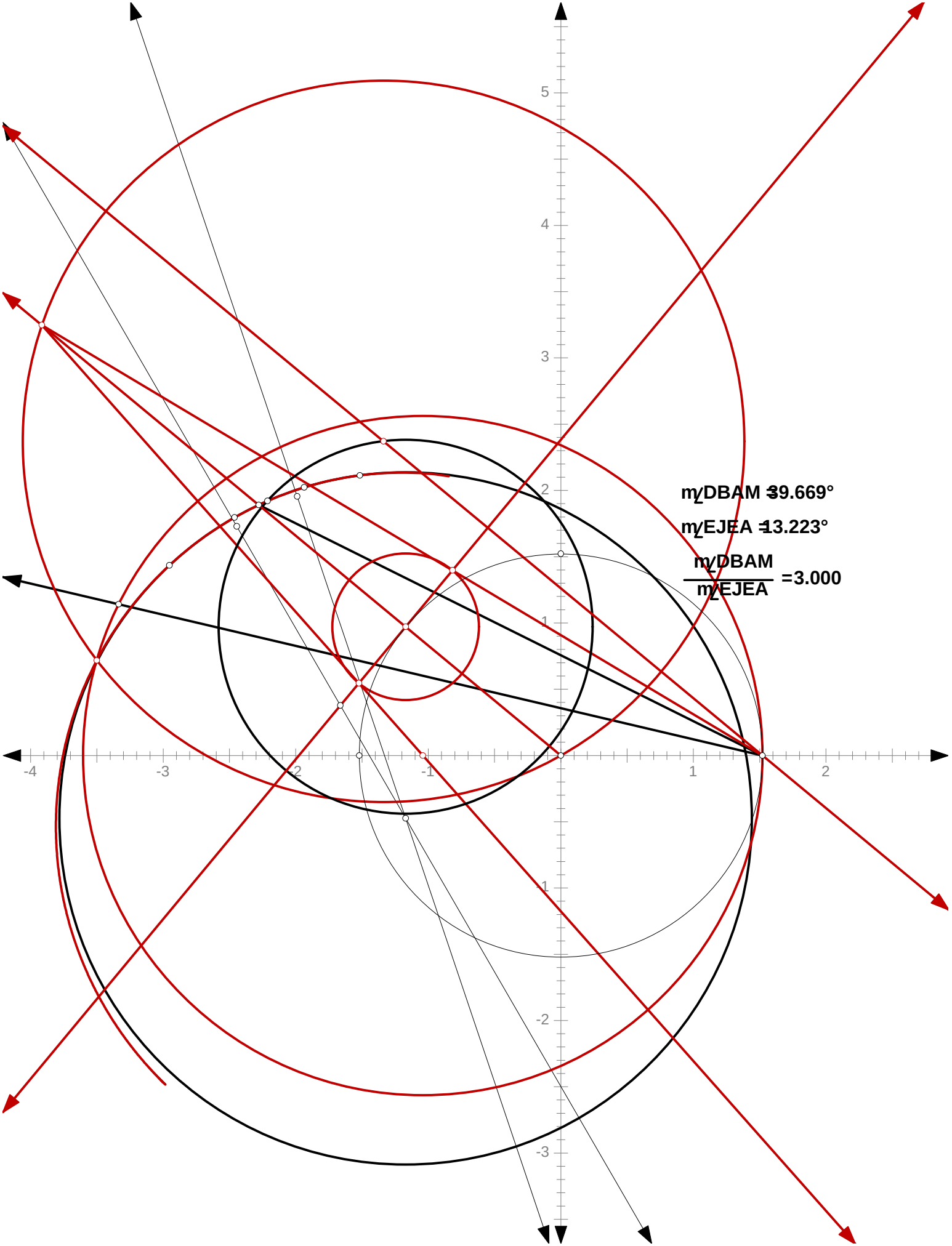

d
b
D
C
B

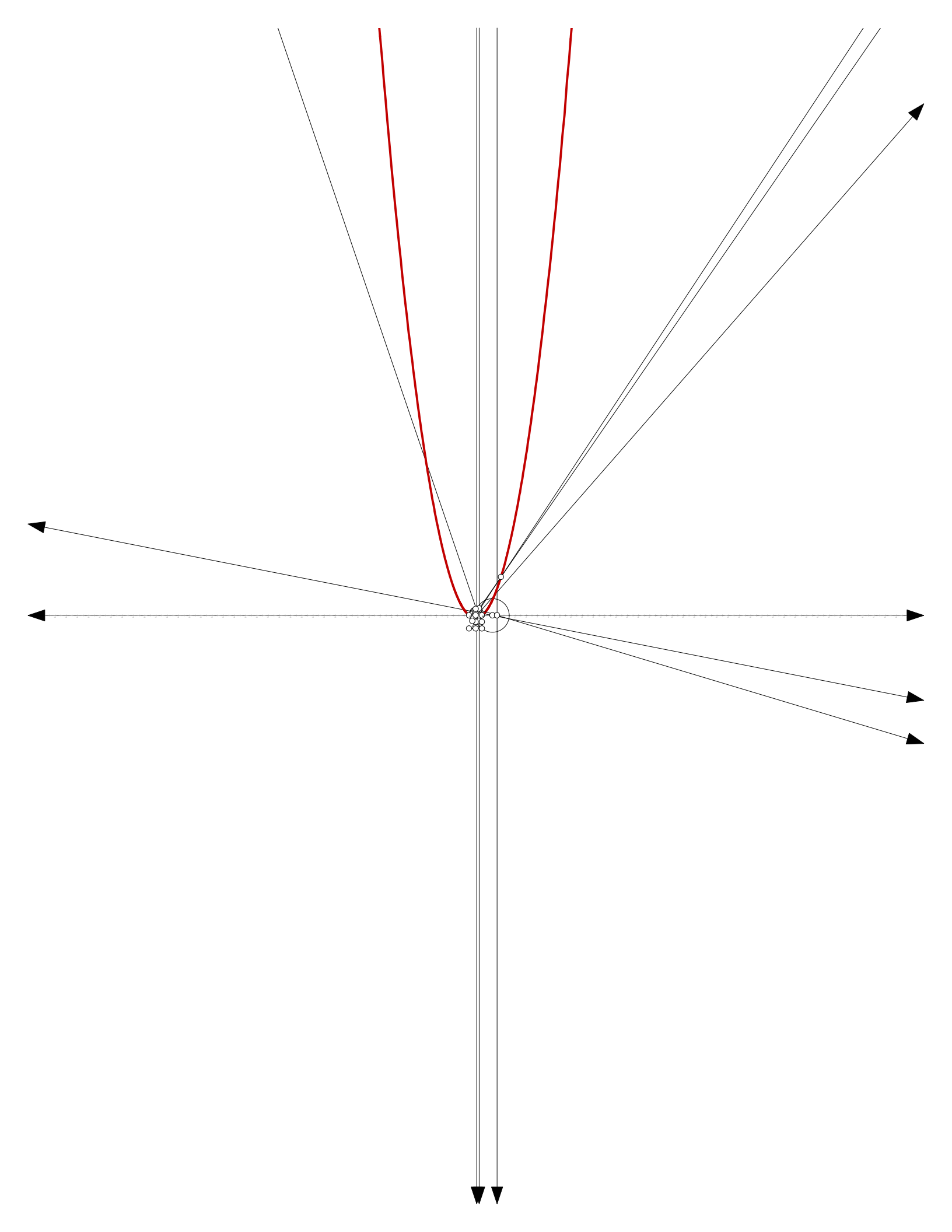

GHGD = 1.553 inches
GDET = 1.553 inches
D
B
-2
-1
1
2
3
4
5

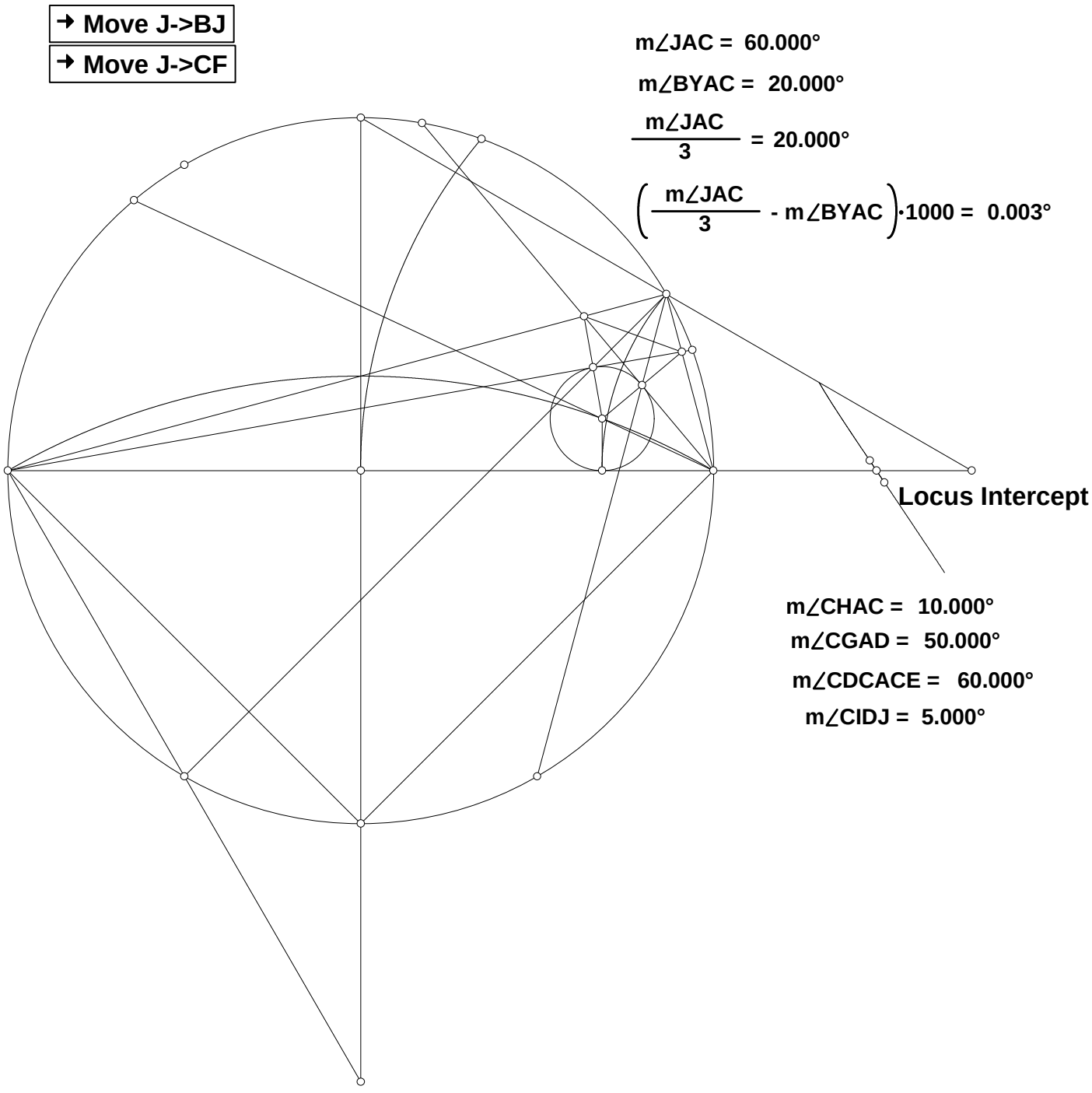

GHGD = 1.733 inches
GDET = 1.733 inches
D
B
-2
-1
1
2
3
4
5
6

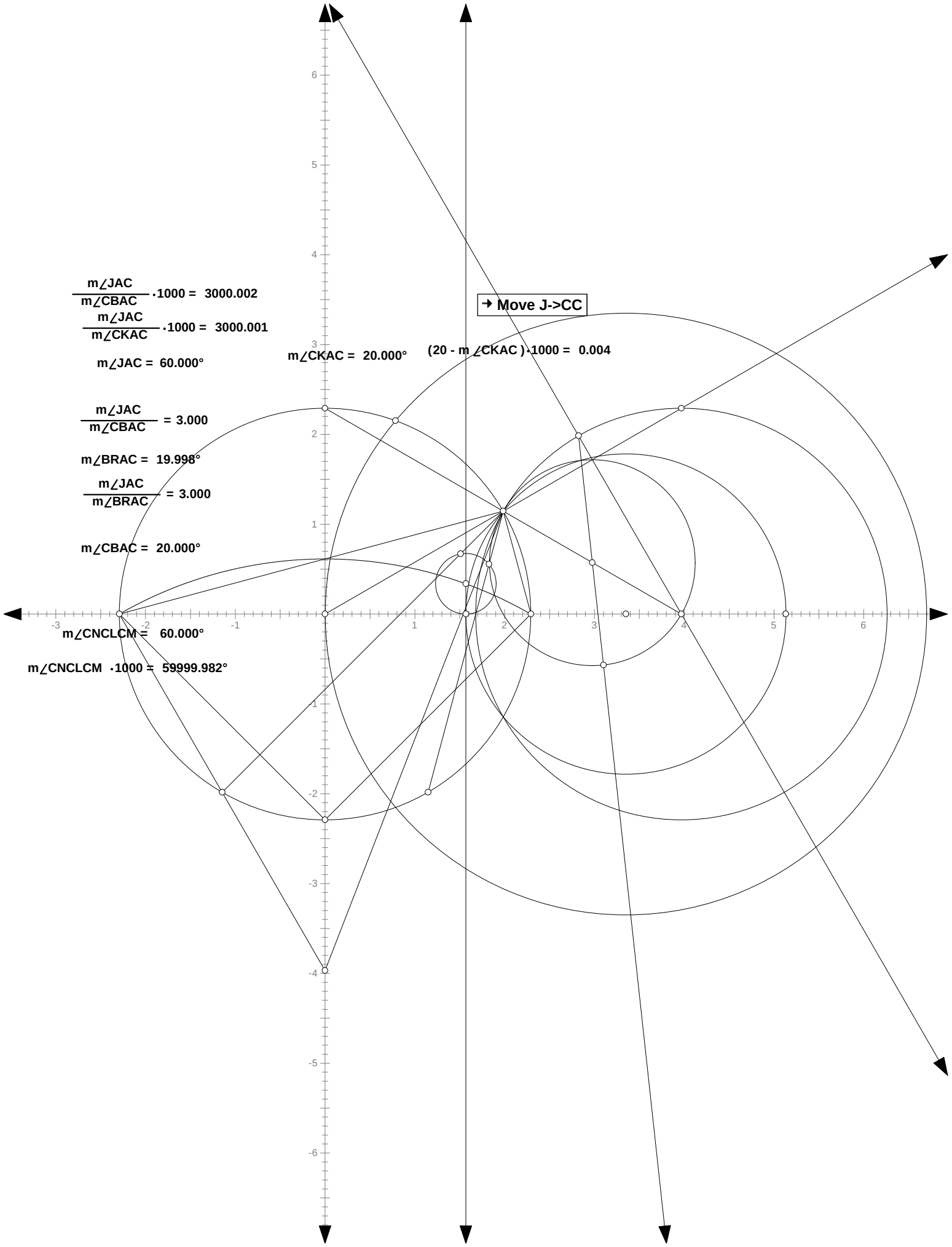

D
B
-2
-1
1
2
3
4
5
6

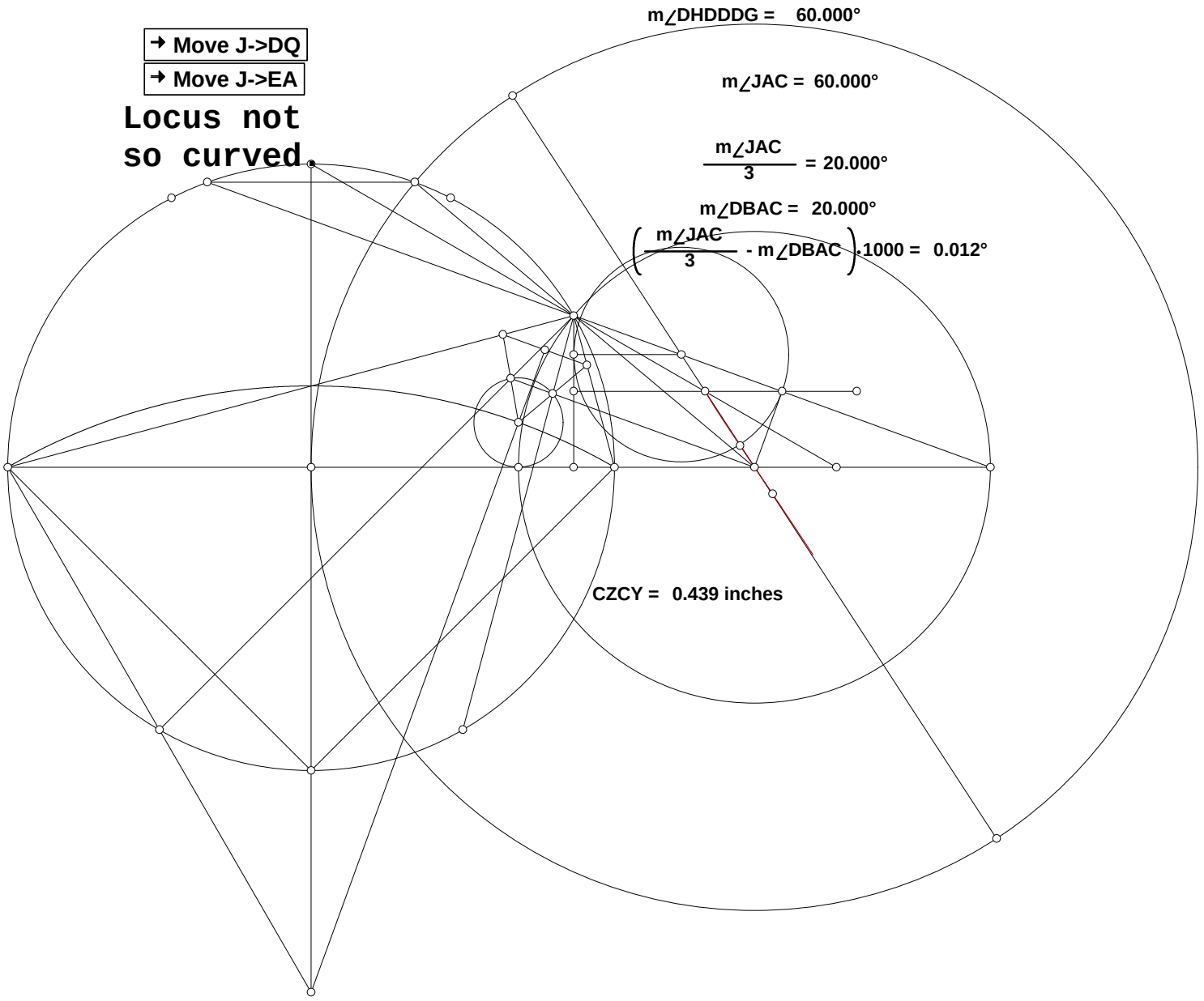

m∠HBC = 51.905°
m∠HFD = 103.810°
m∠HAD = 34.603°
m∠BAHA = 8.651°
m∠AHB = 17.302°
m∠AHB / m∠BAHA = 2.000
m∠BAHB = 25.952°
m∠BAHB / m∠BAHA = 3.000
m∠BECG = 4.325°
m∠HCE / m∠BECG = 16.000
m∠HCE = 69.207°
m∠CHA = 34.603°
m∠CHB = 17.302°
m∠HCE / m∠CHB = 4.000
m∠HBE = 51.905°
m∠HBE / m∠CHB = 3.000
H
G
F
E
D
C
B
A

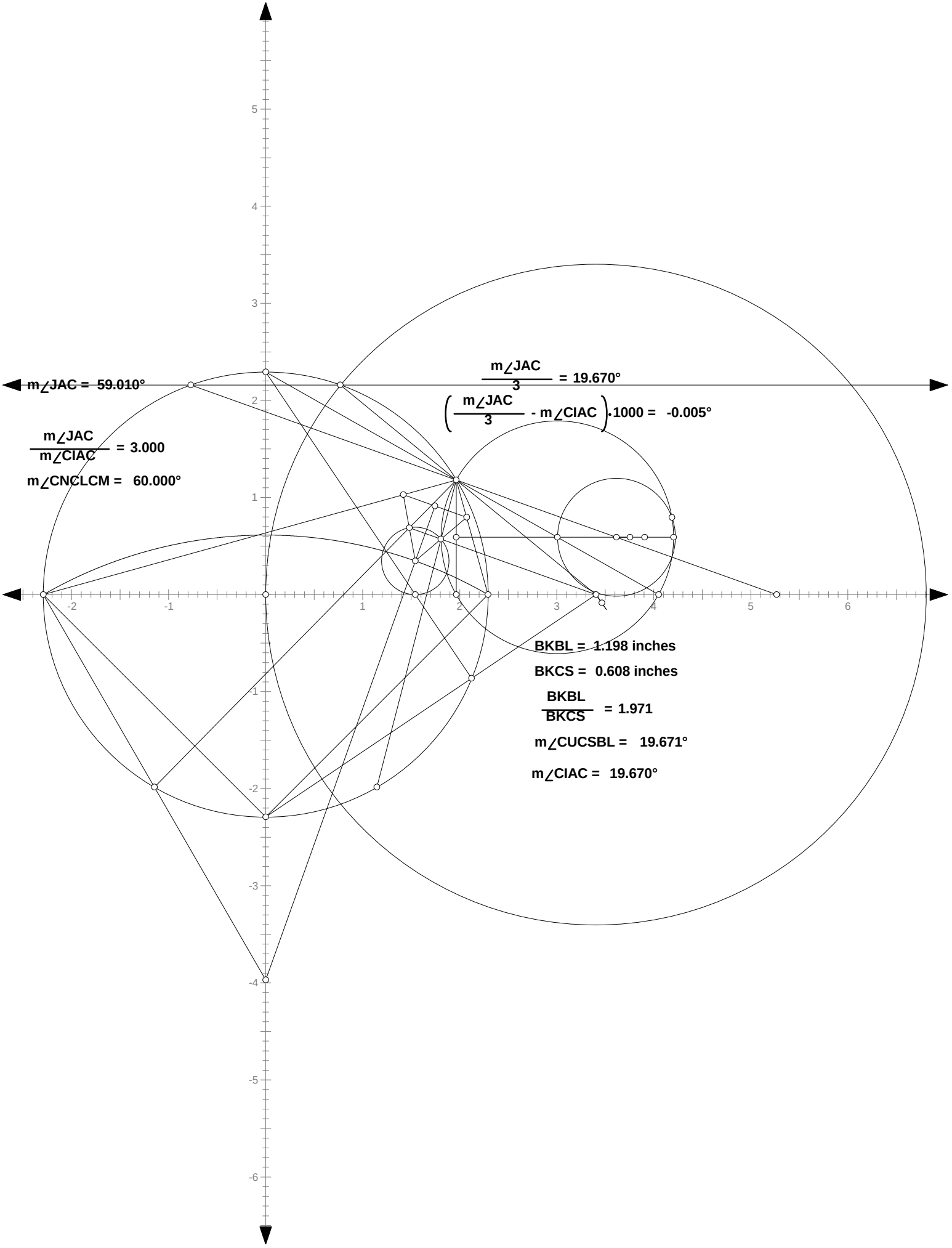

m∠DBAM = 39.669°
m∠EJEA = 13.223°
m∠DBAM / m∠EJEA = 3.000
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
1
2

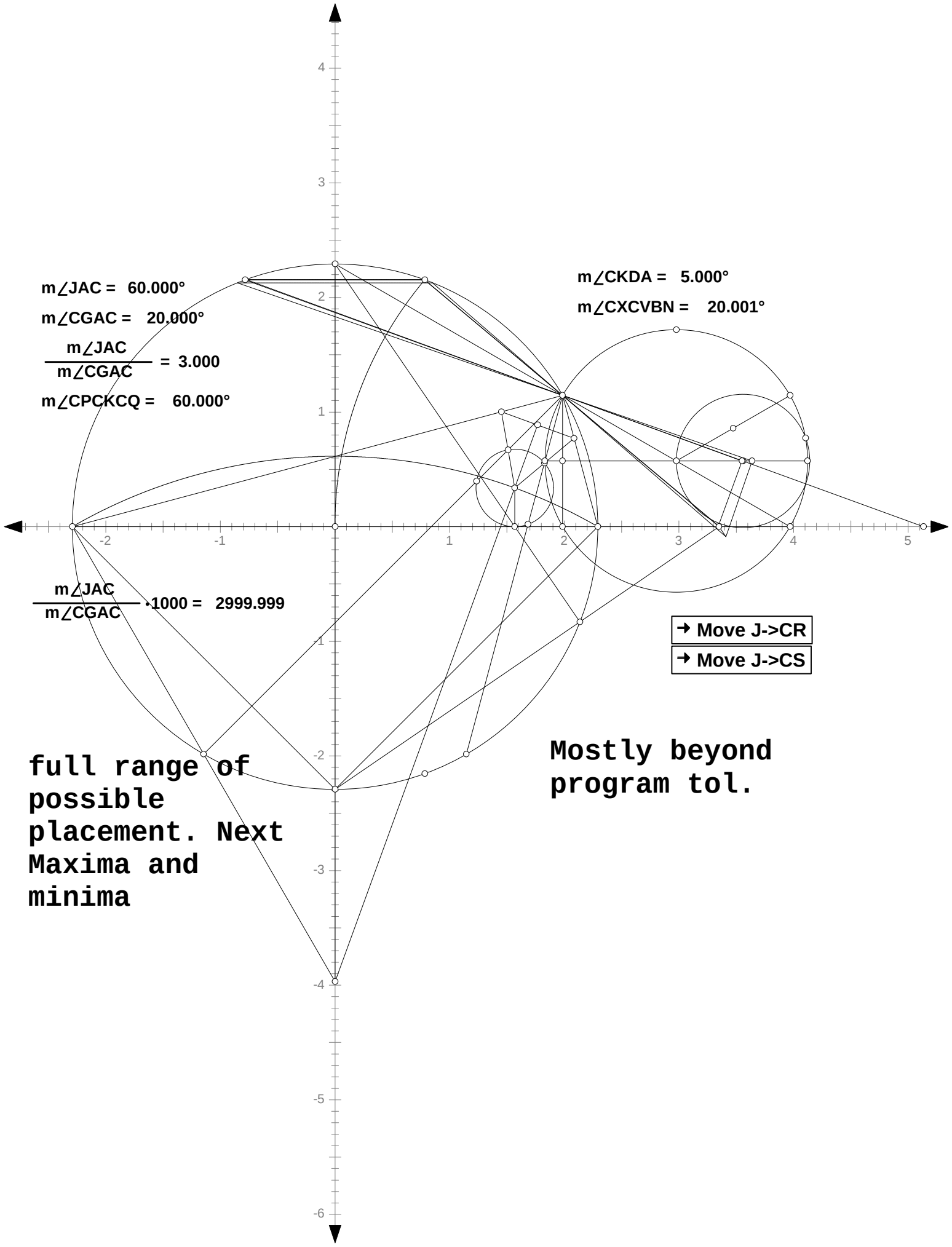


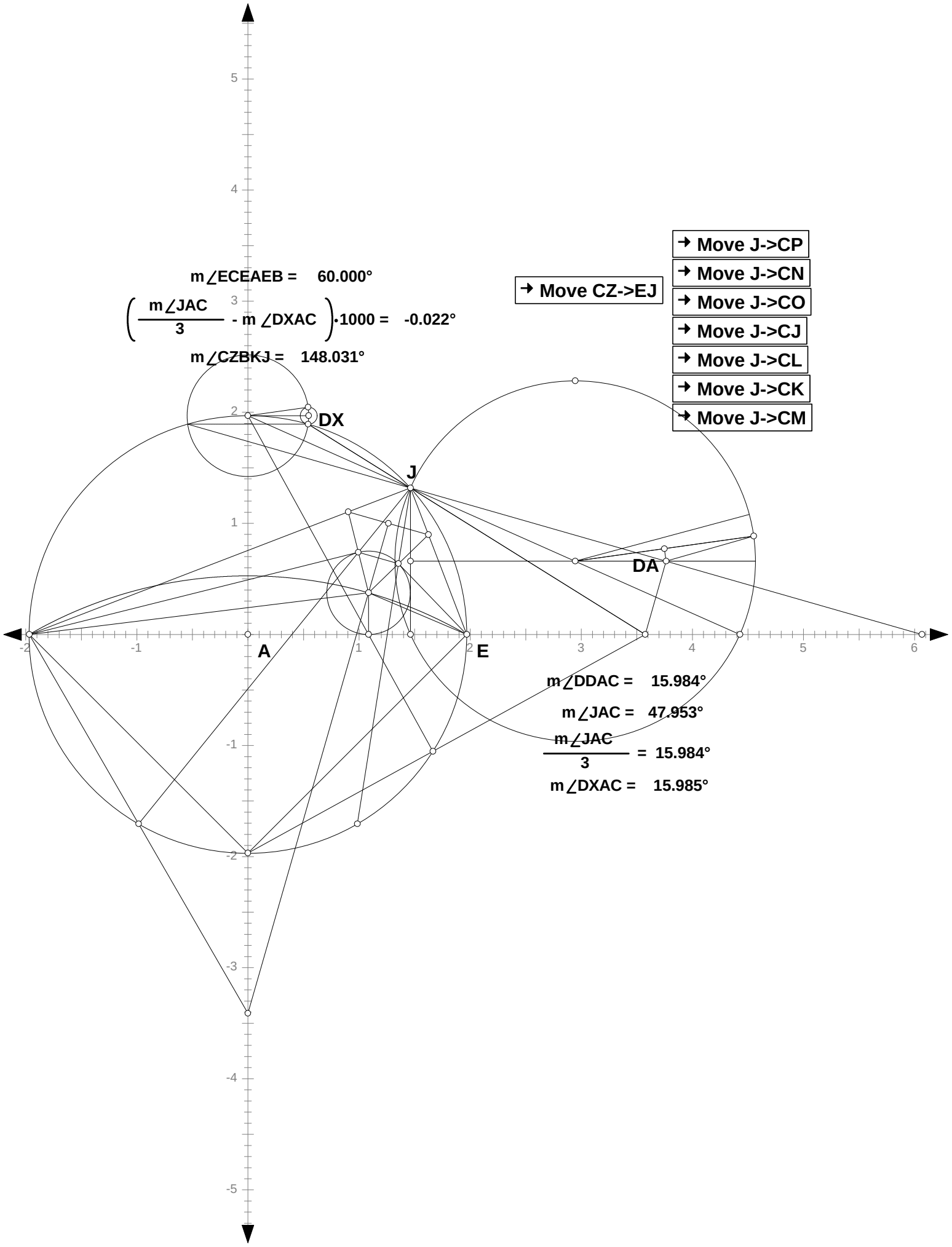

Move J->BJ
Move J->CF
m∠JAC = 60.000°
m∠BYAC = 20.000°
m∠JAC / 3 = 20.000°
(m∠JAC / 3 - m∠BYAC)·1000 = 0.003°
Locus Intercept
m∠CHAC = 10.000°
m∠CGAD = 50.000°
m∠CDCACE = 60.000°
m∠CIDJ = 5.000°

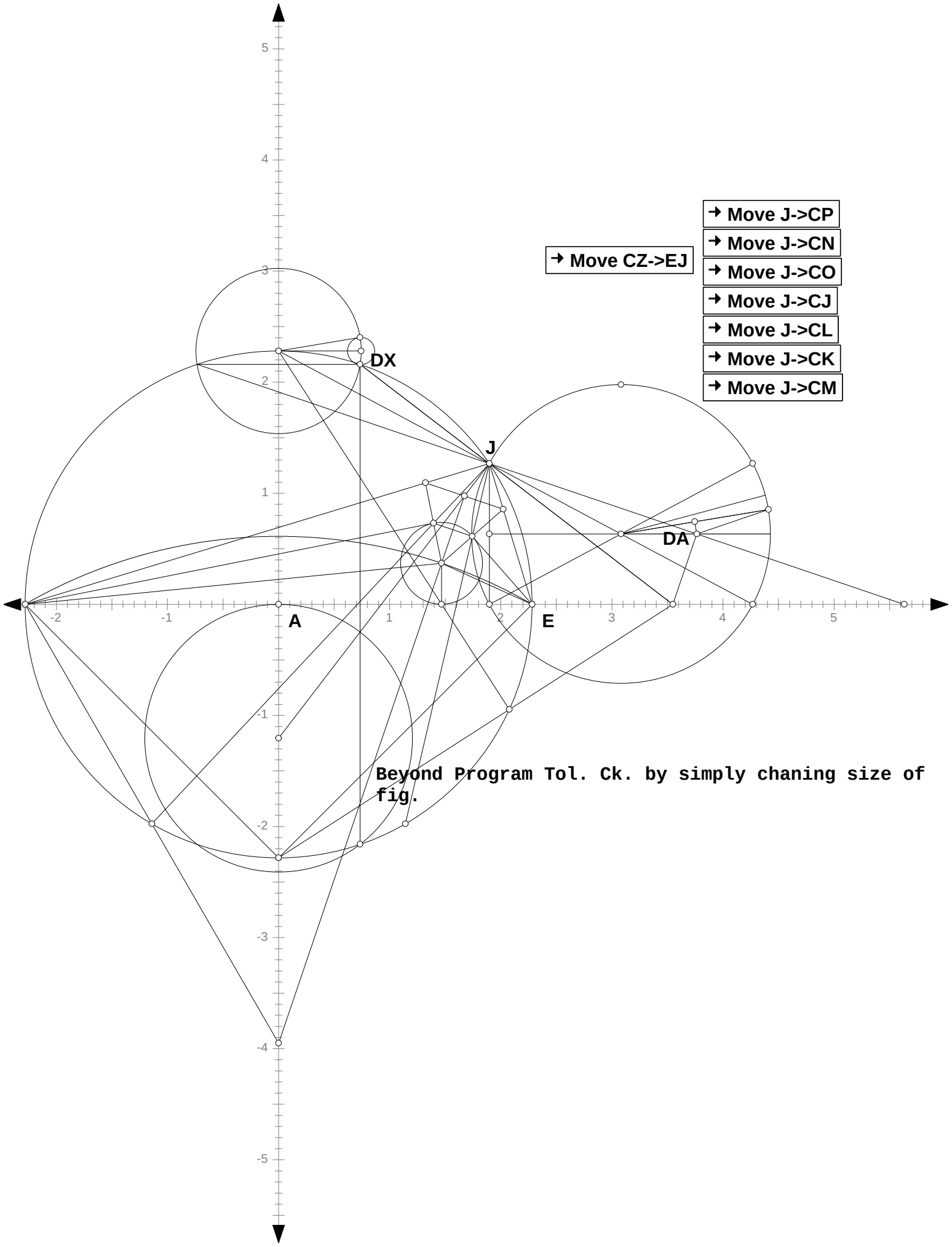

$\frac{m\angle JAC}{m\angle CBAC} \cdot 1000 = 3000.002$
$\frac{m\angle JAC}{m\angle CKAC} \cdot 1000 = 3000.001$
$m\angle JAC = 60.000°$
$\frac{m\angle JAC}{m\angle CBAC} = 3.000$
$m\angle BRAC = 19.998°$
$\frac{m\angle JAC}{m\angle BRAC} = 3.000$
$m\angle CBAC = 20.000°$
$m\angle CKAC = 20.000°$
$(20 - m\angle CKAC) \cdot 1000 = 0.004$
Move J->CC
$m\angle CNCLCM = 60.000°$
$m\angle CNCLCM \cdot 1000 = 59999.982°$

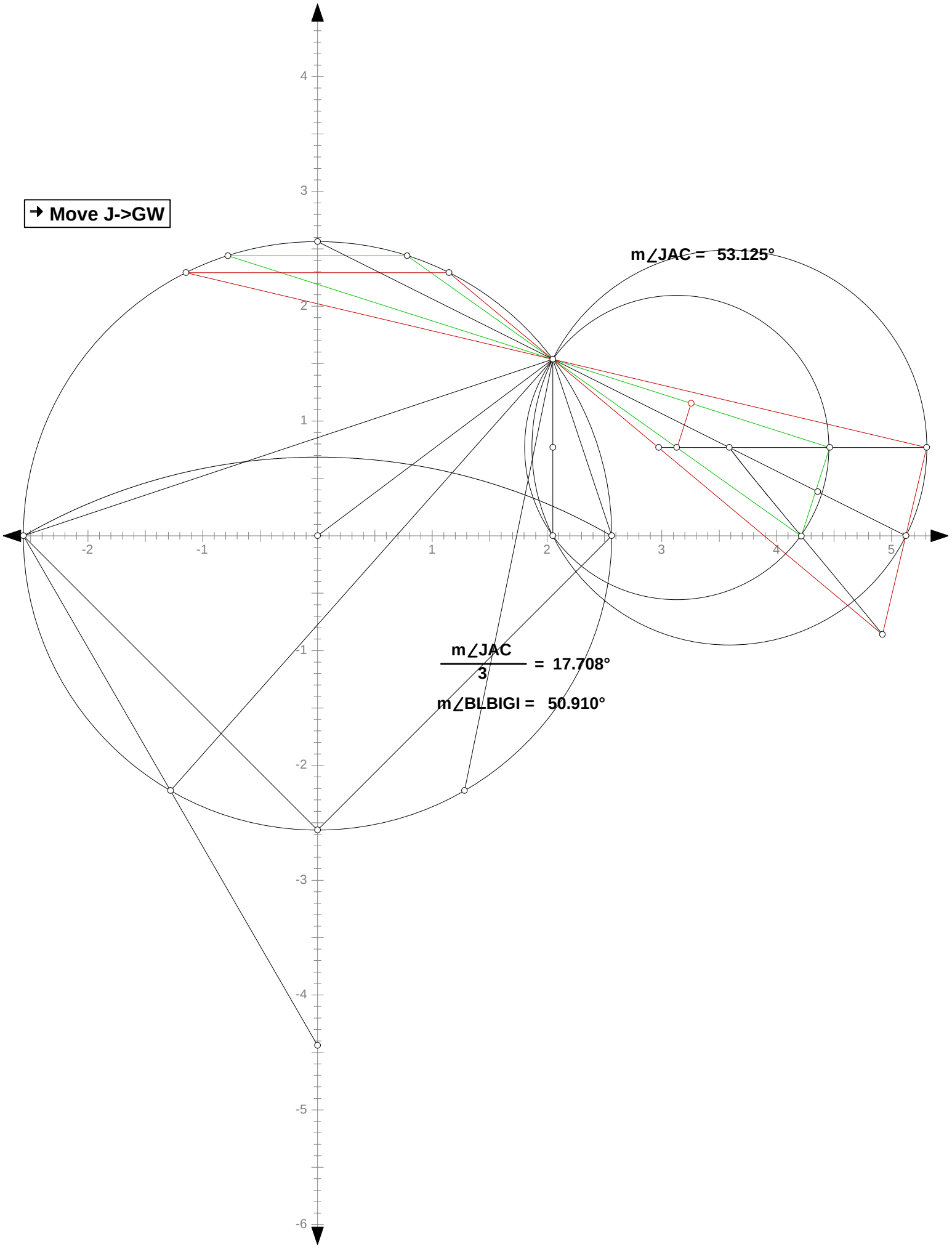

m∠DHDDDG = 60.000°
Move J->DQ
Move J->EA
Locus not so curved
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠DBAC = 20.000°
(m∠JAC / 3 - m∠DBAC) ·1000 = 0.012°
CZCY = 0.439 inches

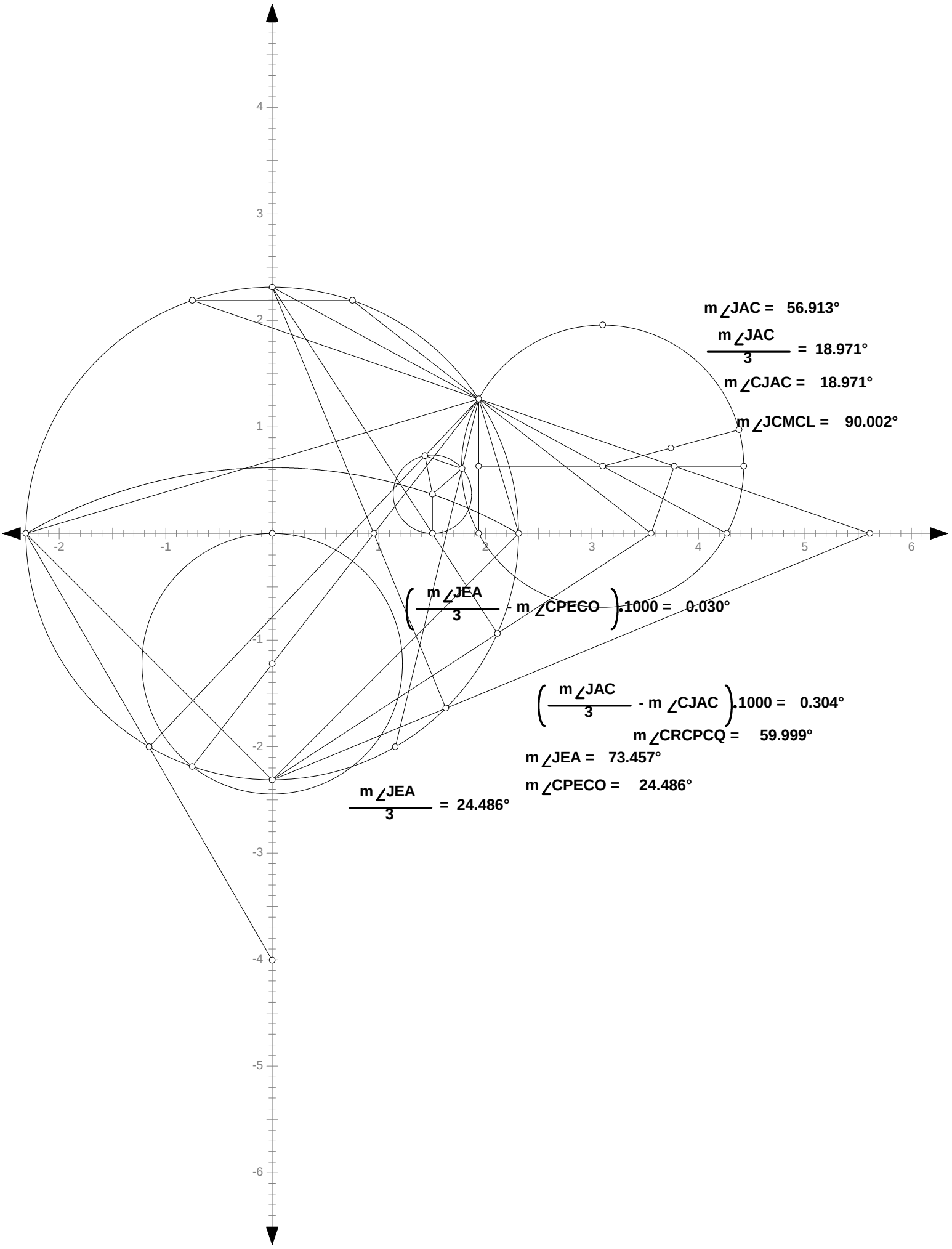

m∠JAC = 59.010°
m∠JAC / 3 = 19.670°
(m∠JAC / 3 - m∠CIAC) ·1000 = -0.005°
m∠JAC / m∠CIAC = 3.000
m∠CNCLCM = 60.000°
BKBL = 1.198 inches
BKCS = 0.608 inches
BKBL / BKCS = 1.971
m∠CUCSBL = 19.671°
m∠CIAC = 19.670°

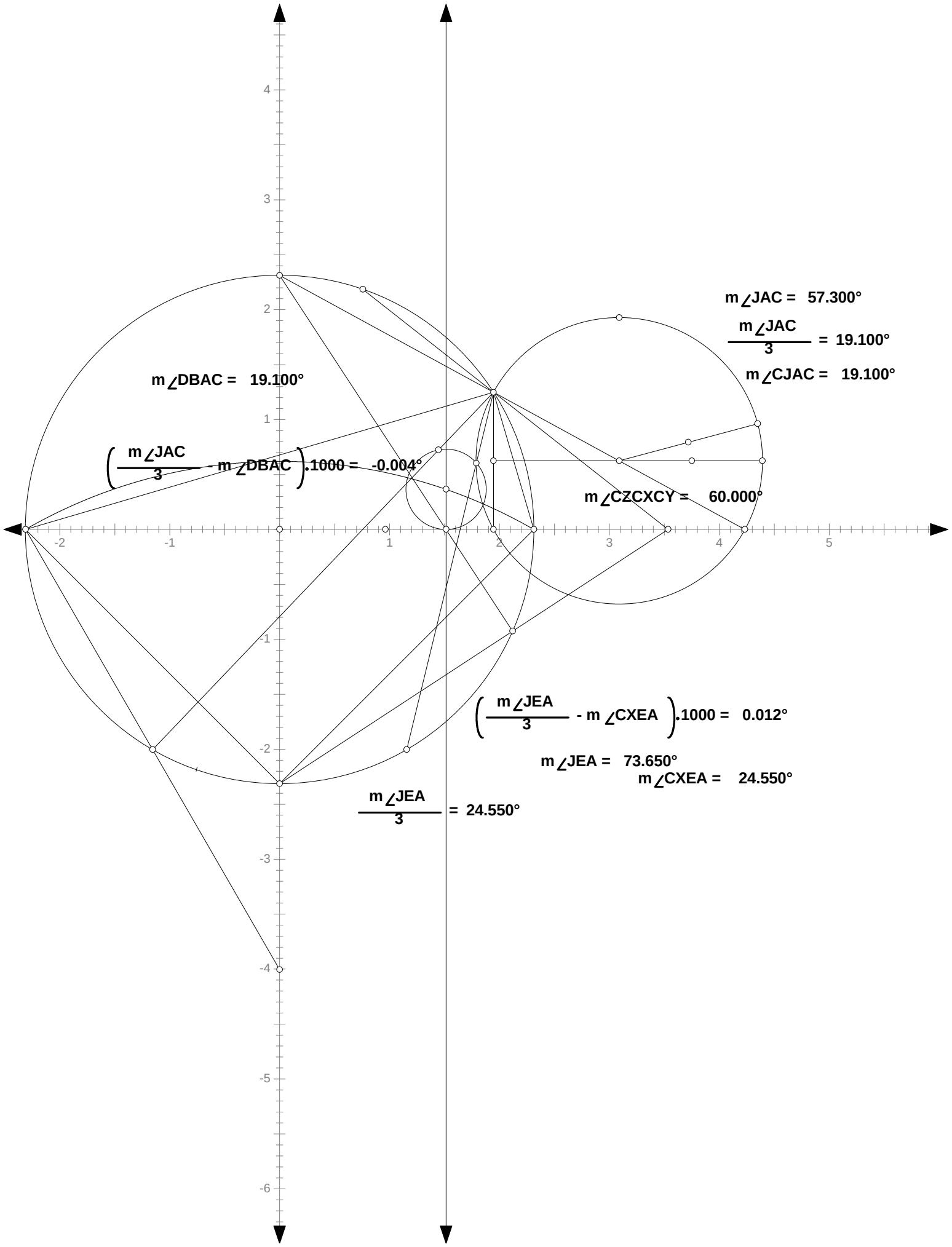

m∠JAC = 60.000°
m∠CGAC = 20.000°
m∠JAC / m∠CGAC = 3.000
m∠CPCKCQ = 60.000°
m∠CKDA = 5.000°
m∠CXCVBN = 20.001°
m∠JAC / m∠CGAC ·1000 = 2999.999
Move J->CR
Move J->CS
full range of possible placement. Next Maxima and minima
Mostly beyond program tol.

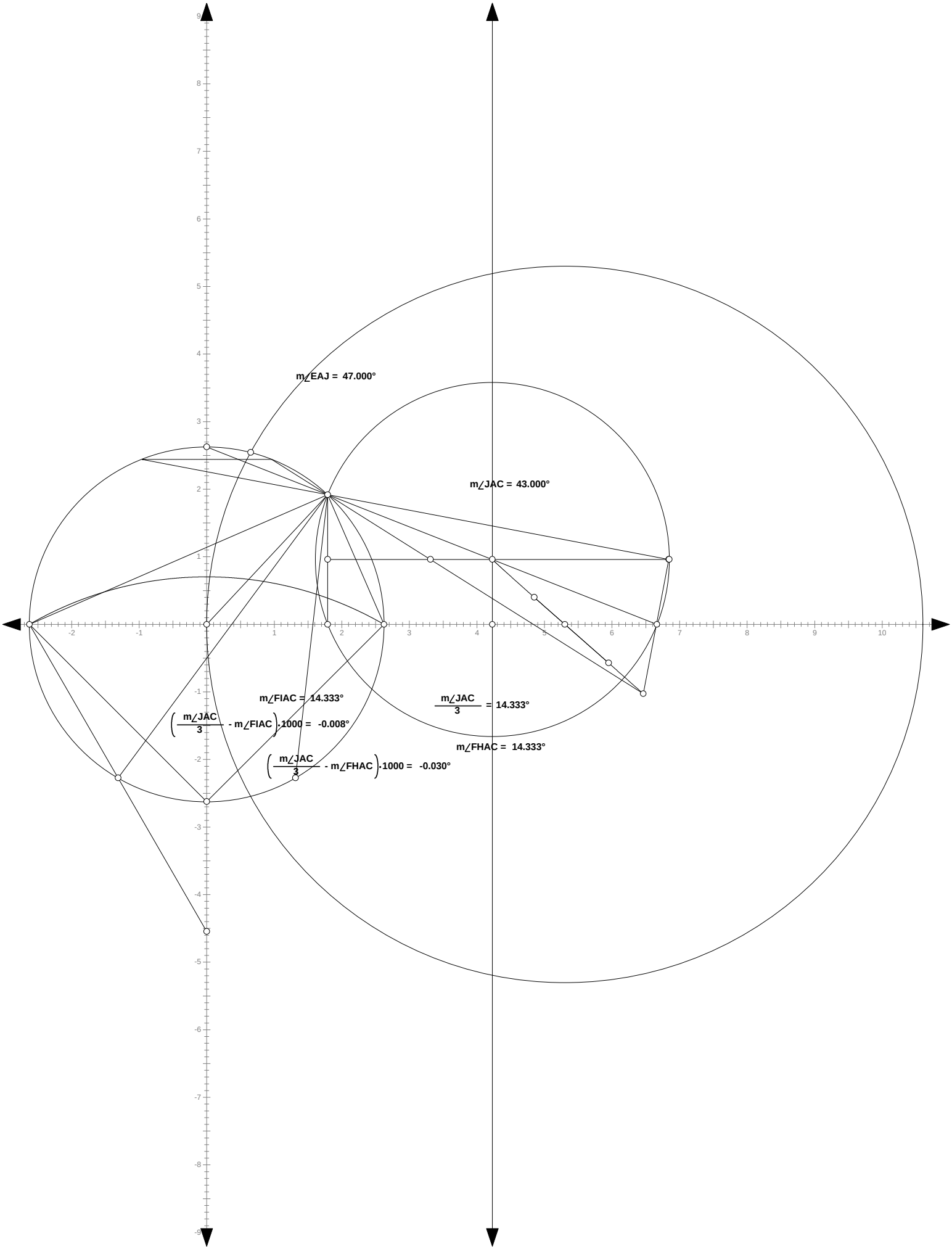

m∠ECEAEB = 60.000°
$\left(\frac{m\angle JAC}{3} - m\angle DXAC\right) \cdot 1000 = -0.022°$
m∠CZBKJ = 148.031°
Move CZ->EJ
Move J->CP
Move J->CN
Move J->CO
Move J->CJ
Move J->CL
Move J->CK
Move J->CM
DX
J
DA
A
E
m∠DDAC = 15.984°
m∠JAC = 47.953°
$\frac{m\angle JAC}{3} = 15.984°$
m∠DXAC = 15.985°

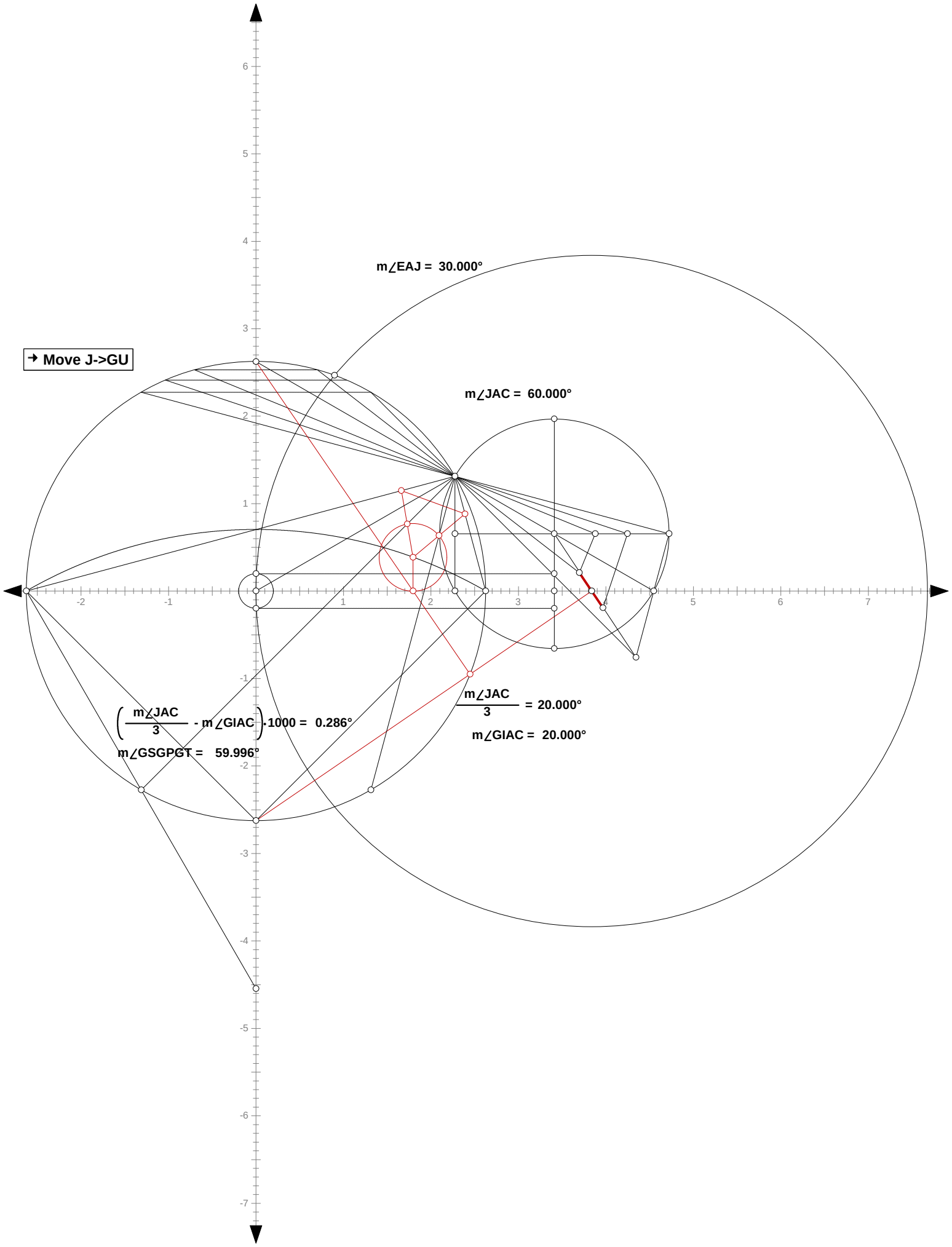

Move J->CP
Move J->CN
Move J->CO
Move J->CJ
Move J->CL
Move J->CK
Move J->CM
Move CZ->EJ
DX
J
DA
A
E
Beyond Program Tol. Ck. by simply chaning size of fig.

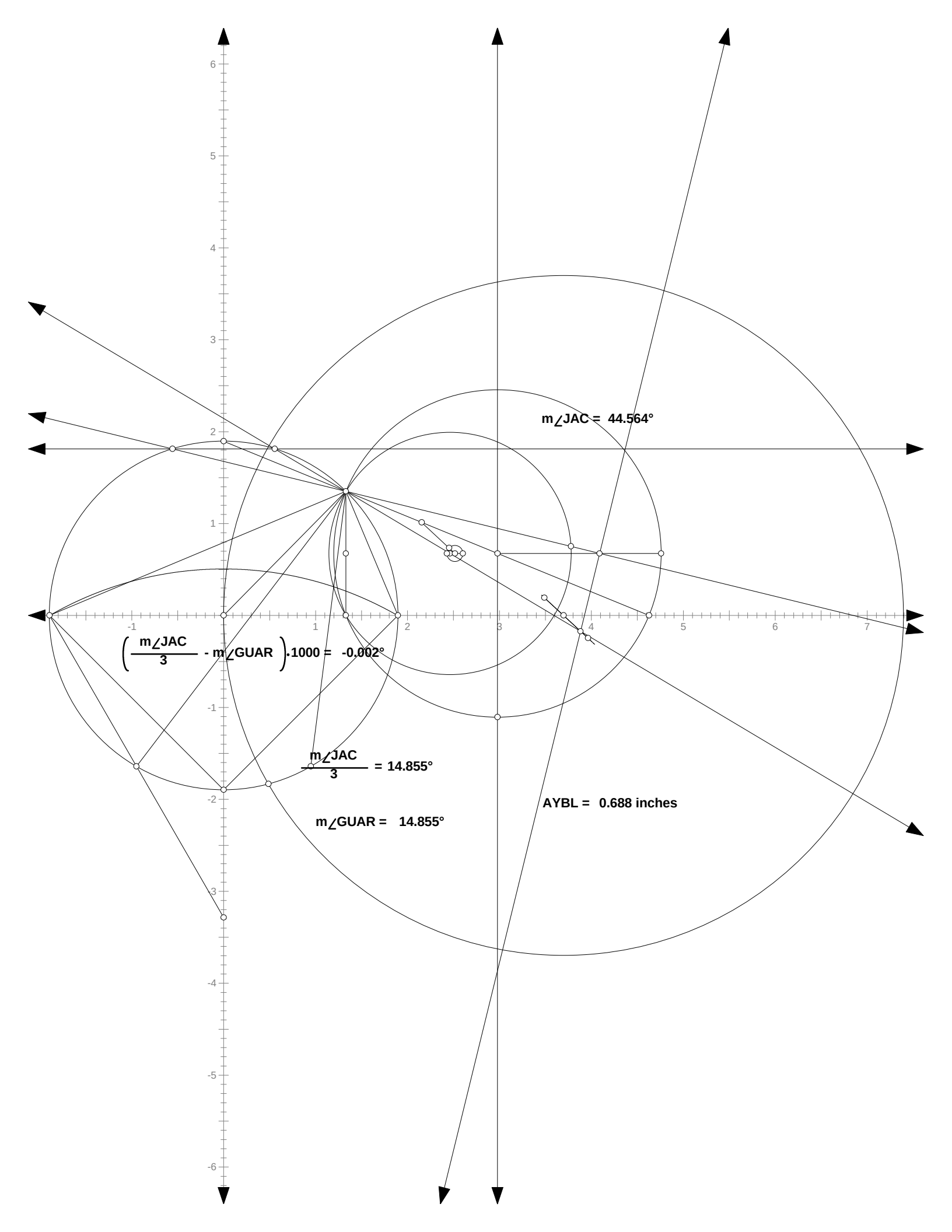

Move J->GW
m∠JAC = 53.125°
m∠JAC / 3 = 17.708°
m∠BLBIGI = 50.910°

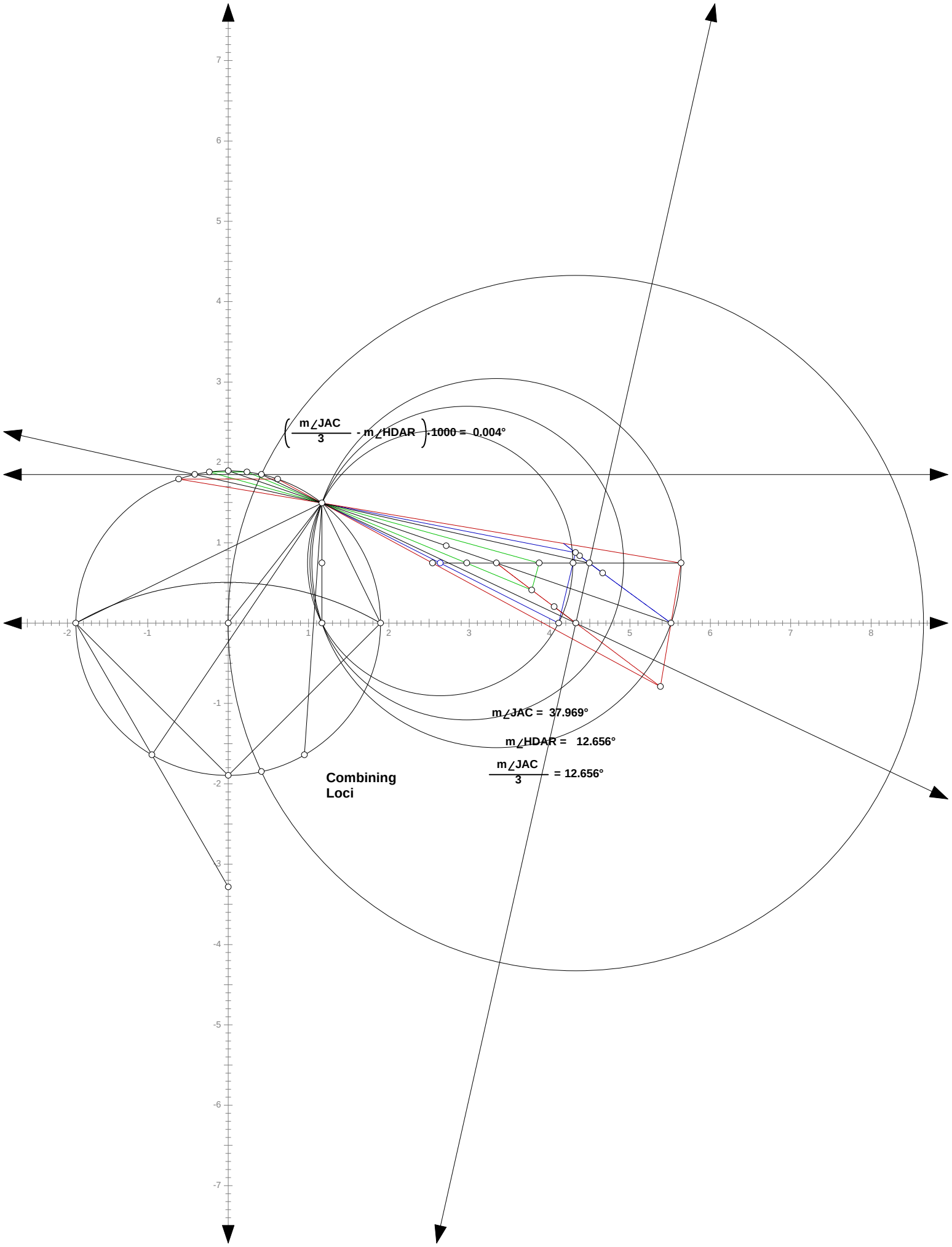

m∠JAC = 56.913°
m∠JAC / 3 = 18.971°
m∠CJAC = 18.971°
m∠JCMCL = 90.002°
(m∠JEA / 3 - m∠CPECO).1000 = 0.030°
(m∠JAC / 3 - m∠CJAC).1000 = 0.304°
m∠CRCPCQ = 59.999°
m∠JEA = 73.457°
m∠CPECO = 24.486°
m∠JEA / 3 = 24.486°


m∠JAC = 57.300°
m∠JAC / 3 = 19.100°
m∠CJAC = 19.100°
m∠DBAC = 19.100°
(m∠JAC / 3 - m∠DBAC).1000 = -0.004°
m∠CZCXCY = 60.000°
(m∠JEA / 3 - m∠CXEA).1000 = 0.012°
m∠JEA = 73.650°
m∠CXEA = 24.550°
m∠JEA / 3 = 24.550°


m∠EAJ = 47.000°
m∠JAC = 43.000°
m∠FIAC = 14.333°
m∠JAC / 3 = 14.333°
(m∠JAC / 3 - m∠FIAC)·1000 = -0.008°
m∠FHAC = 14.333°
(m∠JAC / 3 - m∠FHAC)·1000 = -0.030°


m∠EAJ = 30.000°
Move J->GU
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠GIAC = 20.000°
(m∠JAC / 3 - m∠GIAC) · 1000 = 0.286°
m∠GSGPGT = 59.996°


m∠JAC = 44.564°
$\left(\frac{m\angle JAC}{3} - m\angle GUAR\right) \cdot 1000 = -0.002°$
$\frac{m\angle JAC}{3} = 14.855°$
m∠GUAR = 14.855°
AYBL = 0.688 inches


$\left(\frac{m\angle JAC}{3} - m\angle HDAR\right) \cdot 1000 = 0.004°$
$m\angle JAC = 37.969°$
$m\angle HDAR = 12.656°$
$\frac{m\angle JAC}{3} = 12.656°$
Combining Loci

Since the trisection is perfect, one can examine the program limitations.

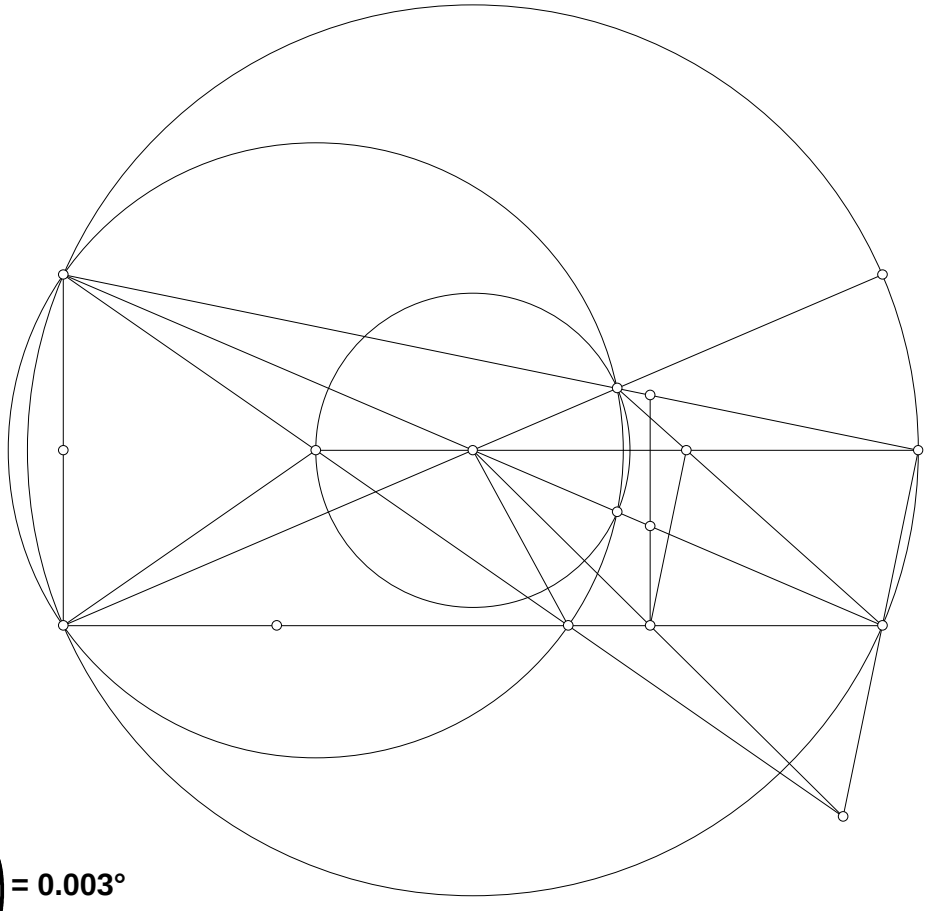
$m\angle UCE = 11.601°$

$\frac{m\angle DUC}{3} = 11.601°$

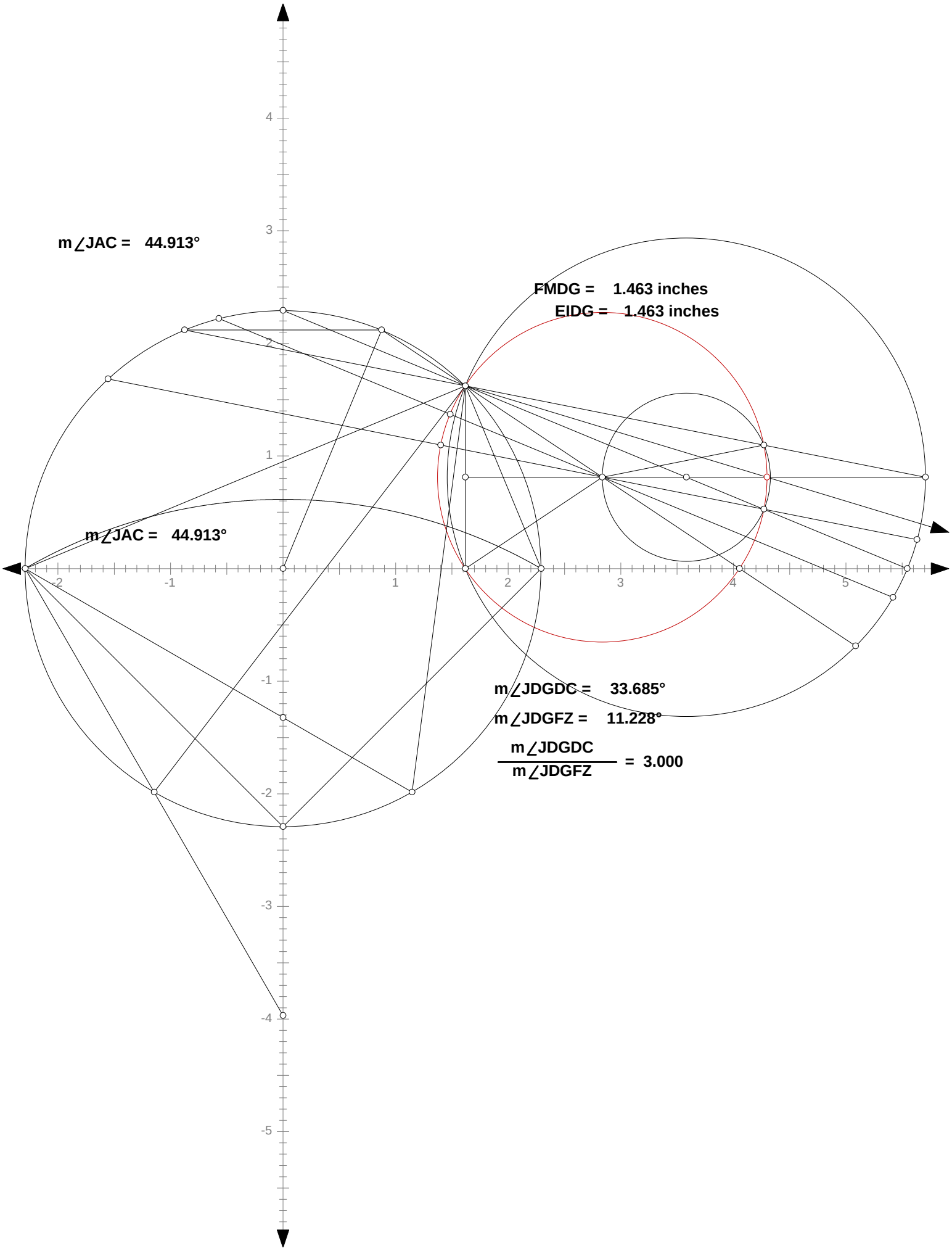
$m\angle ATAMAS = 11.601°$

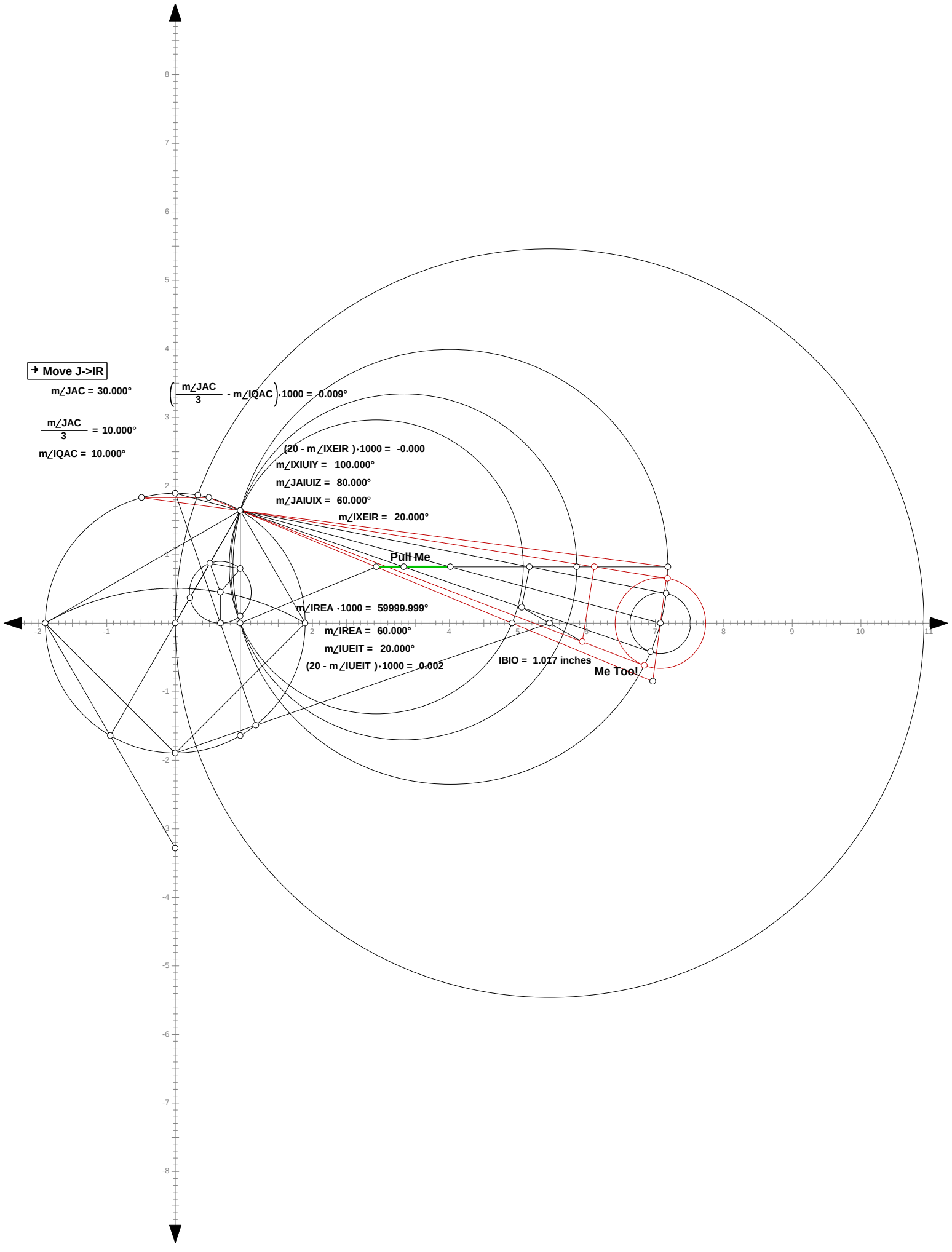
$m\angle DUC = 34.802°$

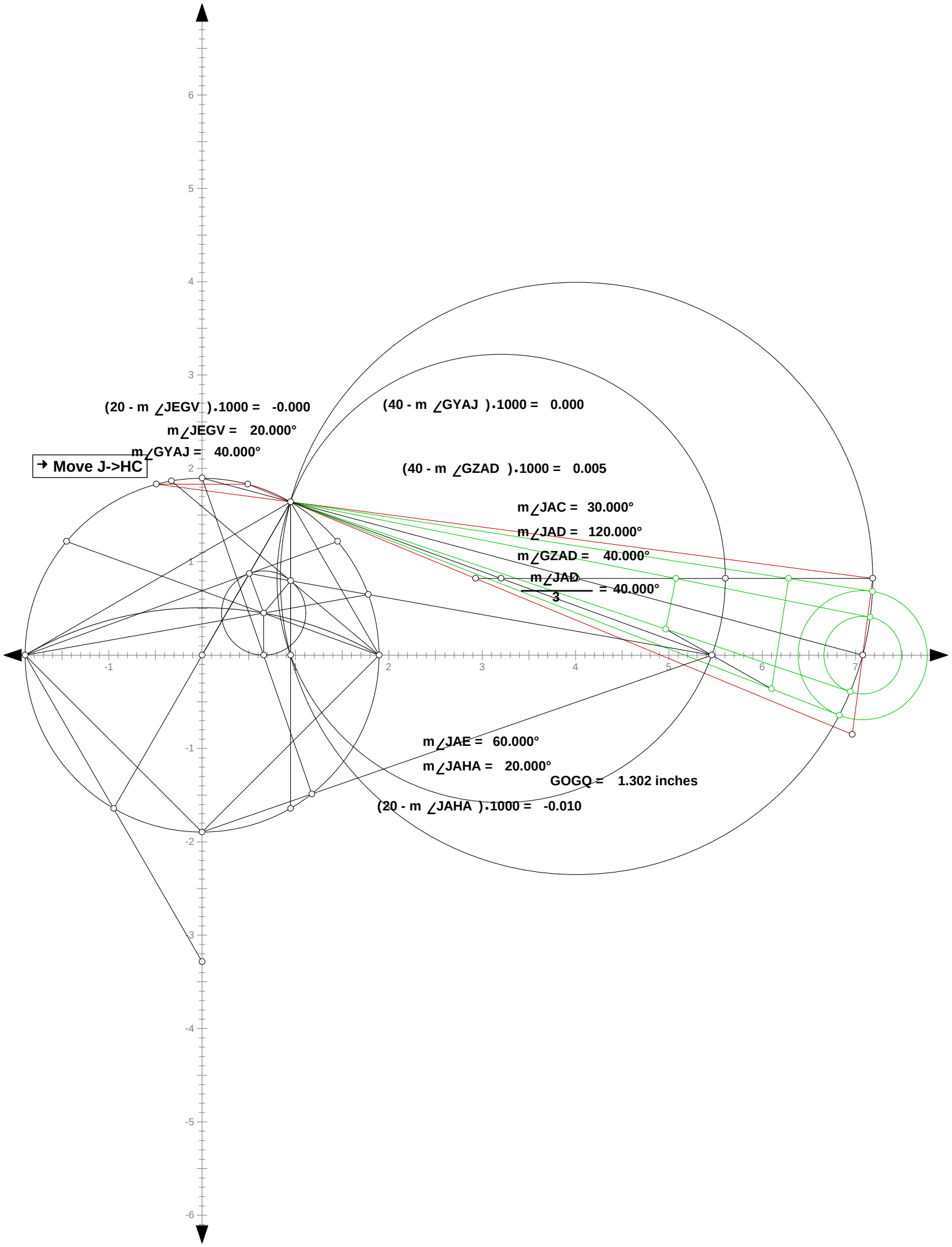
$\frac{m\angle DUC}{m\angle ATAMAS} = 3.000$

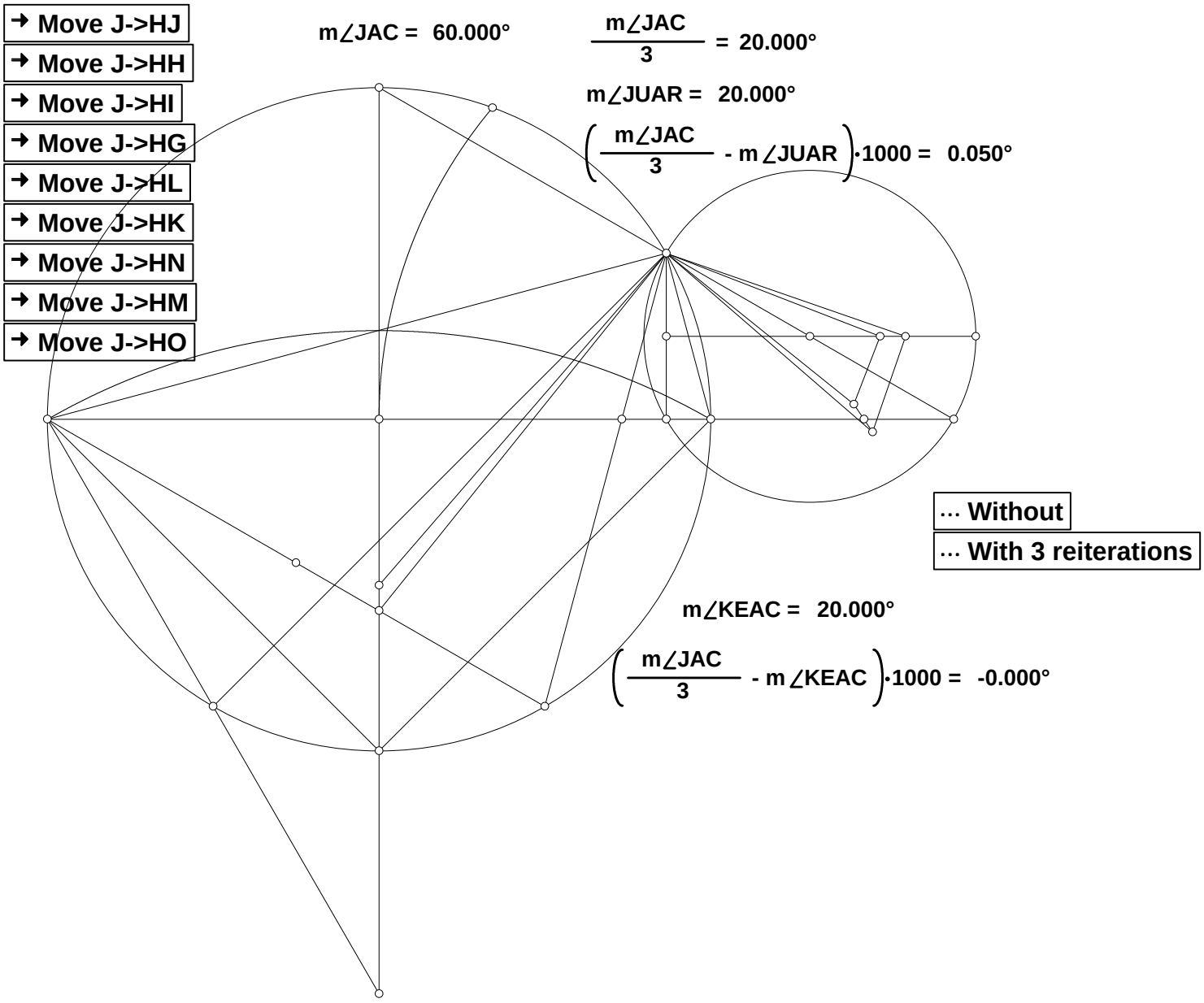


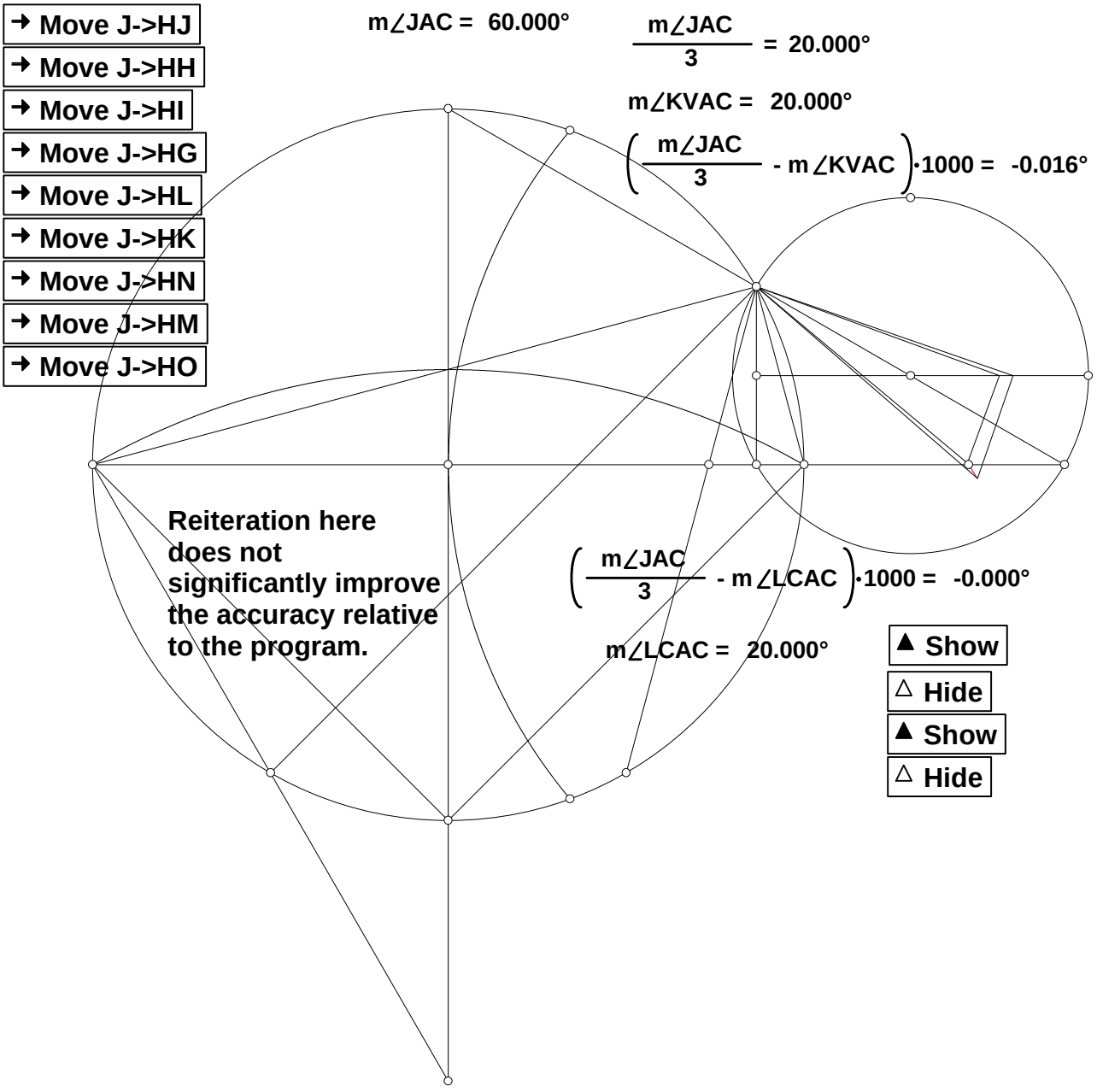
$\left(\left(\frac{m\angle DUC}{3} - m\angle ATAMAS\right).1000\right) = 0.003°$


m∠JAC = 44.913°
FMDG = 1.463 inches
EIDG = 1.463 inches
m∠JAC = 44.913°
m∠JDGDC = 33.685°
m∠JDGFZ = 11.228°
m∠JDGDC / m∠JDGFZ = 3.000


Move J->IR
m∠JAC = 30.000°
m∠JAC / 3 = 10.000°
m∠IQAC = 10.000°
(m∠JAC / 3 - m∠IQAC)·1000 = 0.009°
(20 - m∠IXEIR)·1000 = -0.000
m∠IXIUIY = 100.000°
m∠JAIUIZ = 80.000°
m∠JAIUIX = 60.000°
m∠IXEIR = 20.000°
Pull Me
m∠IREA ·1000 = 59999.999°
m∠IREA = 60.000°
m∠IUEIT = 20.000°
(20 - m∠IUEIT)·1000 = 0.002
IBIO = 1.017 inches
Me Too!


(20 - m ∠JEGV).1000 = -0.000
m∠JEGV = 20.000°
m∠GYAJ = 40.000°
Move J->HC
(40 - m ∠GYAJ).1000 = 0.000
(40 - m ∠GZAD).1000 = 0.005
m∠JAC = 30.000°
m∠JAD = 120.000°
m∠GZAD = 40.000°
m∠JAD / 3 = 40.000°
m∠JAE = 60.000°
m∠JAHA = 20.000°
GOGQ = 1.302 inches
(20 - m ∠JAHA).1000 = -0.010


→ Move J->HJ
→ Move J->HH
→ Move J->HI
→ Move J->HG
→ Move J->HL
→ Move J->HK
→ Move J->HN
→ Move J->HM
→ Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠JUAR = 20.000°
(m∠JAC / 3 - m∠JUAR)·1000 = 0.050°
··· Without
··· With 3 reiterations
m∠KEAC = 20.000°
(m∠JAC / 3 - m∠KEAC)·1000 = -0.000°

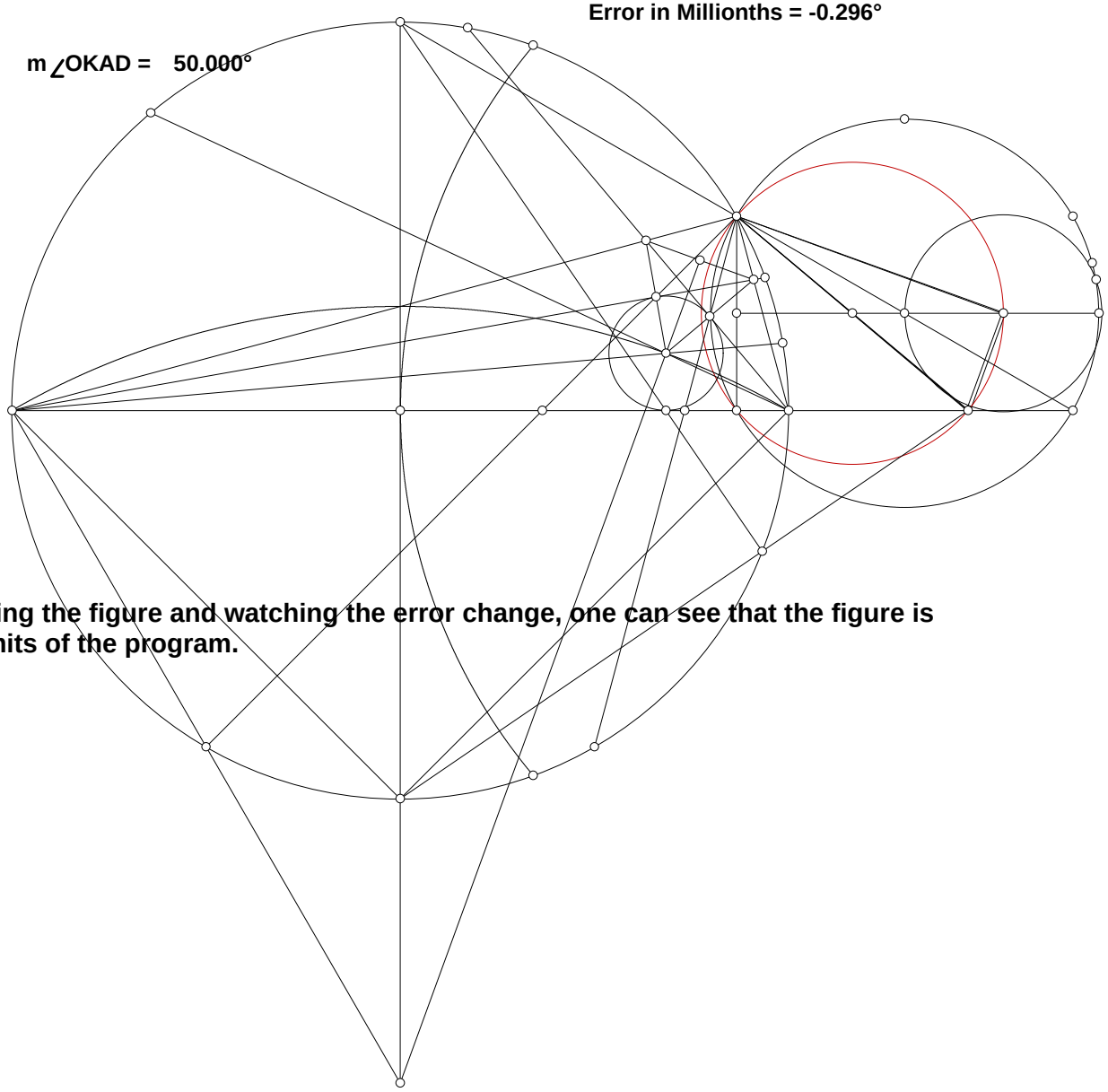

➜ Move J->HJ
➜ Move J->HH
➜ Move J->HI
➜ Move J->HG
➜ Move J->HL
➜ Move J->HK
➜ Move J->HN
➜ Move J->HM
➜ Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠KVAC = 20.000°
(m∠JAC / 3 - m∠KVAC)·1000 = -0.016°
Reiteration here does not significantly improve the accuracy relative to the program.
(m∠JAC / 3 - m∠LCAC)·1000 = -0.000°
m∠LCAC = 20.000°
▲ Show
△ Hide
▲ Show
△ Hide

- Move J->HJ
- Move J->HH
- Move J->HI
- Move J->HG
- Move J->HL
- Move J->HK
- Move J->HN
- Move J->HM
- Move J->HO

$m\angle JAC = 60.000°$ $\quad \frac{m\angle JAC}{3} = 20.000°$ $\quad m\angle NYAC = 20.000°$

Error in Millionths = -0.296°

$m\angle OKAD = 50.000°$



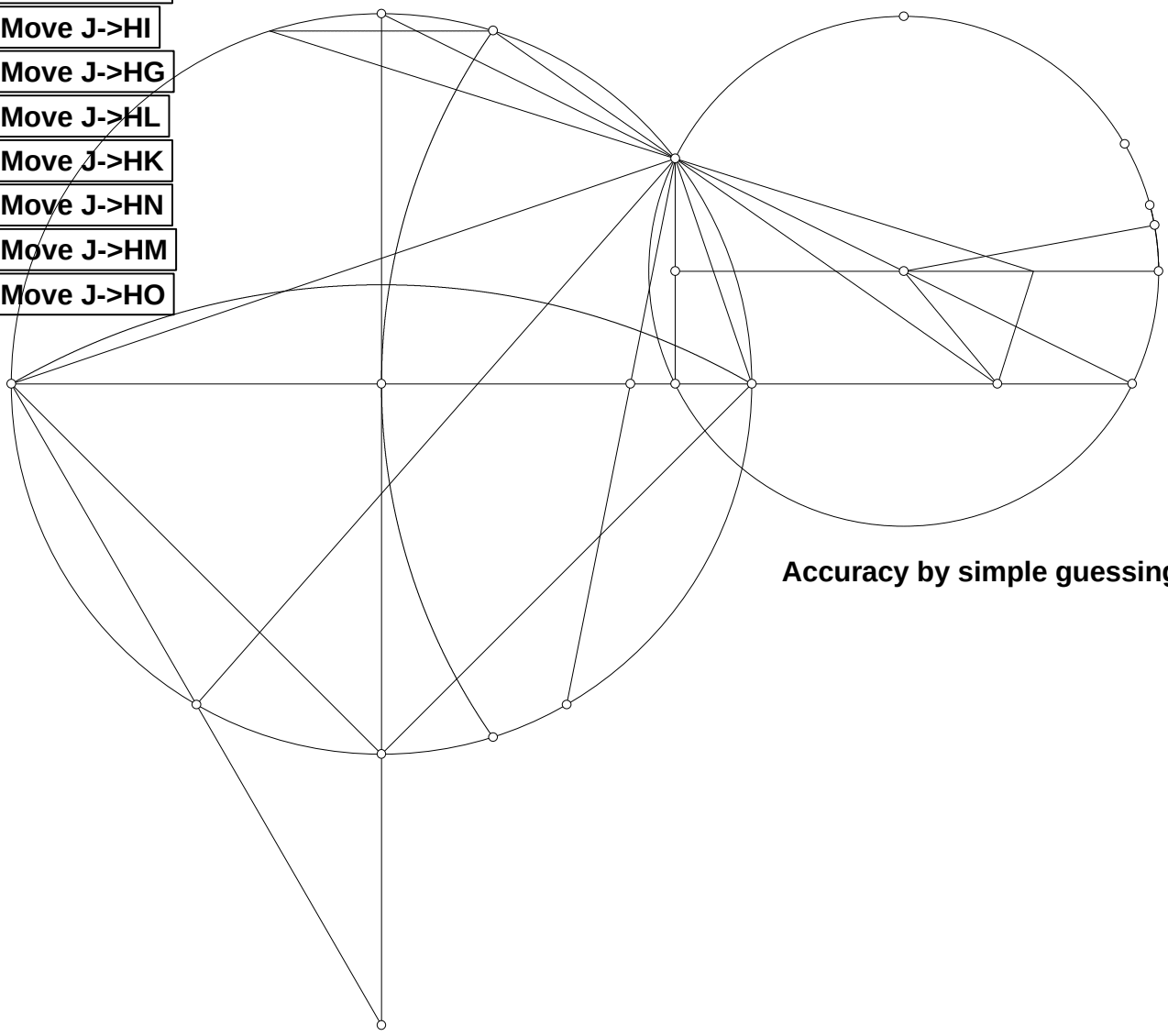
By proportioning the figure and watching the error change, one can see that the figure is beyond the limits of the program.

$m\angle JGAC = 17.500°$

$m\angle JAC = 52.500°$

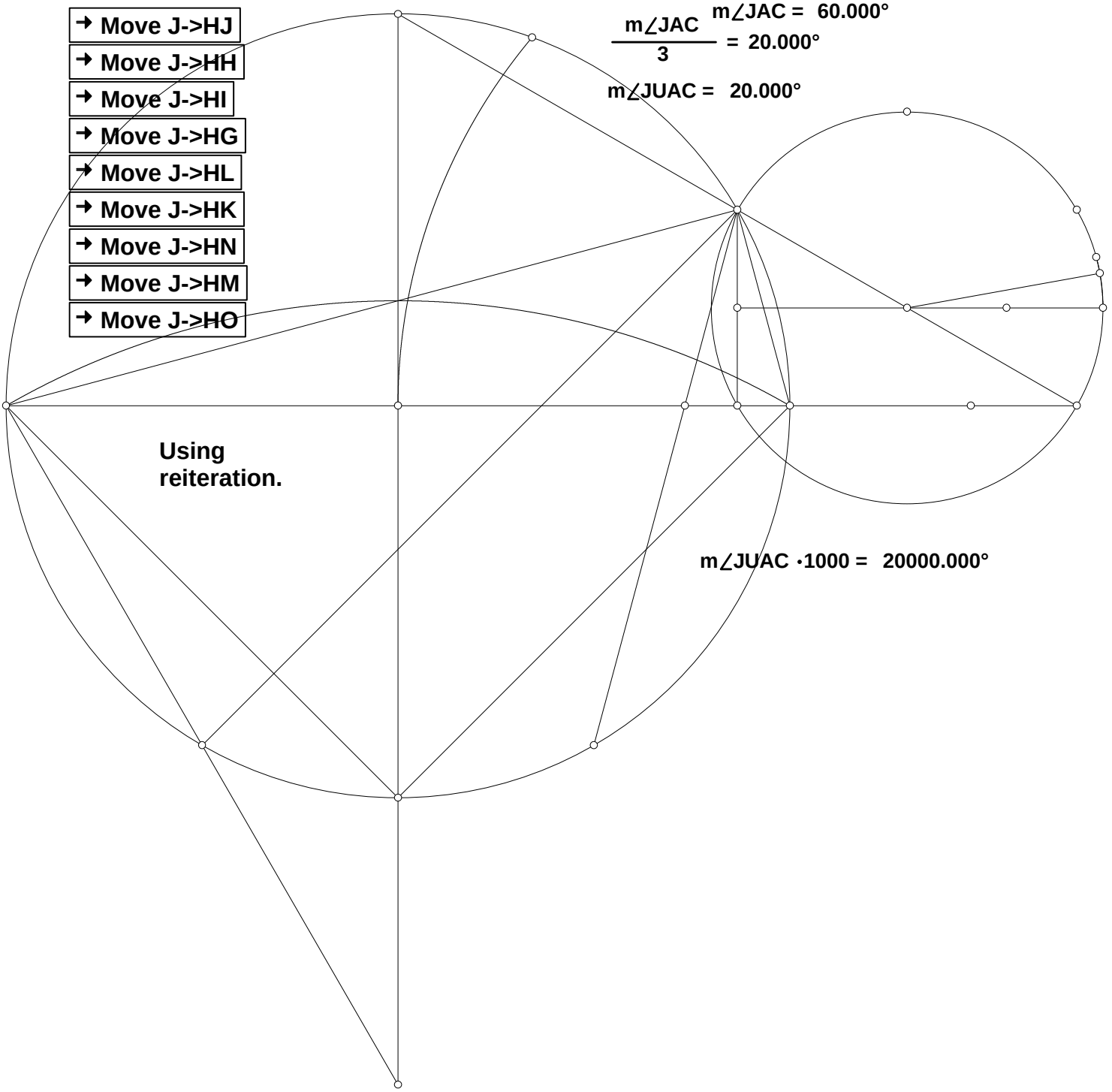
$\frac{m\angle JAC}{3} = 17.500°$

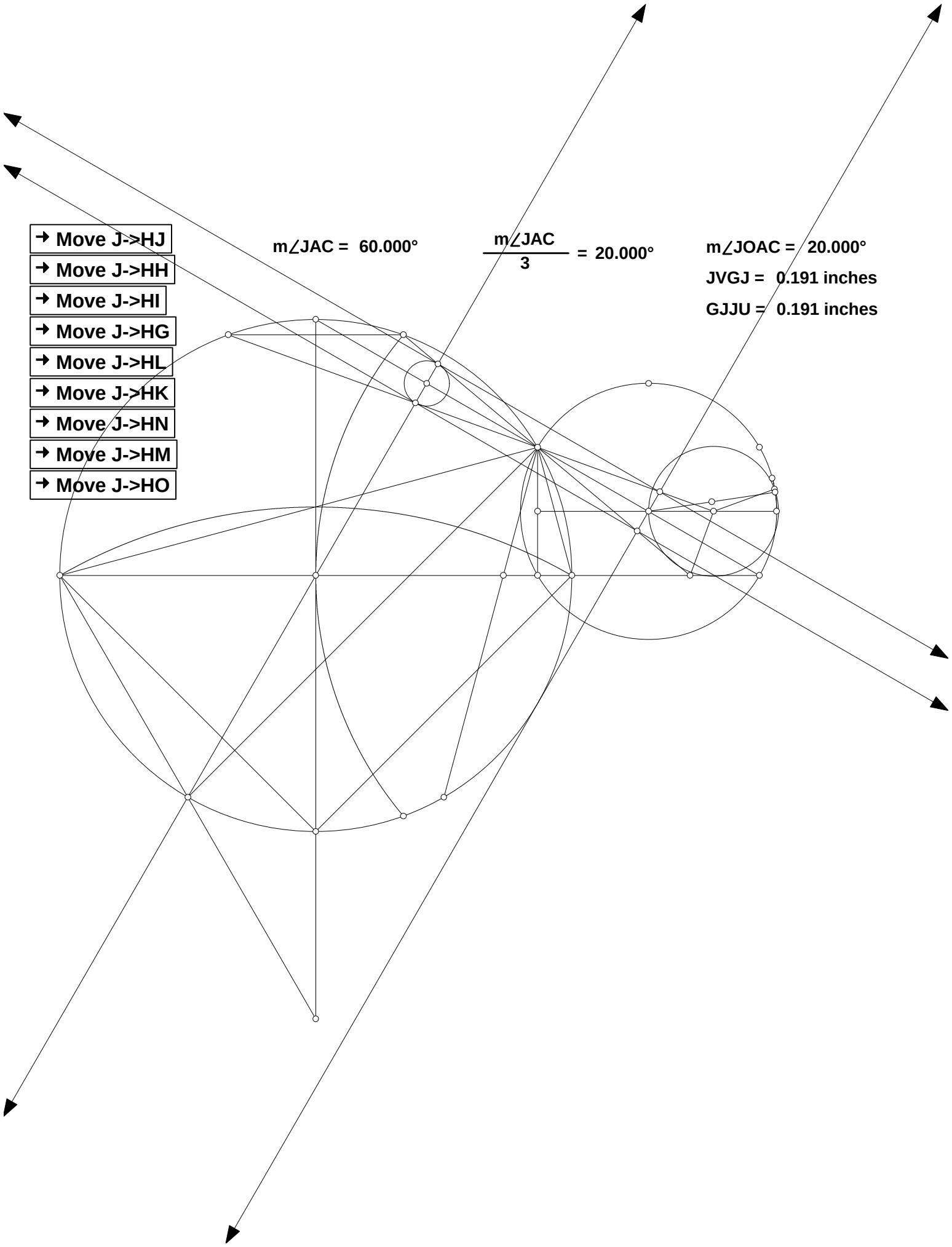
- Move J->HJ
- Move J->HH
- Move J->HI
- Move J->HG
- Move J->HL
- Move J->HK
- Move J->HN
- Move J->HM
- Move J->HO



Guess Me

Accuracy by simple guessing.


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠JUAC = 20.000°
Using reiteration.
m∠JUAC ·1000 = 20000.000°

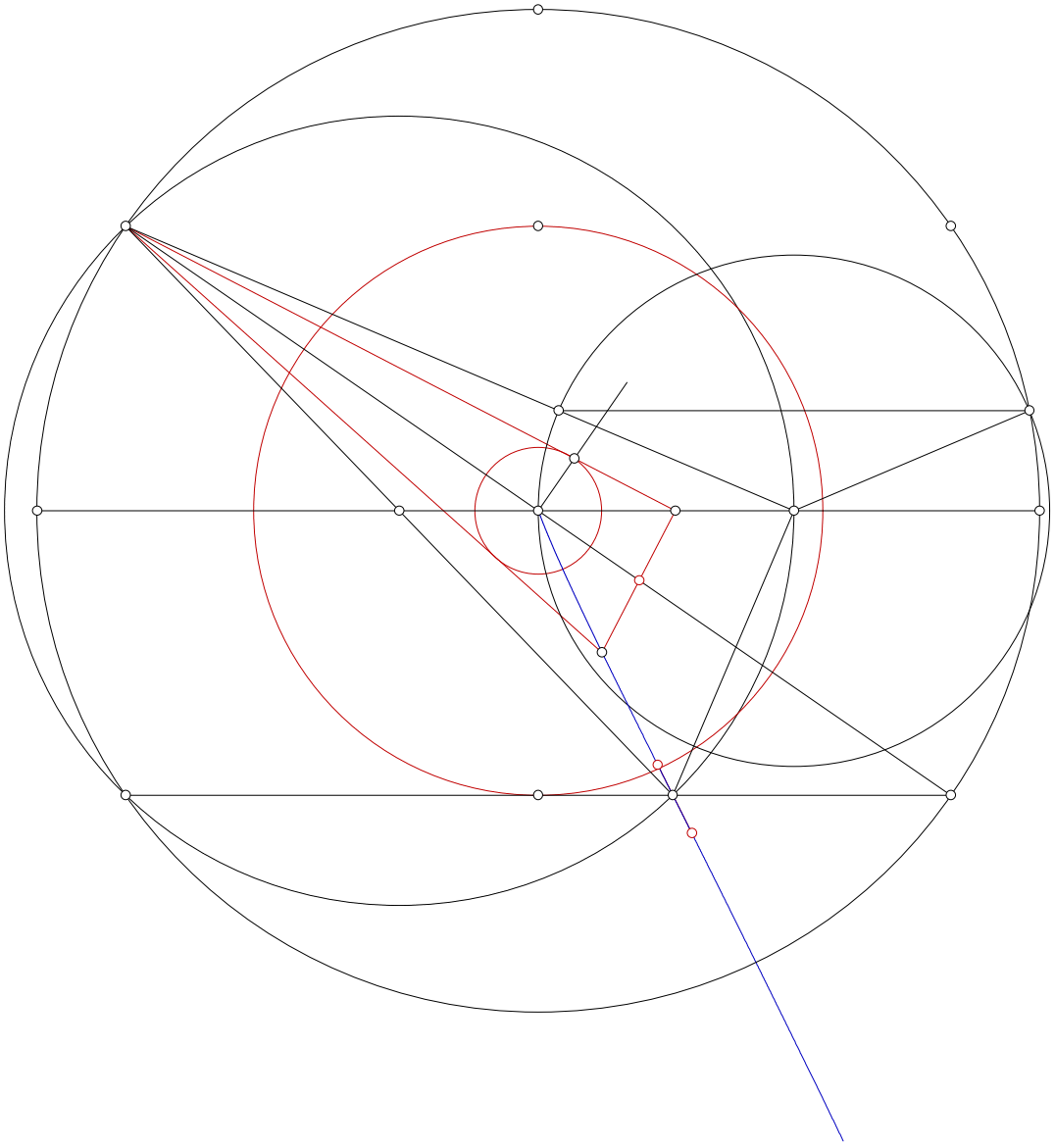

Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠JOAC = 20.000°
JVGJ = 0.191 inches
GJJU = 0.191 inches

$$\frac{m\angle HAG}{m\angle AGHAC} = 3.000$$

$m\angle HAG = 69.181°$

$m\angle HAGAC = 90.000°$

$m\angle AGHAC = 23.060°$

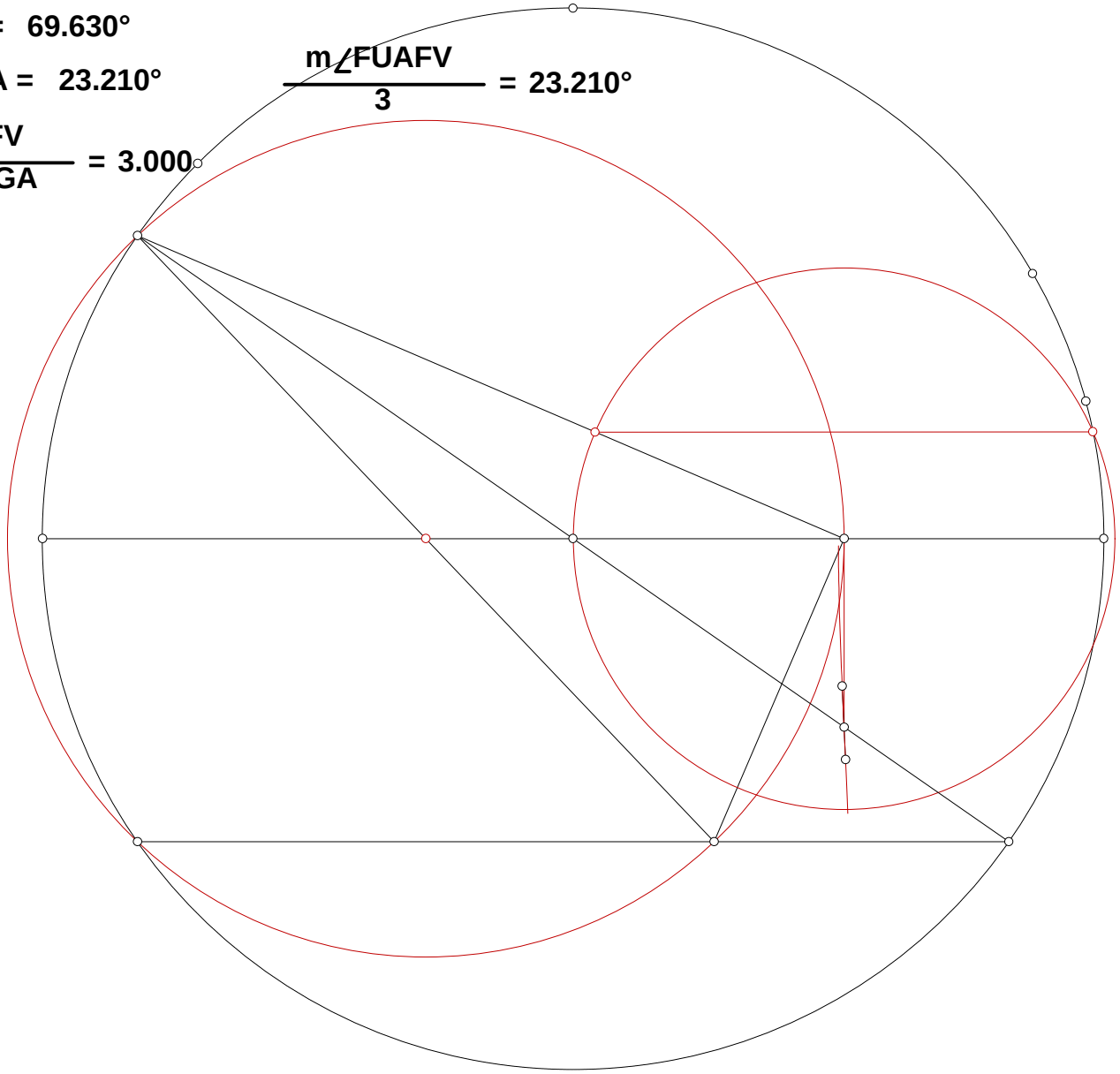


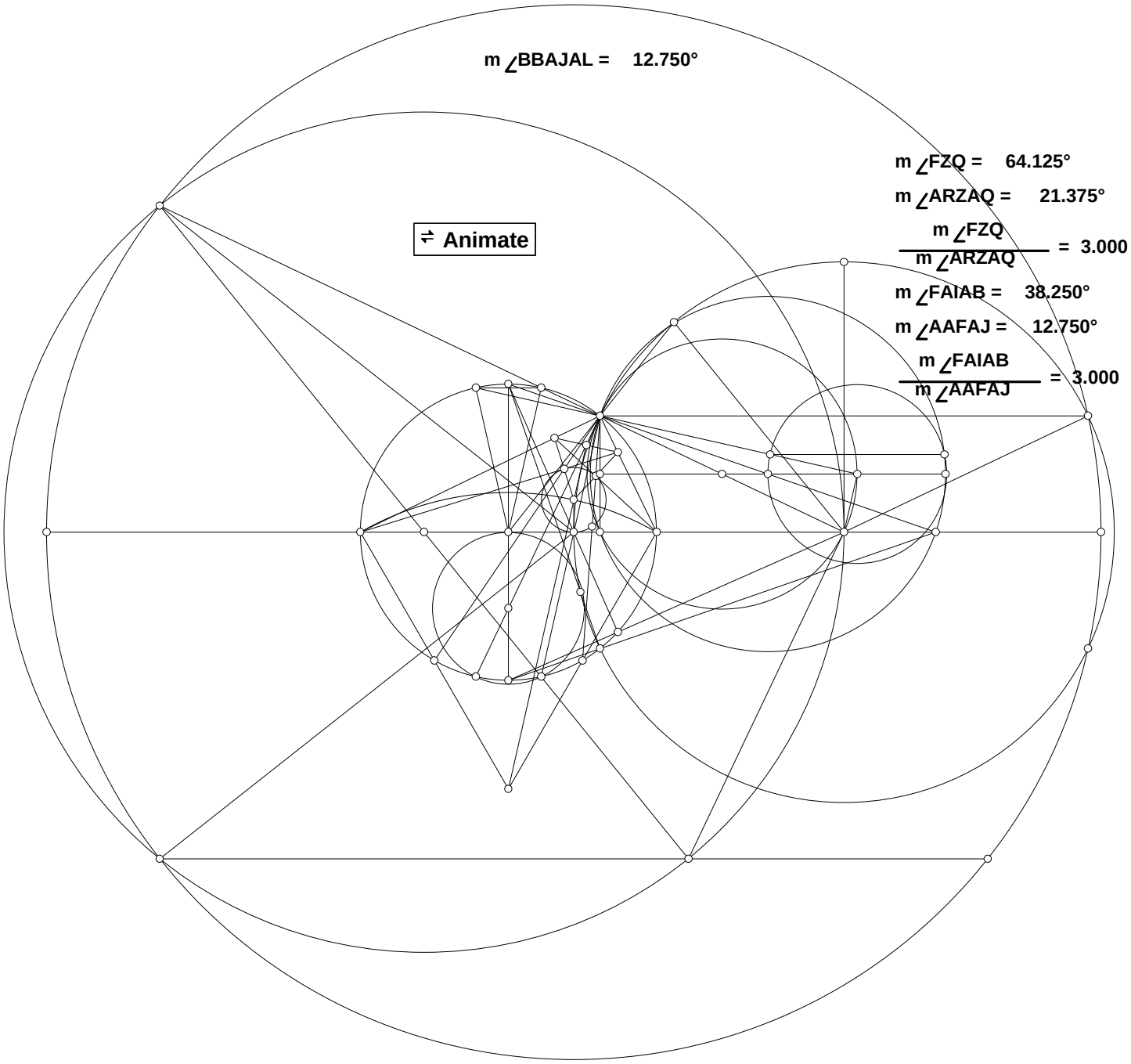
$m\angle FUAFV = 69.630°$

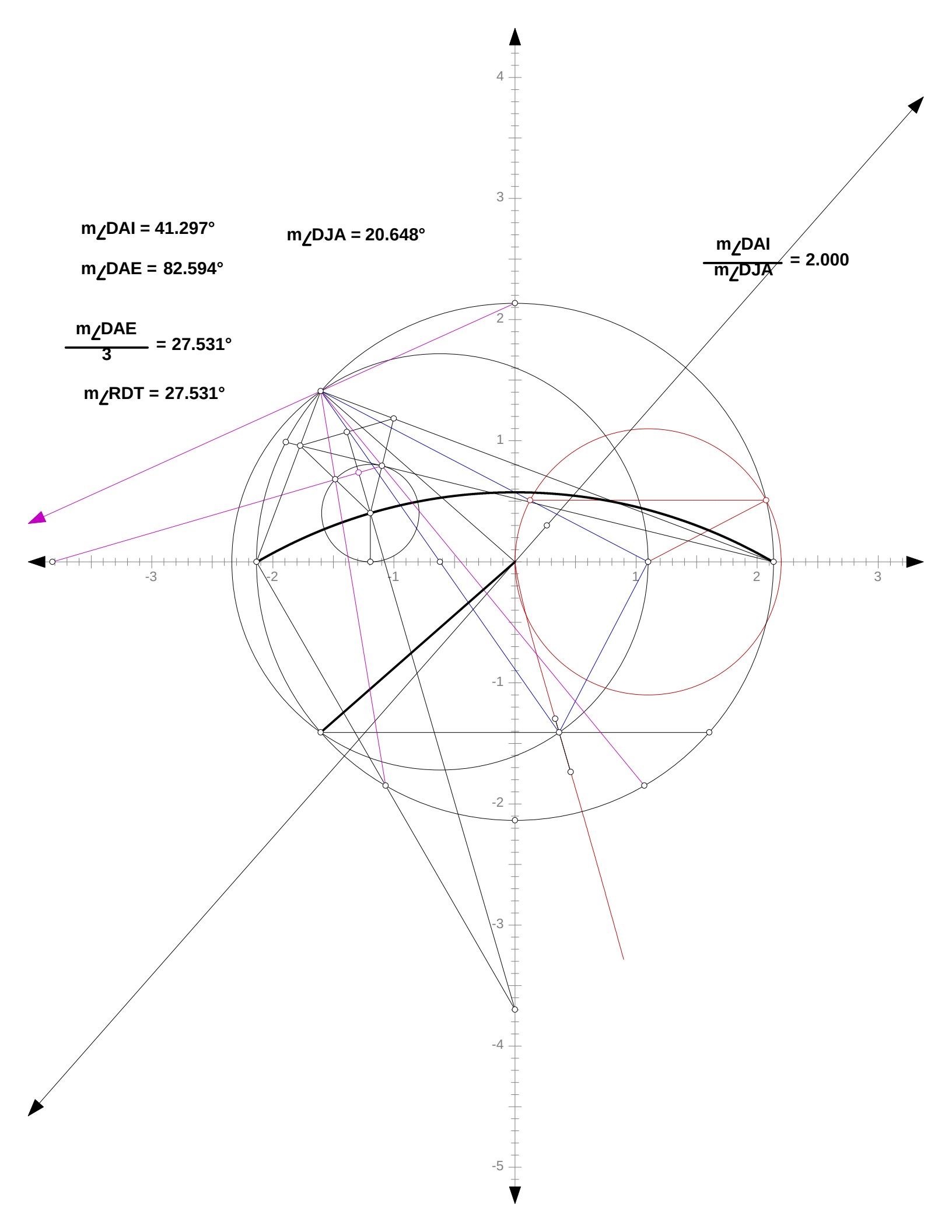
$m\angle GBFUGA = 23.210°$

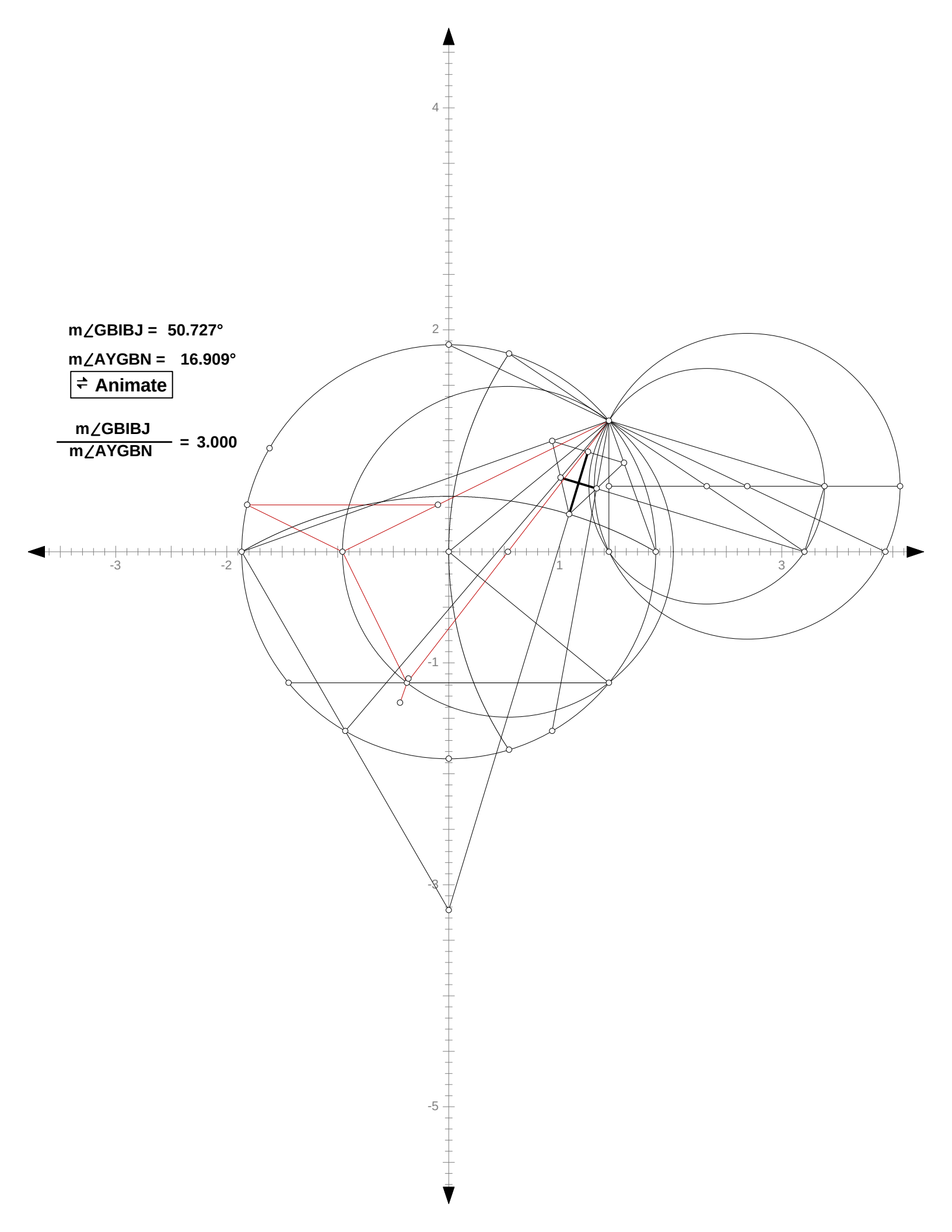
$\frac{m\angle FUAFV}{m\angle GBFUGA} = 3.000$

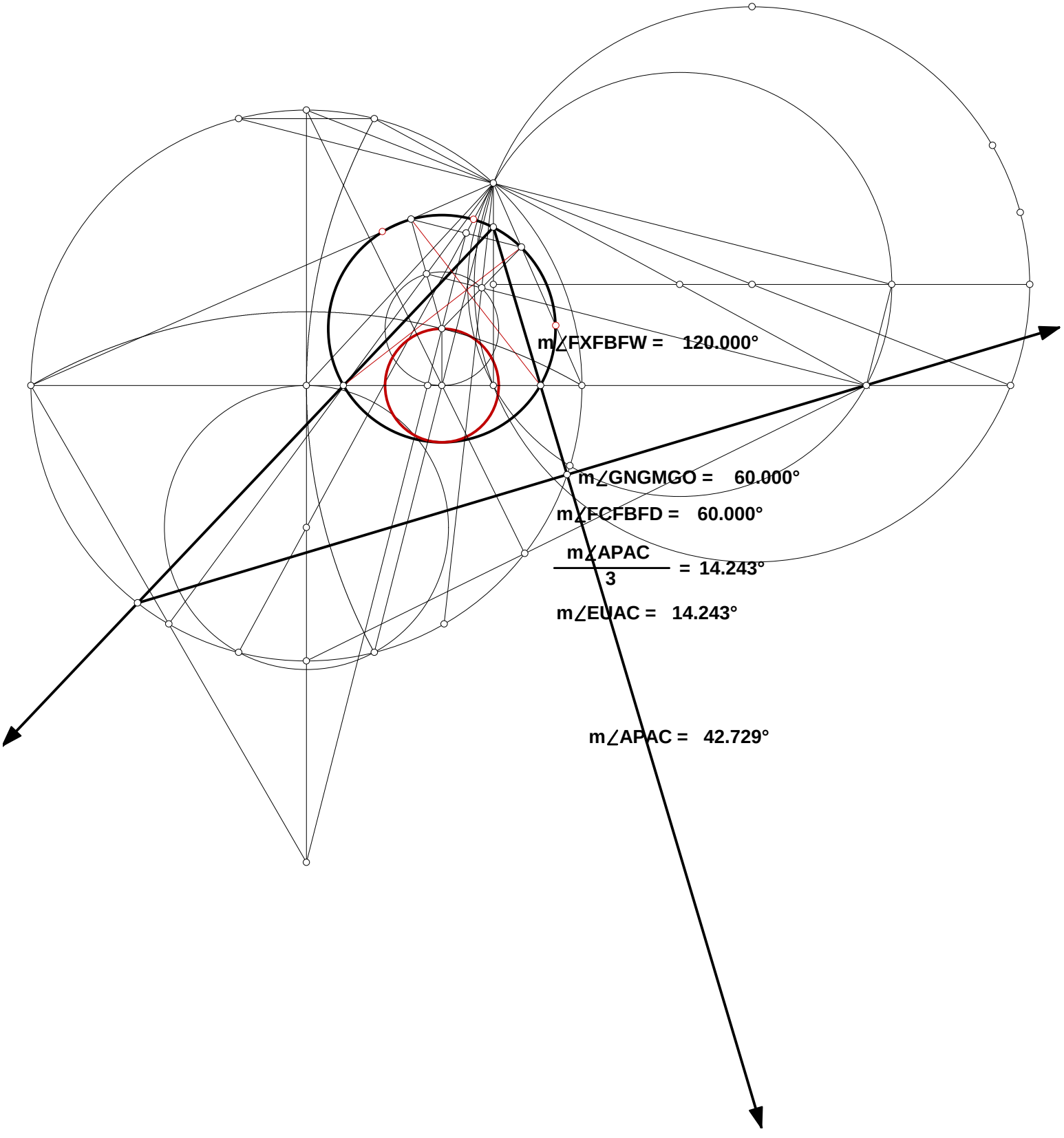
$\frac{m\angle FUAFV}{3} = 23.210°$

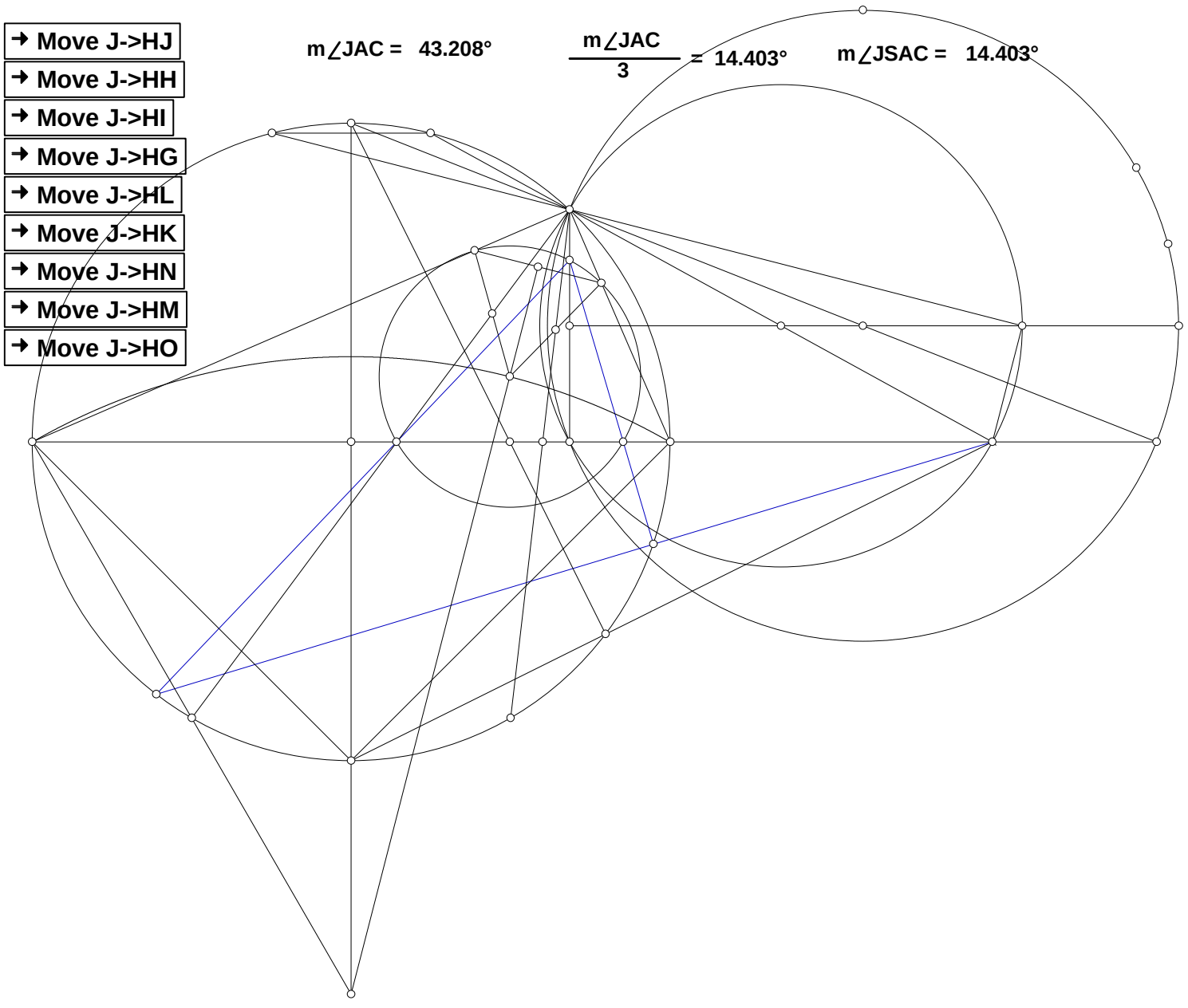


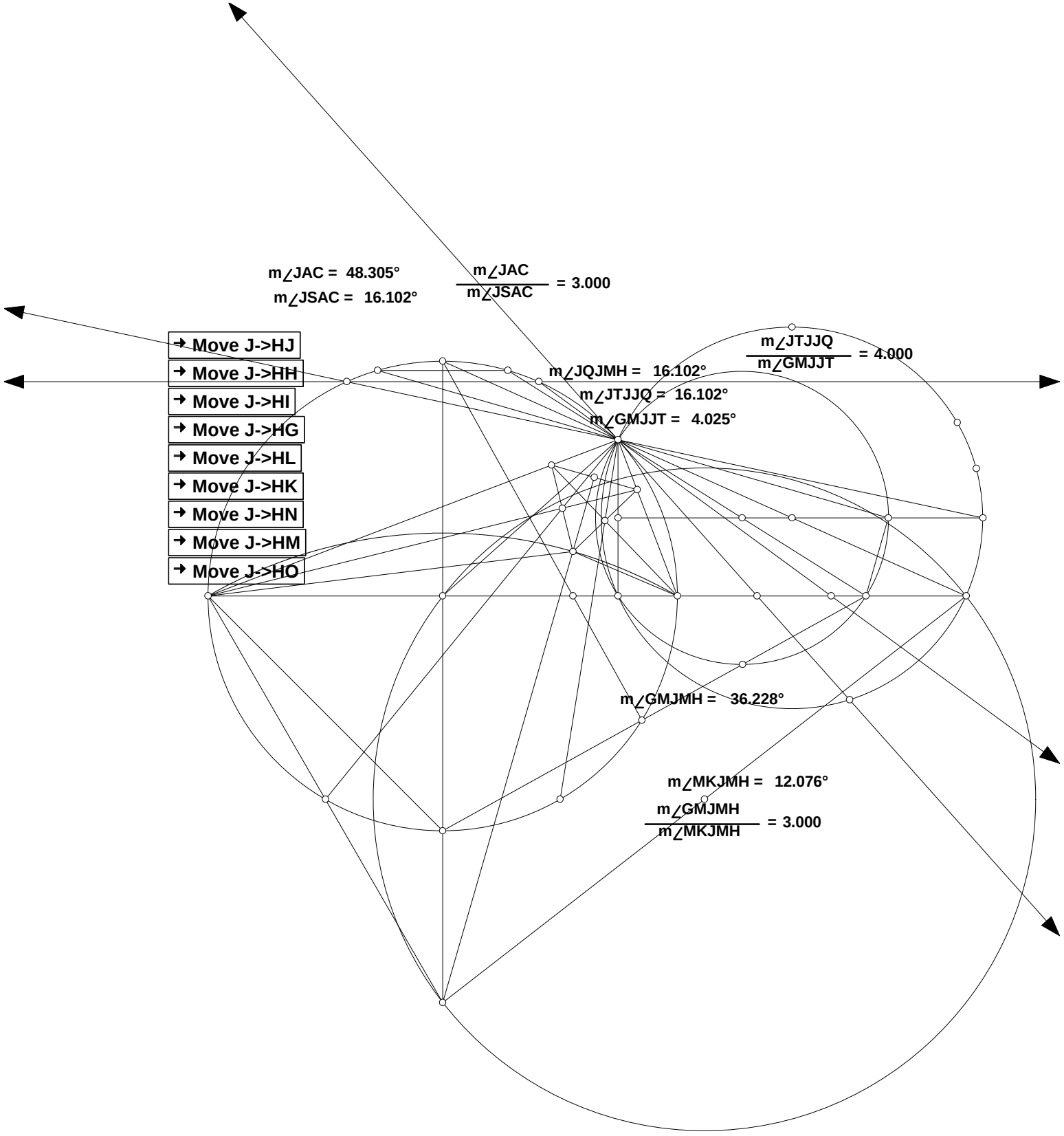

m ∠BBAJAL = 12.750°
m ∠FZQ = 64.125°
m ∠ARZAQ = 21.375°
m ∠FZQ / m ∠ARZAQ = 3.000
m ∠FAIAB = 38.250°
m ∠AAFAJ = 12.750°
m ∠FAIAB / m ∠AAFAJ = 3.000
Animate


m∠DAI = 41.297°
m∠DJA = 20.648°
m∠DAE = 82.594°
m∠DAE / 3 = 27.531°
m∠RDT = 27.531°
m∠DAI / m∠DJA = 2.000
4
3
2
1
-1
-2
-3
-4
-5
-3
-2
-1
1
2
3


m∠GBIBJ = 50.727°
m∠AYGBN = 16.909°
⇄ Animate
m∠GBIBJ / m∠AYGBN = 3.000
4
2
-1
-3
-5
-3
-2
1
3


m∠FXFBFW = 120.000°
m∠GNGMGO = 60.000°
m∠FCFBFD = 60.000°
m∠APAC / 3 = 14.243°
m∠EUAC = 14.243°
m∠APAC = 42.729°


→ Move J->HJ
→ Move J->HH
→ Move J->HI
→ Move J->HG
→ Move J->HL
→ Move J->HK
→ Move J->HN
→ Move J->HM
→ Move J->HO
m∠JAC = 43.208°
m∠JAC / 3 = 14.403°
m∠JSAC = 14.403°


m∠JAC = 48.305°
m∠JSAC = 16.102°
m∠JAC / m∠JSAC = 3.000
m∠JTJJQ / m∠GMJJT = 4.000
m∠JQJMH = 16.102°
m∠JTJJQ = 16.102°
m∠GMJJT = 4.025°
Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠GMJMH = 36.228°
m∠MKJMH = 12.076°
m∠GMJMH / m∠MKJMH = 3.000

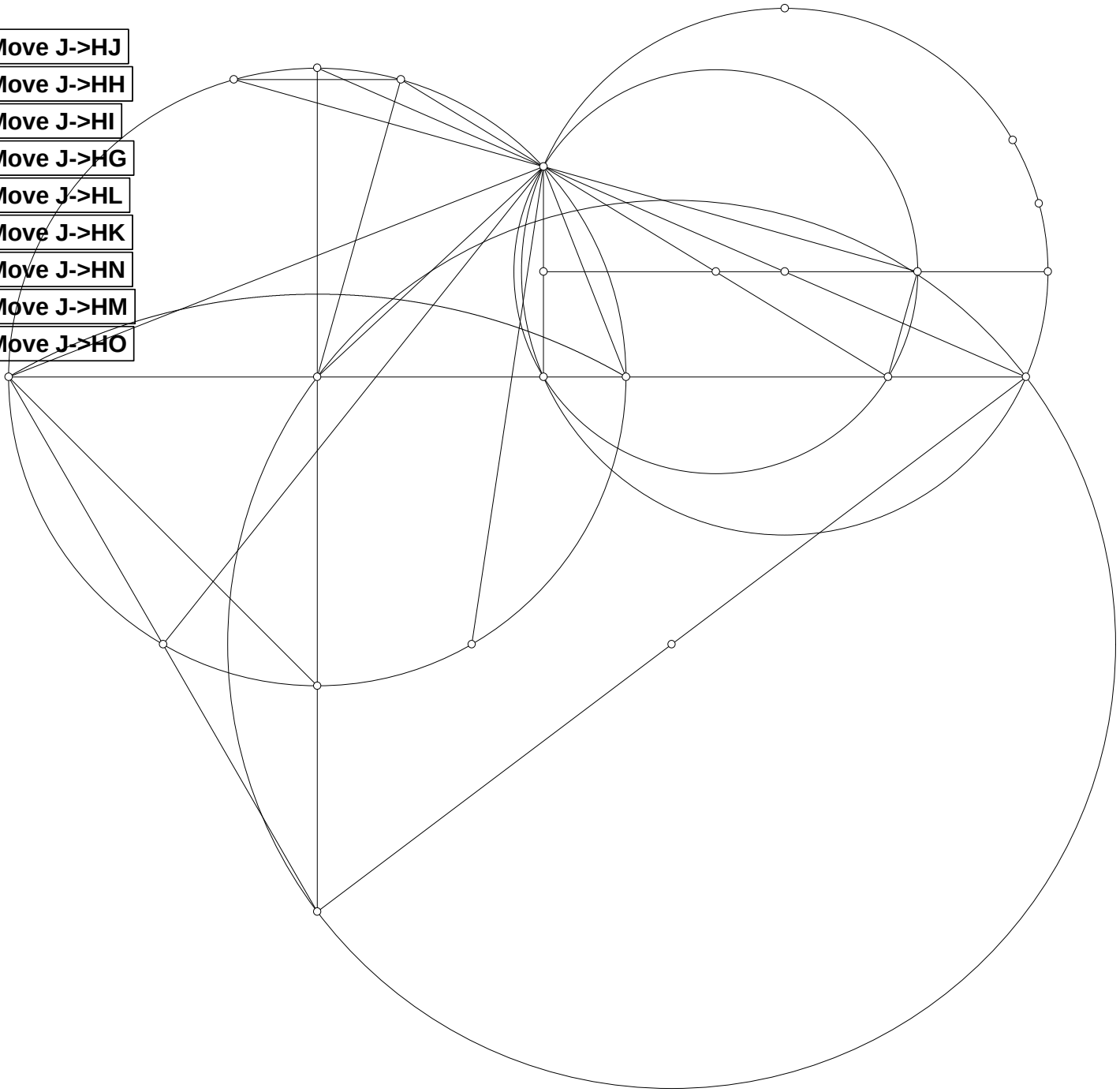
m∠JAC = 47.080°

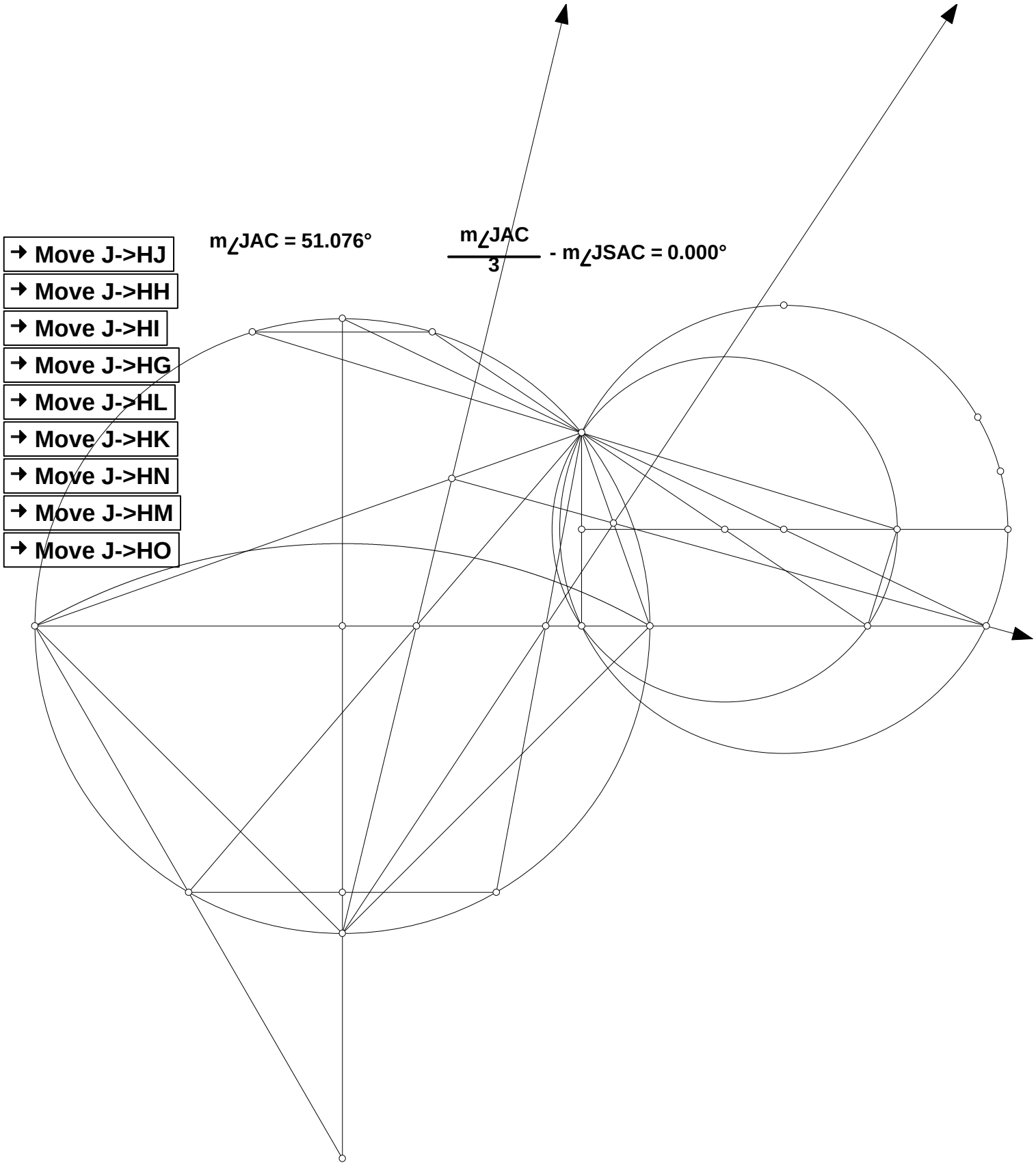
m∠JSAC = 15.693°

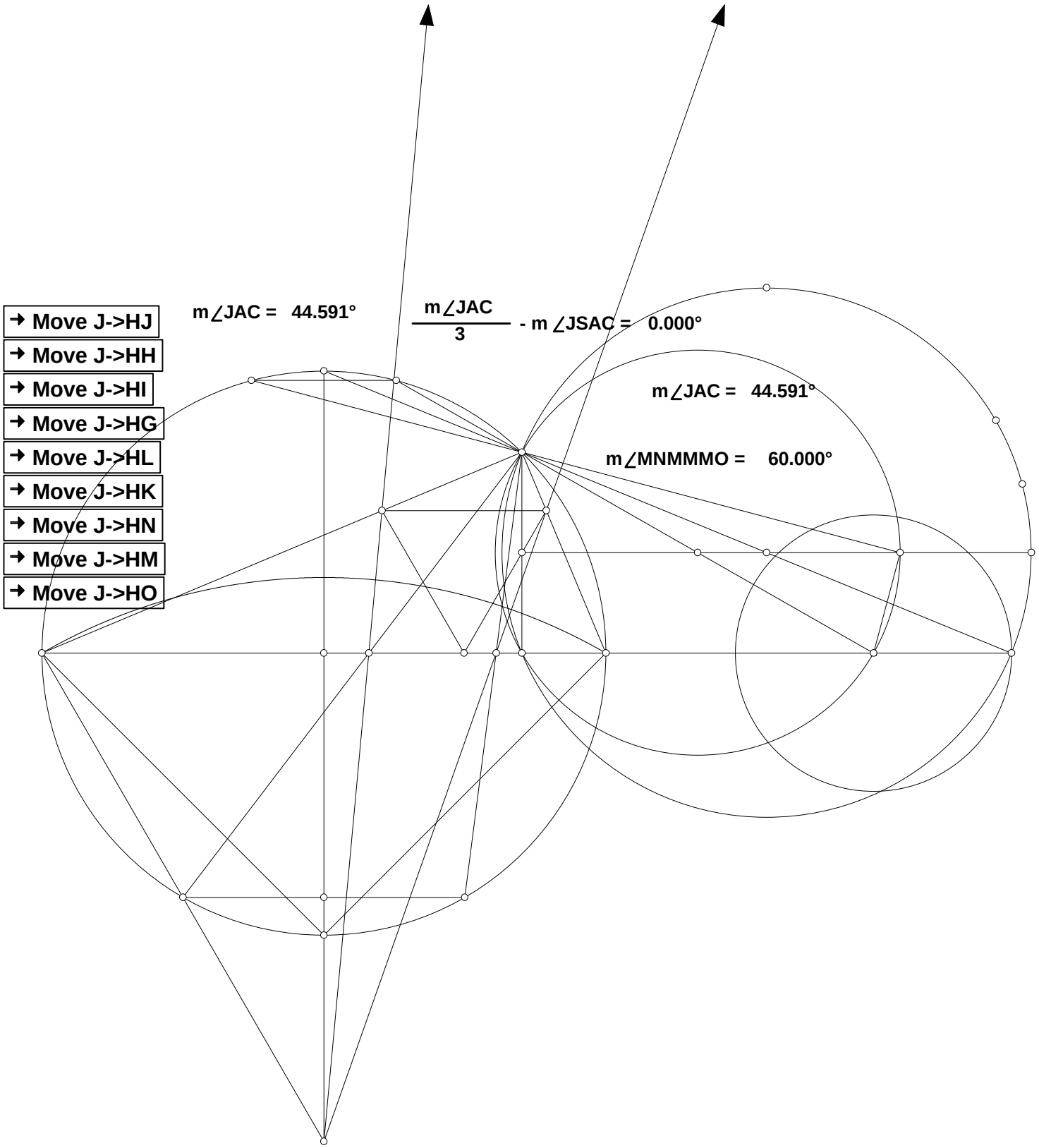
$$\frac{m\angle JAC}{m\angle JSAC} = 3.000$$

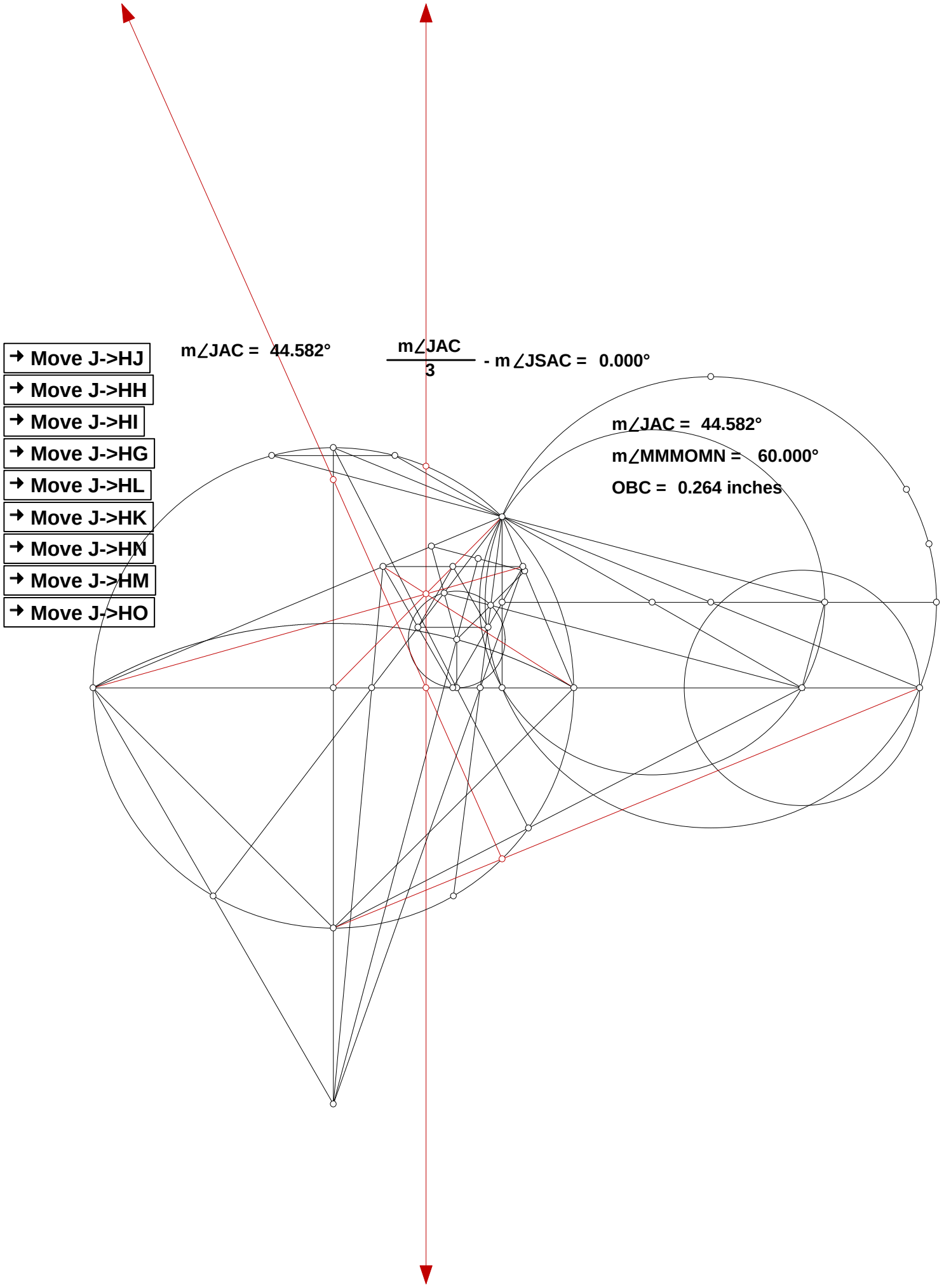
m∠JAC = 47.080°

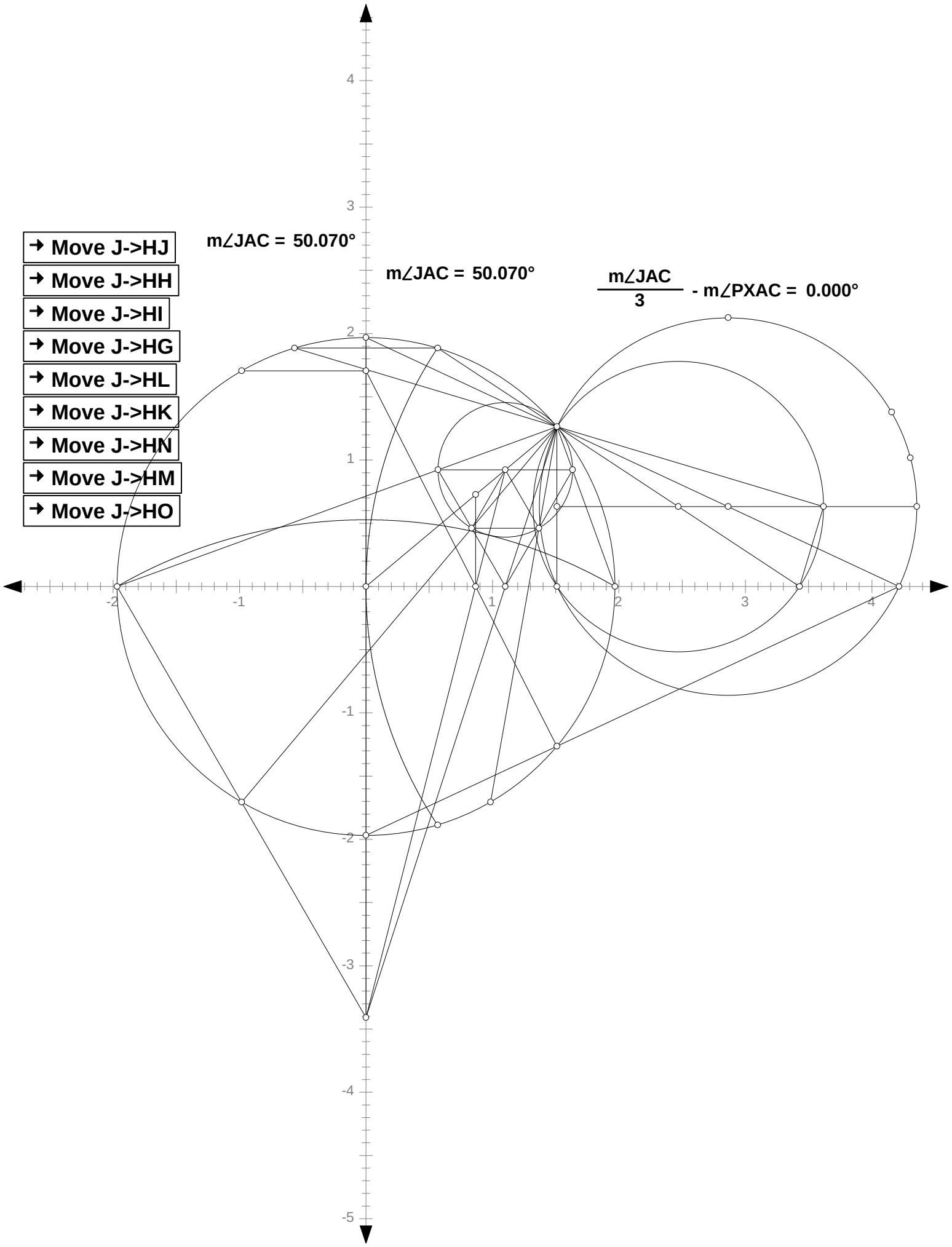
- Move J->HJ
- Move J->HH
- Move J->HI
- Move J->HG
- Move J->HL
- Move J->HK
- Move J->HN
- Move J->HM
- Move J->HO

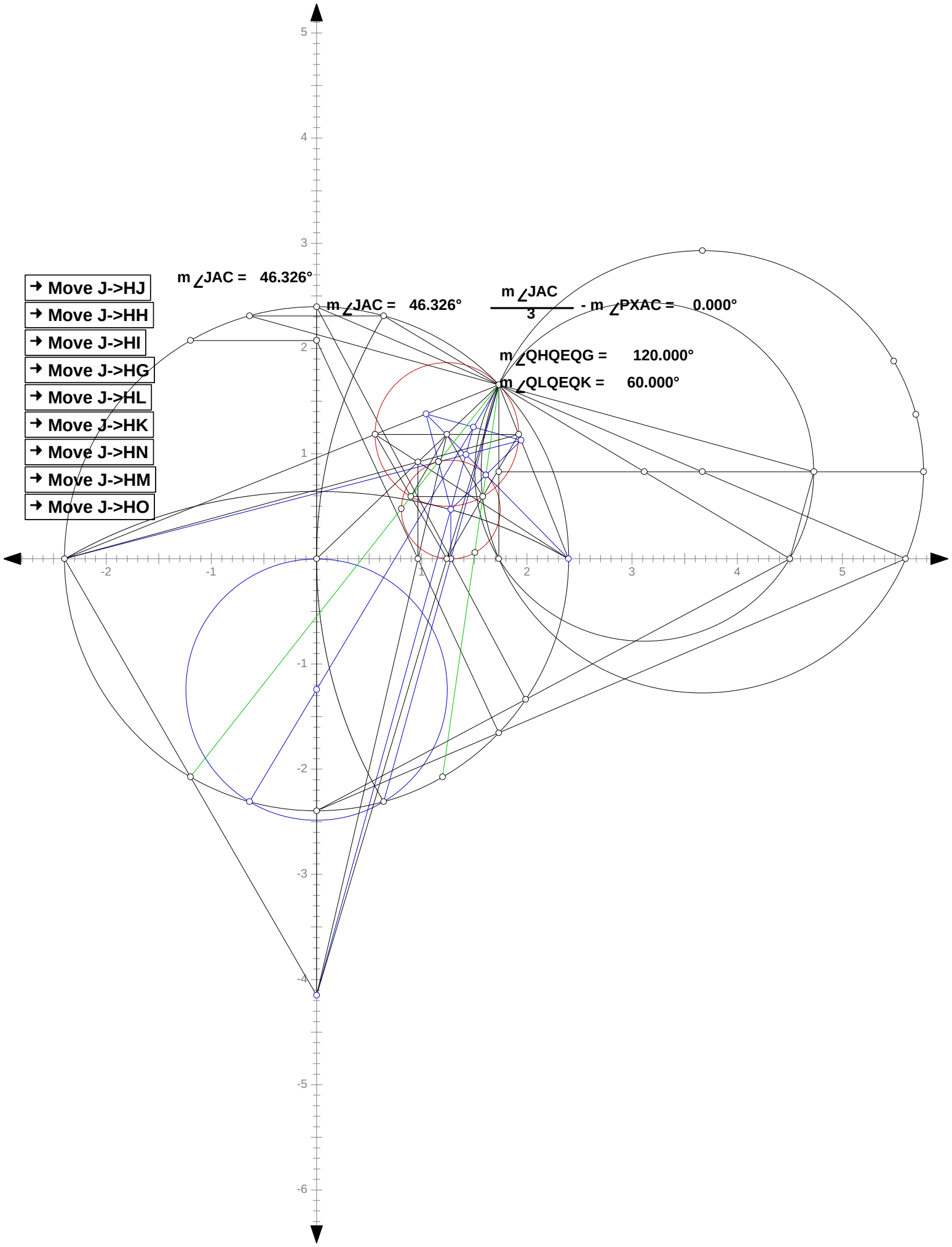


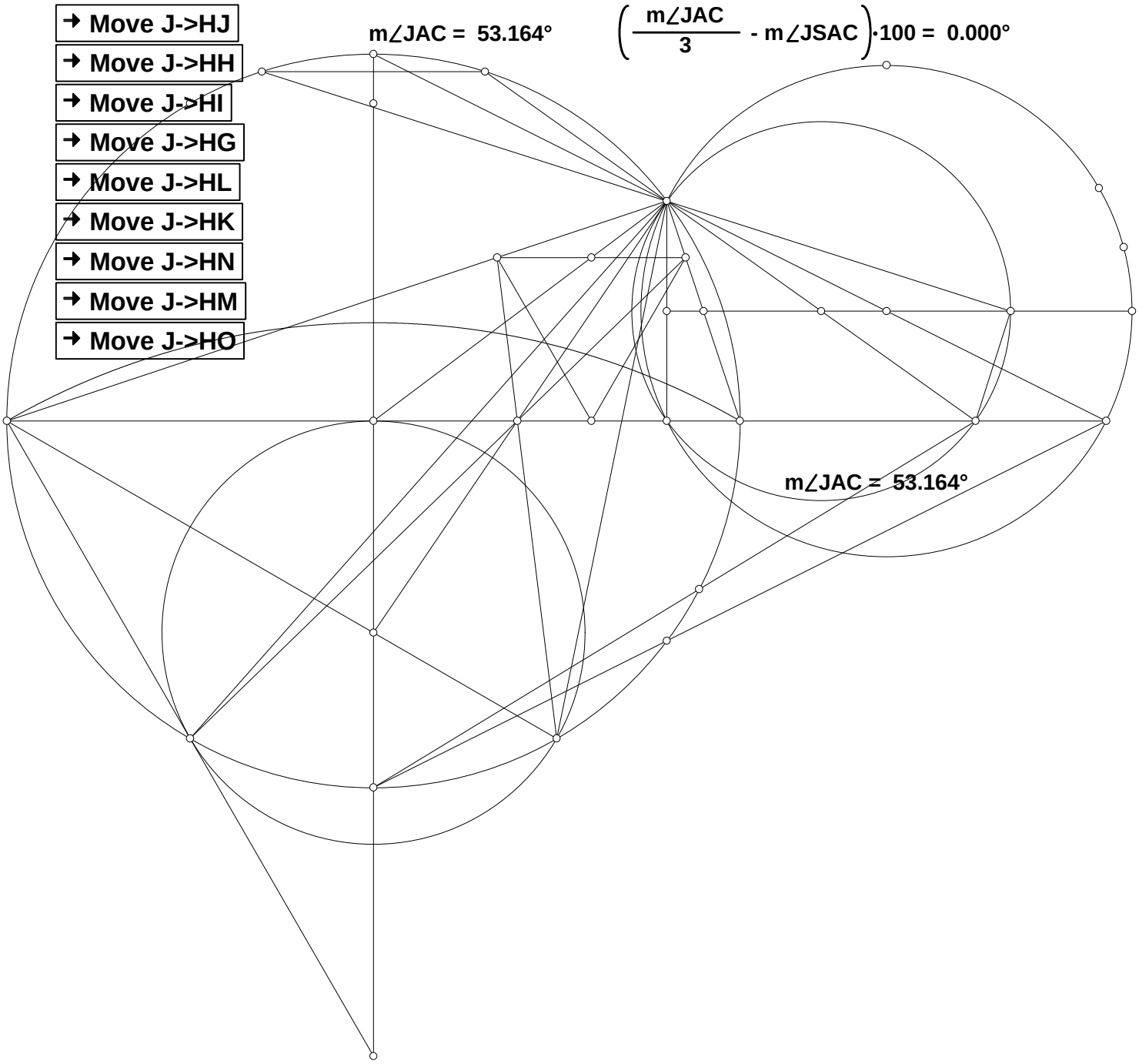

Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 51.076°
m∠JAC / 3 - m∠JSAC = 0.000°

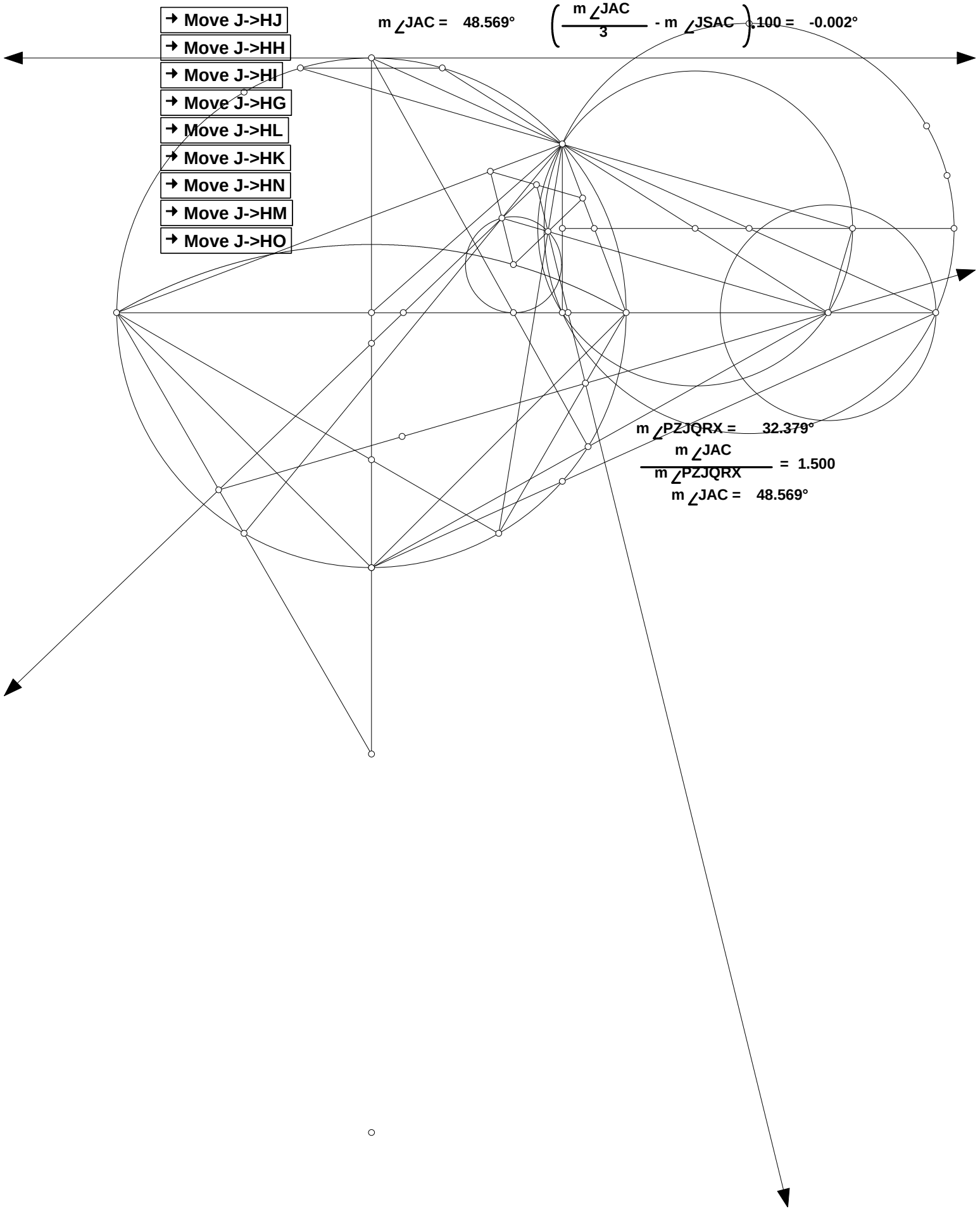

➜ Move J->HJ
➜ Move J->HH
➜ Move J->HI
➜ Move J->HG
➜ Move J->HL
➜ Move J->HK
➜ Move J->HN
➜ Move J->HM
➜ Move J->HO
m∠JAC = 44.591°
m∠JAC / 3 - m∠JSAC = 0.000°
m∠JAC = 44.591°
m∠MNMMMO = 60.000°

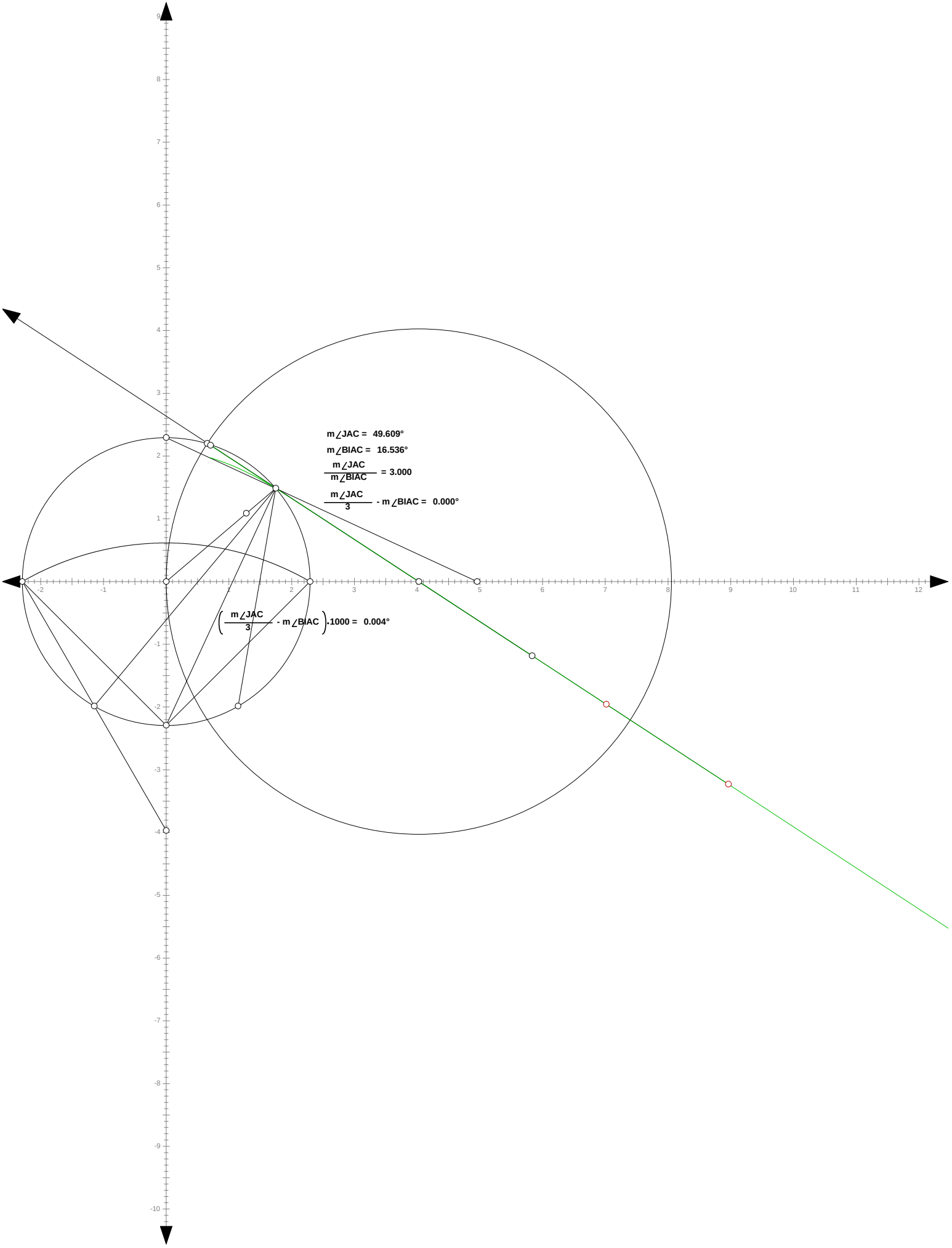

Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 44.582°
m∠JAC / 3 - m∠JSAC = 0.000°
m∠JAC = 44.582°
m∠MMMOMN = 60.000°
OBC = 0.264 inches


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 50.070°
m∠JAC = 50.070°
m∠JAC / 3 - m∠PXAC = 0.000°


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 46.326°
m∠JAC = 46.326°
m∠JAC / 3 - m∠PXAC = 0.000°
m∠QHQEQG = 120.000°
m∠QLQEQK = 60.000°


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 53.164°
$\left(\frac{m\angle JAC}{3} - m\angle JSAC\right)\cdot 100 = 0.000°$
m∠JAC = 53.164°


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 48.569°
(m∠JAC / 3 - m∠JSAC) ∘100 = -0.002°
m∠PZJQRX = 32.379°
m∠JAC / m∠PZJQRX = 1.500
m∠JAC = 48.569°


m∠JAC = 49.609°
m∠BIAC = 16.536°
m∠JAC / m∠BIAC = 3.000
m∠JAC / 3 - m∠BIAC = 0.000°
(m∠JAC / 3 - m∠BIAC) • 1000 = 0.004°